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$$L_2 = \{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$$

- ▶ FSOC suppose L_2 is context free
- ▶ Let p be the pumping length.
- ▶ Select $s = a^p b^p c^p$
 - ▶ $s \in L_2$ and also $|s| \geq p$

▶ Partition $s = uvxyz$

if v, y
single
type
of symbol

- ▶ Do v, y have no a s?
 - ▶ uxz has more a than b or c
- ▶ Do v, y have no b s?
 - ▶ Are any in v, y ? uv^2xy^2z
 - ▶ Else uxz has lost c s not b s
- ▶ Do v, y have no c s?

uv^2xy^2z has
more a or b than c

multiple symbols
in v and/or y

uv^2xy^2z is
out of order

$$L_3 = \{ ww \mid w \in \{a, b\}^* \}$$

► FSOC suppose L_3 is context free

► Let p be the pumping length.

► Select $s = \underline{a^p b^p} a^p b^p$

► $s \in L_3$ and also $|s| \geq p$

► Partition $s = uvxyz$

IS vxy entirely contained within 1st half?

entirely contained within 2nd half?

if vxy is "in middle"?

$$|vxy| \leq p$$

uv^2xy^2z begins a,
2nd half begins b

uv^2xy^2z ends b,
1st half ends a

uxz is $a^p b^i a^j b^p$
 $i \neq p$ or $j \neq p$ (or both)

CompSci 162
Spring 2023 Lecture 13:
Closures of Languages
Regular and Context Free

Regular Languages: Closed under Union

If L_1 and L_2 are regular, so is $L_3 = L_1 \cup L_2$

- ▶ Take NFAs N_1 and N_2
- ▶ Create a new start state
- ▶ ε transition this to N_1 .start and N_2 .start

← We will build an
NFA for N_3

Regular Languages: Closed under Concatenation

If L_1 and L_2 are regular, so is

$$L_4 = \{w_1 w_2 : w_1 \in L_1, w_2 \in L_2\}$$

- ▶ Take NFAs N_1 and N_2
- ▶ N_1 's accept states stop being accept states
- ▶ N_1 's accept states gain ε to N_2 .start
- ▶ L_4 .start = N_1 .start

If L_1 is regular, so is $L_5 = L_1^*$

- ▶ Take NFAs N_1
- ▶ Create a new start state
 - ▶ This is an accept state
- ▶ ε transition this to N_1 .start
- ▶ For each $a \in N_1$.accept
 - ▶ Add ε to N_1 .start

7 Regular Languages: Closed under Complement

If L_1 is regular, so is $L_6 = \overline{L_1}$

- ▶ L_1 has a DFA D_1
- ▶ Invert all accept/non-accept states of D_1

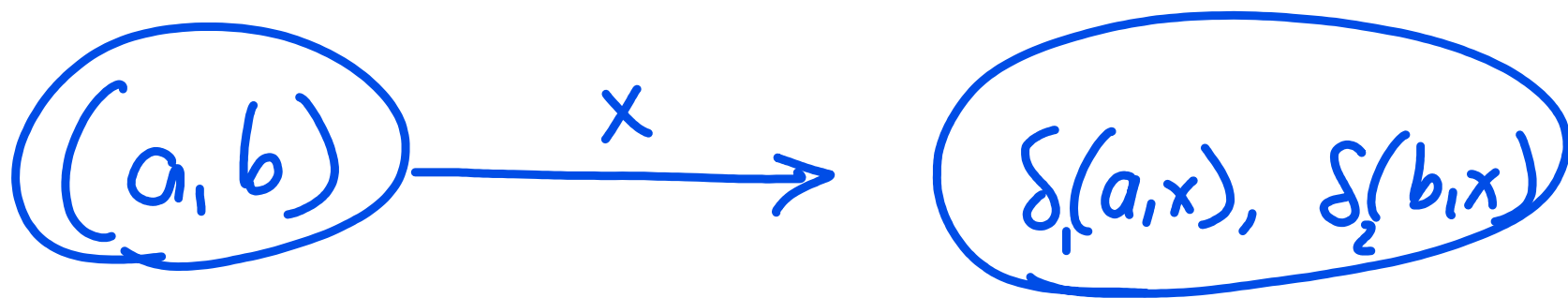
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Regular Languages: Closed under Intersection

If L_1 and L_2 are regular, so is $L_7 = L_1 \cap L_2$ D_1 D_2 : DFAs for L_1, L_2

Build DFA w/ $|D_1 \text{ states}| \times |D_2 \text{ states}|$ states

for all $x \in \Sigma$ Each is $(s_1 \in D_1, s_2 \in D_2)$



accept in (p, q) iff p, q both accept

2nd proof: $L_7 = \overline{L_1} \cup \overline{L_2}$