

CompSci 162  
Spring 2023 Lecture 14:  
Closures of Context-Free Languages

# CFLs: Closed Under Union

If  $L_1$  and  $L_2$  are CFLs, then so is  $L_3 = L_1 \cup L_2$

►  $G_1 = (V_1, \Sigma_1, R_1, S_1)$  for  $L_1$

$S_3 \rightarrow S_1 / S_2$

►  $G_2 = (V_2, \Sigma_2, R_2, S_2)$  for  $L_2$

$G_3 = (V_1 \cup V_2, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup$

$\cup \{S_3\})$

$\{S_3 \rightarrow S_1, S_3 \rightarrow S_2\},$   
 $S_3 \}$

# CFLs: Closed Under Concatenation

If  $L_1$  and  $L_2$  are CFLs, then so is  $L_4 = L_1 L_2$ .

Proof looks a lot like Union:

- ▶  $G_1 = (V_1, \Sigma_1, R_1, S_1)$
- ▶  $G_2 = (V_2, \Sigma_2, R_2, S_2)$
- ▶  $G_4 = (V_1 \cup V_2 \cup \{S_4\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{\}, S_4)$

$S_4 \rightarrow S_1 S_2$

# CFLs: Closed Under Kleene Star

If  $L_1$  is a CFL, then so is  $L_5 = L_1^*$ .

Proof looks a lot like Union:

- ▶  $G_1 = (V_1, \Sigma_1, R_1, S_1)$
- ▶  $G_5 = (V_1 \cup \{S_5\}, \Sigma_1 \cup \{\epsilon\}, R_1 \cup \{S_5 \rightarrow S_5 S_5, S_5, \epsilon\}, S_5)$

$$S_5 \rightarrow S_5 S_5 \mid S_5 \mid \epsilon$$

## Intersection of CFLs and Regular Languages

Sheep  $\rightarrow$  baaSheep  $\rightarrow$  baa Sheep

The intersection of a CFL and a Regular Language

► PDA  $M_1 = (K_1, \underline{\Sigma}, \Gamma_1, \Delta_1, s_1, F_1)$

► DFA  $M_2 = (K_2, \underline{\Sigma}, \delta, s_2, F_2)$

► Result PDA is  $M = (K, \underline{\Sigma}, \Gamma, \Delta, s, F)$  where:

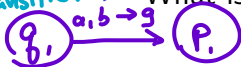
States ►  $K = K_1 \times K_2$  (all pairs)

Stack alph ►  $\Gamma = \Gamma_1$

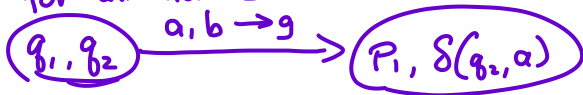
start ►  $s = (s_1, s_2)$

accepts ►  $F = F_1 \times F_2$

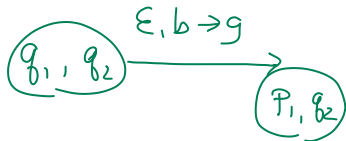
transition ► What is  $\Delta$ ?



for all non- $\epsilon$  tr. in  $M_1$ ,



for all  $\epsilon$ -tr. in  $M_1$ ,  
 $q_1 \xrightarrow{\epsilon, b \rightarrow g} p_1$  in  $M_1$



# Are CFLs closed under intersection?

If  $L_1$  and  $L_2$  are CFLs, is  $L_1 \cap L_2$  CFL? *No.*

Suppose FSOC that **yes**.

$$L_1 = \{ a^n b^n c^m : m, n \in \mathbb{N} \}$$

$$L_2 = \{ a^m b^n c^n : m, n \in \mathbb{N} \}$$

$$L_1 \cap L_2 = \{ a^n b^n c^n : n \in \mathbb{N} \},$$

but that is not CF  $\rightarrow \leftarrow$

# Are CFLs closed under complement?

If  $L_1$  is a CFL, is  $L_2 = \overline{L_1}$ ?

$$\frac{\overline{L_1} \text{ and } \overline{L_2}}{\overline{L_1} \cup \overline{L_2}} \quad \text{if yes} = L_1 \cap L_2 \quad \ddot{!}$$

So no.  $\rightarrow \leftarrow$