Lec 17

Undecidability

Review

- ▷ Church – Turing Thesis
- ▷ High Level Describe Turing Machines
  - ▷ and subroutines
- ▷ Turing – recog v Turing-decidable

$A_{TM} = \{ \langle M, w \rangle : M \text{ is T.M. and } M \text{ accepts } w \}$

Turing - recognizable

How many TMs [can] exist?

finite Σ

Σ^* countable

Set of languages?

$\Sigma^* = \{ S_0, S_1, \ldots \}$

Ea lang $\subseteq \Sigma^*$: binary encoded (bitmap)

How many inf large binary sequences? $\mathcal{B}$

$\mathcal{B} \longleftrightarrow 2^{N}$ ← strictly larger than N

i.e. each $b \in \mathcal{B}$ is a subset of natural #s

So un countably infinite.

A_{TM} is undecidable

aside: Quine program

FSOC $\exists$ decider H for A_{TM}

$H(\langle M, w \rangle) \rightarrow$ H accepts iff M accepts w

H rejects iff M rejects or loops forever

build D. Input: $\langle M \rangle$

1. Run $H(\langle M, M \rangle)$
2. If H accepts, loop forever

if H rejects, ~~else~~ then → accept

Conceptual

Where is D?

	$\langle M_1 \rangle$	$\langle M_2 \rangle$	$\langle M_3 \rangle$	$\langle M_4 \rangle$	$\langle M_5 \rangle$	$\ldots$	$\langle D \rangle$
M_1	reject	accept					
M_2	loop forever	accept					
M_3							
M_4							
M_5							
$\vdots$							
D							????

run D w/ input $\langle D \rangle$??

Contradiction: H cannot exist

Cannot exist...

How?

T- recog and co-Turing-recog

$\overline{A_{TM}}$