

CompSci 162
Spring 2023 Lecture 1and 2
Sets and Functions

Let's see some functions

Given sets A and B ,

► What does $|A| = |B|$ mean?

- Cardinality same

- equinumerable

- $\exists$ bijective $f: A \leftrightarrow B$

$f(\text{guacamole}) = \text{Colossus of Rhodes}$

$|A| \leq |B|$? injective, possibly bijective

$|A| < |B|$? injective not bijective

How many naturals?

in $\mathbb{N}$, $0 \in \mathbb{N}$

► $\mathbb{N} = \{0, 1, 2, \dots\}$

► $|\mathbb{N}| = \aleph_0$ "countably infinite"

► How many odd numbers are there?

$$f(x) = 2x+1$$

$$x \in \mathbb{N}$$

odd

~~injective~~

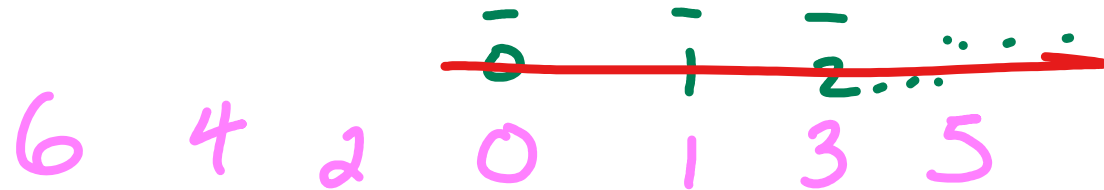
bijjective

injective: $f(i) = f(j) \Rightarrow i = j?$

surjective: $\frac{x-1}{2} \in \mathbb{N}$

How about $\mathbb{Z}$?

$$\mathbb{Z} = \{ \dots -2, -1, 0, 1, 2 \dots \}$$



$$\left. \begin{array}{l} f(2k) = -k \\ f(2k+1) = k+1 \end{array} \right\} \text{one function in two pieces}$$

$$\exists \text{ bijective "one piece" function } f(n) = \frac{1 + (-1)^n(2n+1)}{4}$$

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How about $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$?

$$P_i = \{ (x, y, z) : x \in \mathbb{N}, y \in \mathbb{N}, z \in \mathbb{N} \text{ and } x+y+z=i \}$$

Sort order: by i , then within P_i
 Sort within P_i by x (y breaks ties)

$$i = x + y + z$$

Within P_i , where is (x, y, z) in order?

$$= (\# x' < x) + (\# x' = x \text{ and } y' < y)$$

$$\sum_{j=0}^{x-1} (i+1-j) + y \rightsquigarrow \frac{x(2i-x+3)}{2} + y$$

How about $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$? (continued)

We saw that (x, y, z) is the $\frac{x(2i-x+3)}{2} + y$ th element in P_i , where $i = x + y + z$.

But how many total elements are in $P_0, P_1, \dots, P_{i-1}$?
 $= \# \text{ ways } x+y+z \leq i-1$

We need to add the above value to that count

$i-1$ pennies

4 jars

$C = \text{place penny} : i-1$

$N = \text{next jar} : 3$

$$\binom{i-1+3}{3} = \binom{i-1+3}{i-1}$$

$$= \binom{i+2}{3} = \frac{(i+2)(i+1)i}{6}$$

$$f(x, y, z) = \frac{x(2i-x+3)}{2} + y + \frac{(i+2)(i+1)i}{6}$$

How about $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$?

Take two, but this time, with Schröder-Bernstein Theorem

$$g: \mathbb{N} \rightarrow \mathbb{N} \times \mathbb{N} \times \mathbb{N} \quad \text{injective}$$

$$g(n) = (n, n, n)$$

$$f: \mathbb{N} \times \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N} \quad \text{injective}$$

$$f(x, y, z) = 2^x 3^y 5^z$$

$\mathbb{Q}$ is countable

$q \in \mathbb{Q}$ means $\exists k, m$ such that:

- ▶ $k, m \in \mathbb{Z}$
- ▶ $k \geq 0$
- ▶ k, m are relatively prime
- ▶ $q = k/m$

$$f : \mathbb{Q} \rightarrow \mathbb{N}$$

$$f(k, m) = \begin{cases} 2^k 3^m & m > 0 \\ 2^k 5^{-m} & m < 0 \end{cases}$$

$$g(n) = n/1$$