What we have seen is $\mathcal{NP}$-complete

- Boolean Satisfiability
- 3-SAT
- Independent Set
- Vertex Cover

Test will include a list and definitions

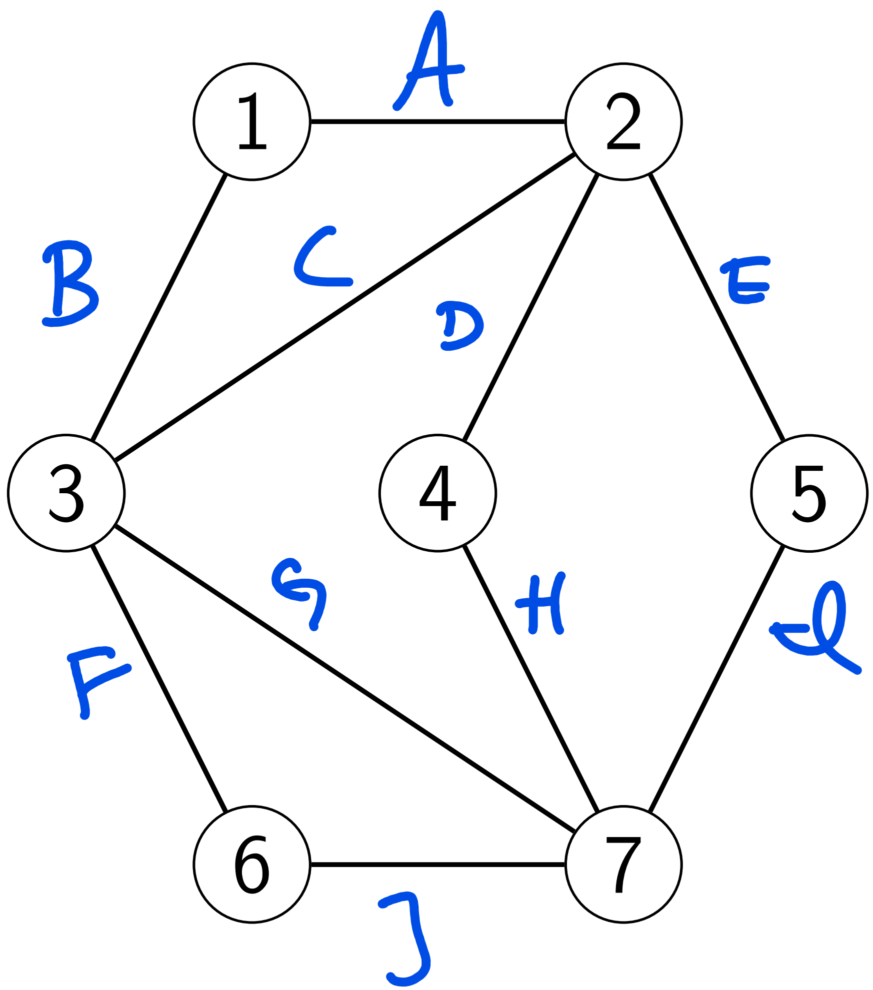
Set Cover

Set Number	Elements
1	A B
2	A C D E
3	B C F I
4	D G
5	E H
6	I J
7	F G H J

Prove Set Cover is in NP

(left as to-do at home)

Idea For Reduction



Vertex-Cover(G,K)
{
Create |V| empty sets
Let E be elements
for each edge e=(u,v)
add e to sets
u,v.
return SetCover(sets,K)
}

CompSci 162
Spring 2023 Lecture 22:
Graph Coloring is $\mathcal{NP}$-complete

3-Color is in $\mathcal{NP}$

DTM :

Certificate: mapping $V \to \{blue, gold, pink\}$

Verifier:

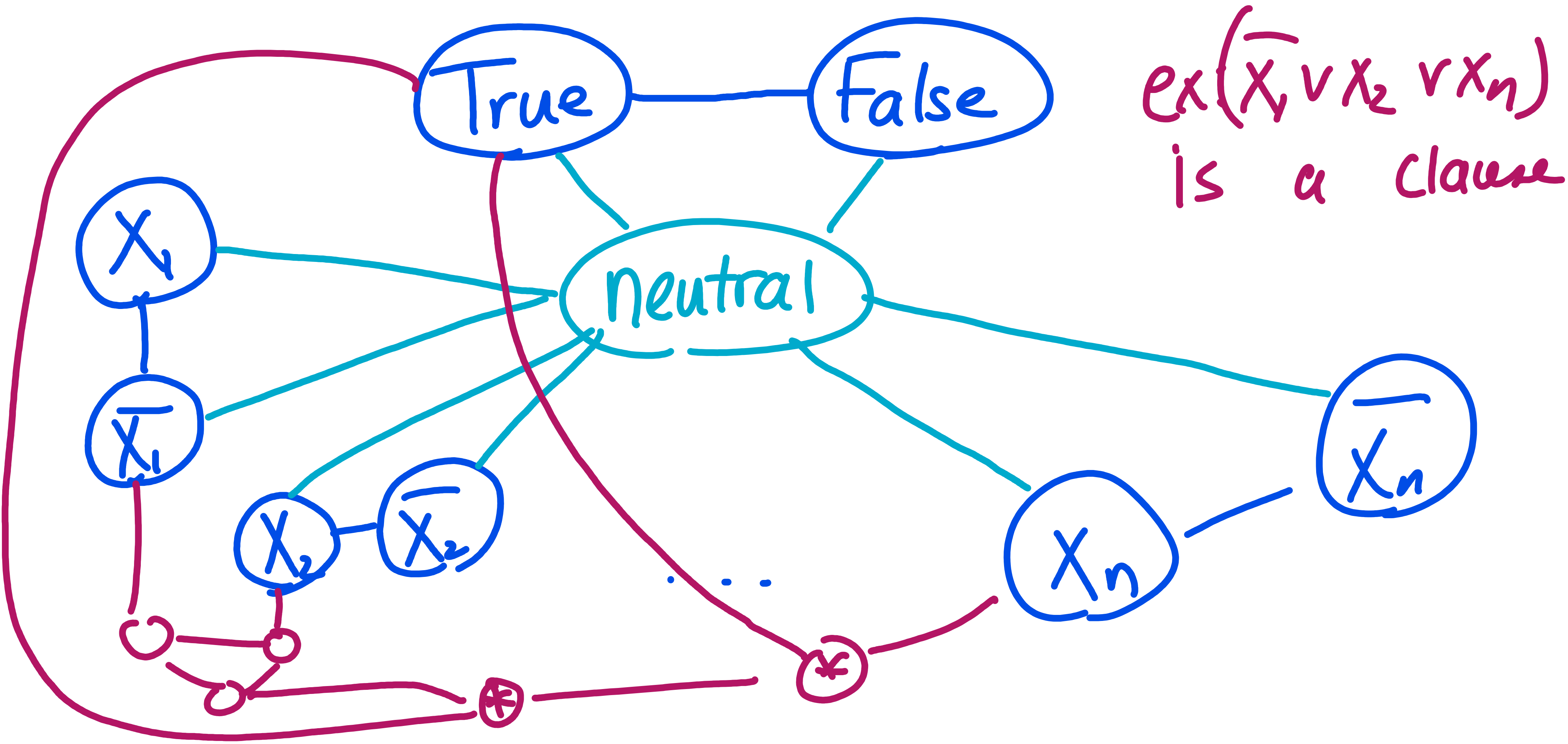
1. ensure every vertex mapped validly

2. for each $e = (u,v)$
   if color(u) = color(v)
   reject

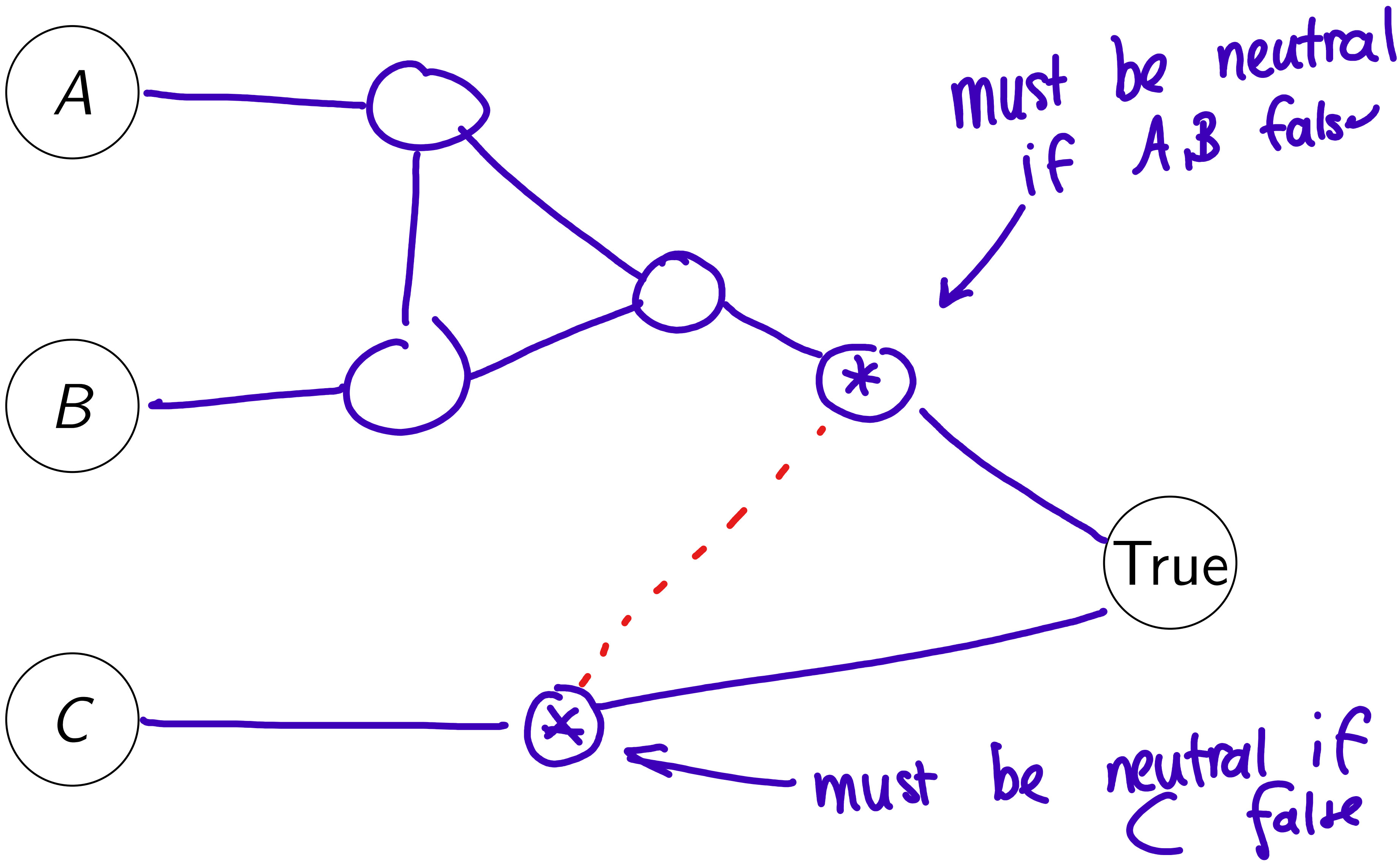
3 accept

Use 3-COLOR to get a truth value assignment on n variables

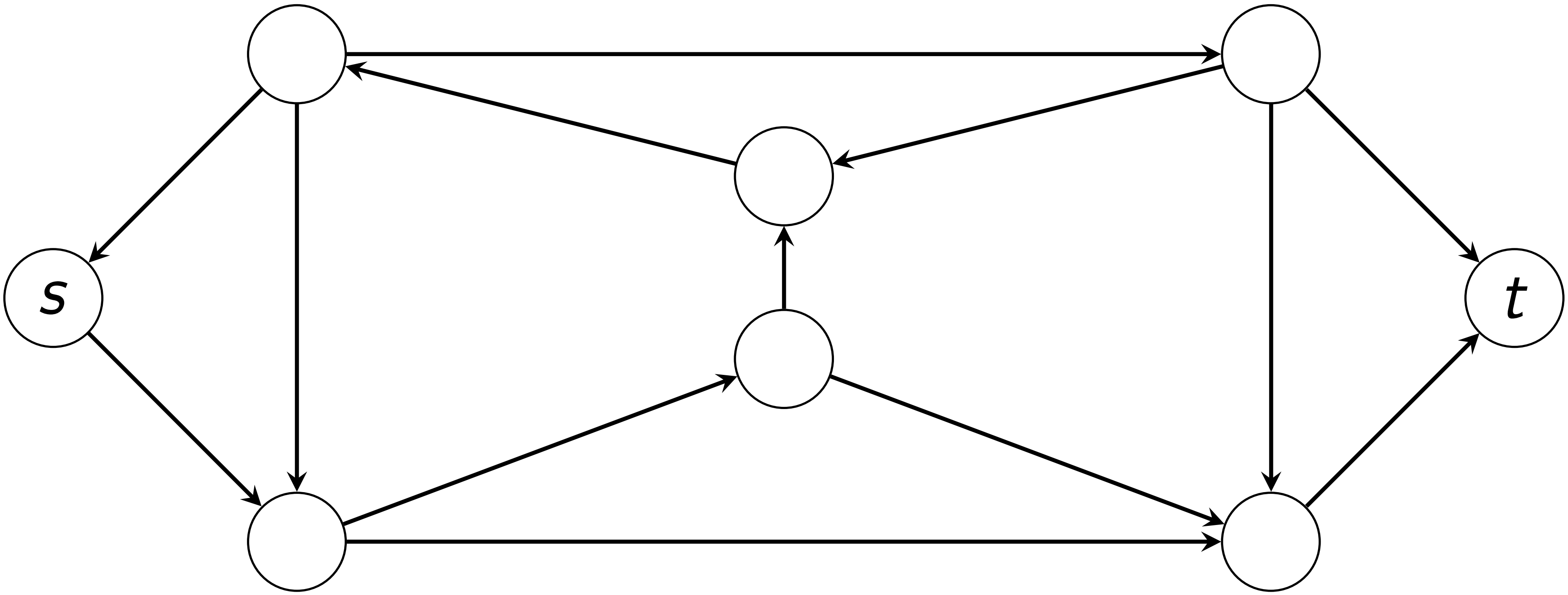
- Remember, all 2^n TVAs should be possible.
- Running time polynomial plus call to 3-COLOR



Amend the 3-COLOR usage to get a satisfying truth value assignment on n variables.
For each clause $A \vee B \vee C$, add as follows:



Hamiltonian Path Problem



G has a path that visits each vertex exactly once?

(Start/End may or may not be specified)

Creating a truth value assignment

- Design a graph so that any Hamiltonian Path in it will correspond to a truth value assignment on n variables. Do not (yet) worry about *satisfying* assignments.
- How many Hamiltonian Paths are in your graph?
- What does any given Hamiltonian Path mean as a truth-value assignment?

Creating a truth value assignment