

CompSci 162
Spring 2023 Lecture 14:
Closures of Context-Free Languages

CFLs: Closed Under Union

If L_1 and L_2 are CFLs, then so is $L_3 = L_1 \cup L_2$

- $G_1 = (V_1, \Sigma_1, R_1, S_1)$ for L_1
- $G_2 = (V_2, \Sigma_2, R_2, S_2)$ for L_2

$$S_3 \rightarrow S_1 | S_2$$

$$G_3 = (V_1 \cup V_2, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2, S_3)$$

$$\boxed{S_3 \rightarrow S_1, S_3 \rightarrow S_2}$$

$$\{ S_3 \rightarrow S_1, S_3 \rightarrow S_2 \}, \\ S_3 \}$$

CFLs: Closed Under Concatenation

If L_1 and L_2 are CFLs, then so is $L_4 = L_1 L_2$.

Proof looks a lot like Union:

- ▶ $G_1 = (V_1, \Sigma_1, R_1, S_1)$
- ▶ $G_2 = (V_2, \Sigma_2, R_2, S_2)$
- ▶ $G_4 = (V_1 \cup V_2 \cup \{S_4\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{\}, S_4)$

$S_4 \rightarrow S_1 S_2$

CFLs: Closed Under Kleene Star

If L_1 is a CFL, then so is $L_5 = L_1^*$.

Proof looks a lot like Union:

- ▶ $G_1 = (V_1, \Sigma_1, R_1, S_1)$
- ▶ $G_5 = (V_1 \cup \{S_5\}, \Sigma_1, R_1, \{\}, S_5)$

$S_5 \rightarrow S_5 S_5 | S_1 | \epsilon$

Sheep \rightarrow baaSheep \rightarrow baa Sheep

The intersection of a CFL and a Regular Language

- ▶ PDA $M_1 = (K_1, \Sigma, \Gamma_1, \Delta_1, s_1, F_1)$
- ▶ DFA $M_2 = (K_2, \Sigma, \delta, s_2, F_2)$
- ▶ Result PDA is $M = (K, \Sigma, \Gamma, \Delta, s, F)$ where:

States

▶ $K = K_1 \times K_2$ (all pairs)

Stack alpha

▶ $\Gamma = \Gamma_1$

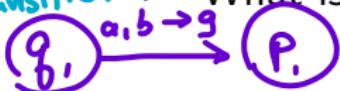
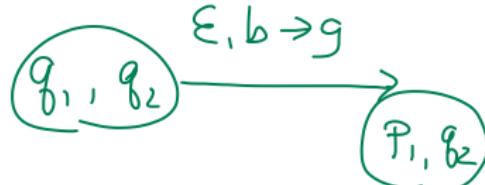
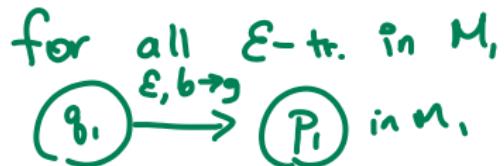
start

▶ $s = (s_1, s_2)$

accepts

▶ $F = F_1 \times F_2$

transition

▶ What is Δ ?for all non- Σ tr. in M_1 

6. Are CFLs closed under intersection?

If L_1 and L_2 are CFLs, is $L_7 = L_1 \cap L_2$ CFL? No.

Suppose FSOC that yes.

$$L_1 = \{a^n b^n c^m : m, n \in \mathbb{N}\}$$

$$L_2 = \{a^m b^n c^n : m, n \in \mathbb{N}\}$$

$$L_1 \cap L_2 = \{a^n b^n c^n : n \in \mathbb{N}\},$$

but that is not CF $\Rightarrow \Leftarrow$

7. Are CFLs closed under complement?

If L_1 is a CFL, is $L_8 = \overline{L_1}$?

$$\overline{L_1} \text{ and } \overline{L_2} \quad \text{if yes}$$
$$\overline{L_1 \cup L_2} = L_1 \cap L_2 \quad \text{if}$$

So no. $\rightarrow \leftarrow$