1

$L_2 = \{a^i b^j c^k \mid 0 \le i \le j \le k\}$

- FSOC suppose L_2 is context free
- Let p be the pumping length.
- Select $s = a^p b^p c^p$
  - $s \in L_2$ and also $|s| \ge p$
- Partition $s = uvxyz$
  - Do v, y have no as?
    - uxz has more a than b or c
  - Do v, y have no bs?
    - Are any as in v, y? uv^2xy^2z
    - Else uxz has lost cs not bs
  - Do v, y have no cs?

uv^2xy^2z has more a or b than c

if v, y single type of symbol

multiple symbols in v and/or y

uv^2xy^2z is out of order

$L_3 = \{ww \mid w \in \{a,b\}^*\}$

- FSOC suppose L_3 is context free
- Let p be the pumping length.
- Select $s = \underbrace{a^p b^p} a^p b^p$
  - $s \in L_3$ and also $|s| \geq p$
- Partition $s = uvxyz$

$|vxy| \leq p$

is vxy entirely contained within 1st half? uv^2xy^2z begins a, 2nd half begins b

entirely contained within 2nd half? uv^2xy^2z ends b, 1st half ends a

if vxy is "in middle"?

uxz is $a^p b^i a^j b^p$

$i \neq p$ or $j \neq p$ (or both)

CompSci 162
Spring 2023 Lecture 13:
Closures of Languages
Regular and Context Free

Regular Languages: Closed under Union

If L_1 and L_2 are regular, so is $L_3 = L_1 \cup L_2$

- Take NFAs N_1 and N_2
- Create a new start state
- ε transition this to N_1.start and N_2.start

We will build an NFA for N_3

Regular Languages: Closed under Concatenation

If L_1 and L_2 are regular, so is
$L_4 = \{w_1 w_2 \ : \ w_1 \in L_1, w_2 \in L_2\}$

- Take NFAs N_1 and N_2
- N_1's accept states stop being accept states
- N_1's accept states gain ε to N_2.start
- L_4.start $=$ N_1.start

Regular Languages: Closed under Kleene Star

If L_1 is regular, so is $L_5 = L_1^*$

- Take NFAs N_1
- Create a new start state
  - This is an accept state
- ε transition this to N_1.start
- For each $a \in N_1$.accept
  - Add ε to N_1.start

Regular Languages: Closed under Complement

If L_1 is regular, so is $L_6 = \overline{L_1}$

- L_1 has a DFA D_1
- Invert all accept/non-accept states of D_1

Regular Languages: Closed under Intersection

D_1 D_2: DFAs for L_1 L_2

If L_1 and L_2 are regular, so is $L_7 = L_1 \cap L_2$

Build DFA w/ $|D_1 \text{ states}| \times |D_2 \text{ states}|$ states

Each is $(s_1 \in D_1,\ s_2 \in D_2)$

for all $x \in \Sigma$

$$(a, b) \xrightarrow{x} (\delta_1(a, x),\ \delta_2(b, x))$$

accept in (p, q) iff p, q both accept

2nd proof: $L_7 = \overline{\overline{L_1} \cup \overline{L_2}}$