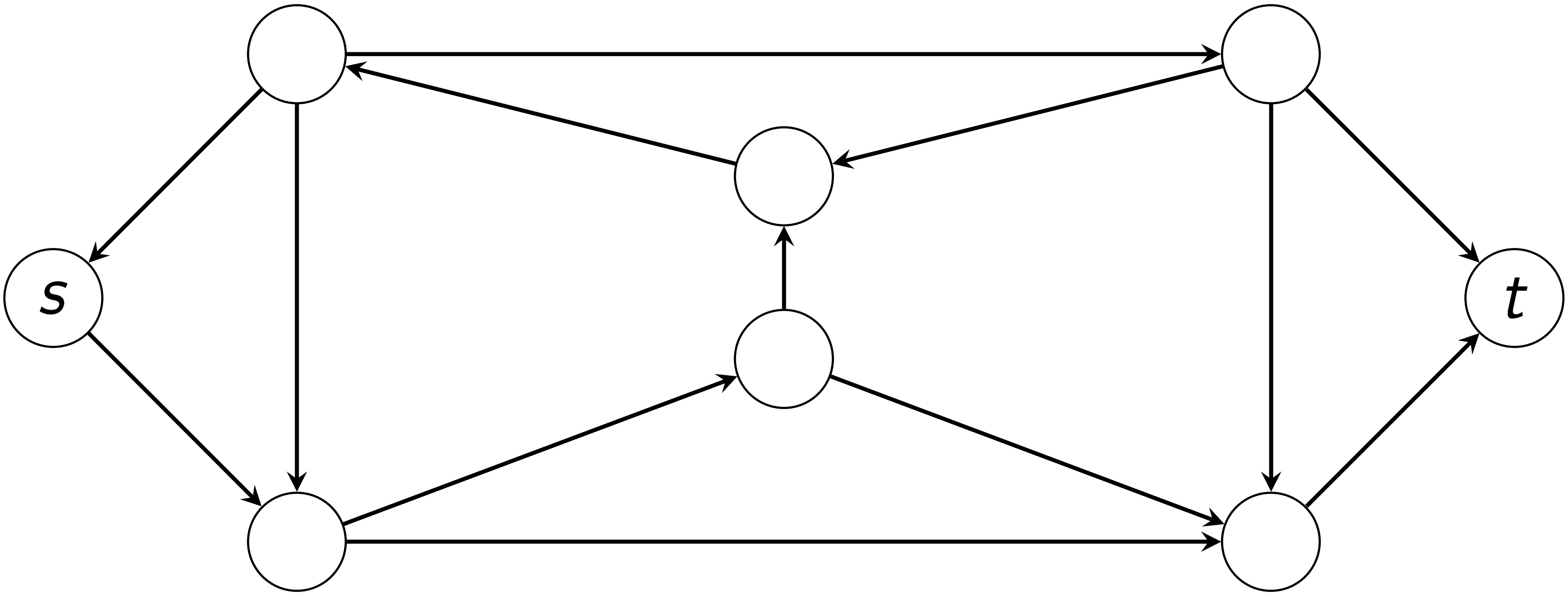
What we have seen is $\mathcal{NP}$-complete

- Boolean Satisfiability
- 3-SAT
- Independent Set
- Vertex Cover
- Set Cover
- 3-Color

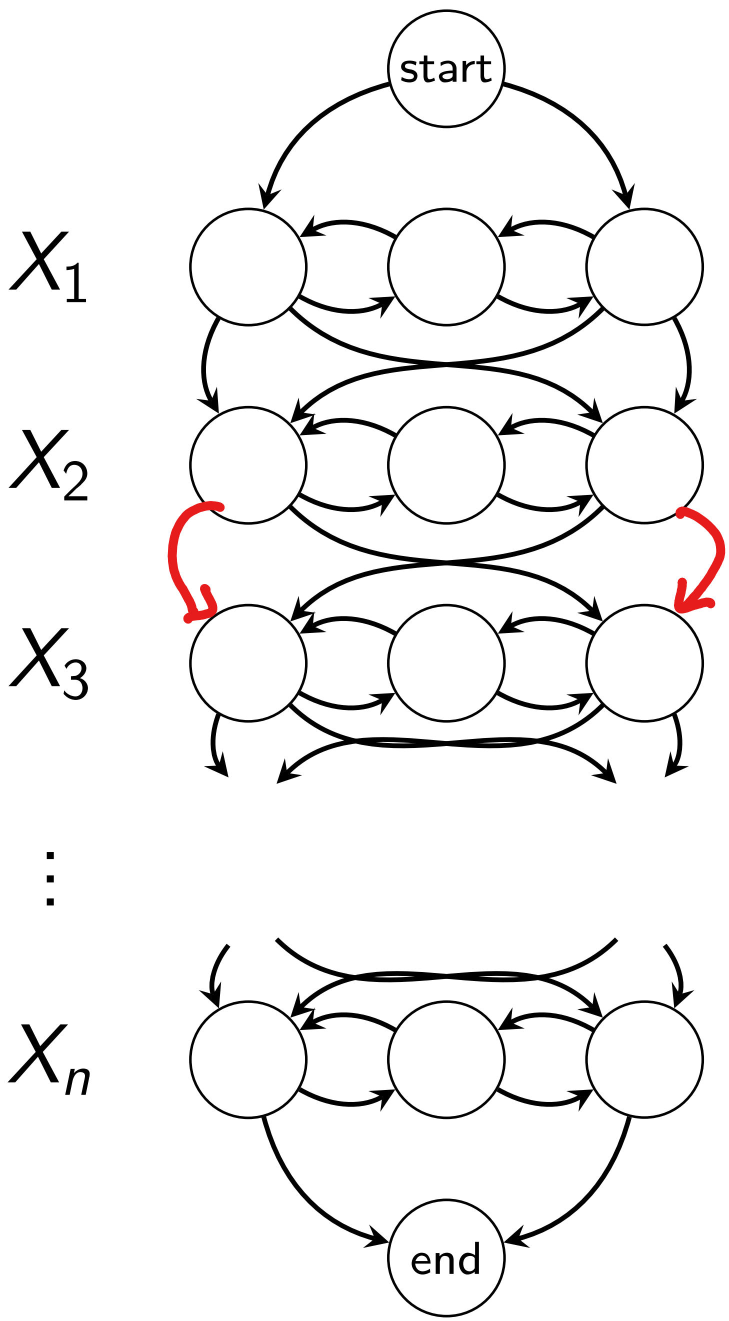
Hamiltonian Path Problem



G has a path that visits each vertex exactly once?

(Start/End may or may not be specified)

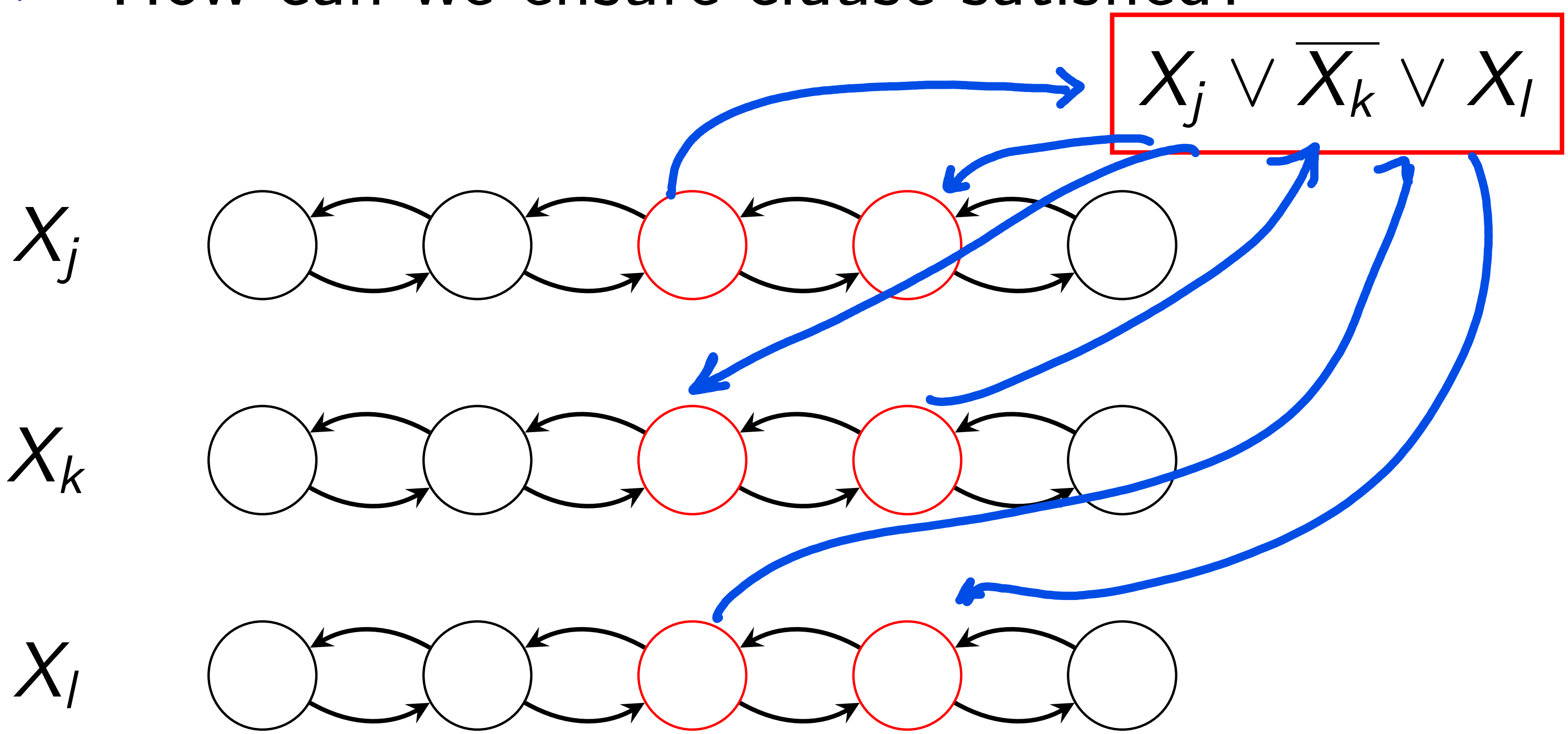
Creating a truth value assignment



Creating a satisfying assignment

For each clause:

- At least one must evaluate to true.
- Add two nodes to "right end" of row for ea var
- Add a "clause node"
- How can we ensure clause satisfied?



The reduction

We want to solve 3-Sat using Hamiltonian Path

- Step 1: Decide a Truth Value Assignment
- Step 2: Require that each clause is satisfied
- Given a Hamiltonian Path in a graph so created, what is the satisfying assignment?
- Given the graph, formed from ϕ, and a valid Truth Value Assignment for ϕ, find a Hamiltonian Path in the graph

Categorizing

Type	Examples
Packing	Independent Set
Covering	Vertex Cover Set Cover
Partitioning	3-COLOR
Sequencing/Permutation	Hamiltonian Path

CompSci 162
Spring 2023 Lecture 23:
Complexity IV: Subset Sum

Definition and Start

Problem Statement: given a set S of numbers and a target T, does a subset of S add up to T?

Example: $\langle \{4, 11, 16, 21, 27\}, 25 \rangle \in$ SUBSET-SUM

Prove that Subset Sum is $\mathcal{NP}$-complete.

Start: it is in $\mathcal{NP}$

Start of Reduction

Use SUBSET SUM to get a TVA on n variables

	x_1	x_2	x_3	x_4	x_5
v_1	1	0	0	0	0
v_1'	1	0	0	0	0
v_2	0	1	0	0	0
v_2'	0	1	0	0	0
v_3	0	0	1	0	0
v_3'	0	0	1	0	0
v_4	0	0	0	1	0
v_4'	0	0	0	1	0
v_5	0	0	0	0	1
v_5'	0	0	0	0	1
T	1	1	1	1	1

→ X_1 = True if v_1 chosen

→ X_1 = False if v_1' chosen

We want a satisfying assignment

$$\phi = (x_2 \vee x_3 \vee x_4)(\overline{x_2} \vee x_3 \vee \overline{x_4})(\overline{x_1} \vee x_3 \vee x_5)\ldots$$

Suppose instead of $\vee$ we wanted exactly one

	x_1	$\ldots$	x_5	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
v_1				0							
v_1'				0							
v_2				1							
v_2'				0							
v_3				1							
v_3'				0							
v_4				1							
v_4'				0							
v_5				0							
v_5'				0							
T	1	1	1	1							