

$$L_2 = \{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$$

- ▶ FSOC suppose  $L_2$  is context free
- ▶ Let  $p$  be the pumping length.
- ▶ Select  $s = a^p b^p c^p$ 
  - ▶  $s \in L_2$  and also  $|s| \geq p$
- ▶ Partition  $s = uvxyz$

if v,y  
single  
type  
of symbol

- ▶ Do  $v, y$  have no  $a$ s?
- ▶  $uxz$  has more  $a$  than  $b$  or  $c$
- ▶ Do  $v, y$  have no  $b$ s?  
as
- ▶ Are any in  $v, y$ ?  $uv^2xy^2z$
- ▶ Else  $uxz$  has lost  $c$ s not  $b$ s
- ▶ Do  $v, y$  have no  $c$ s?

$UV^2XY^2Z$  has  
more  $a$  or  $b$  than  $c$

multiple symbols  
in  $v$  and/or  $y$

$UV^2XY^2Z$  is  
out of order

$$L_3 = \{ww \mid w \in \{a, b\}^*\}$$

- ▶ FSOC suppose  $L_3$  is context free
- ▶ Let  $p$  be the pumping length.

$$|vxy| \leq p$$

- ▶ Select  $s = \underline{a^p b^p}, a^p b^p$
- ▶  $s \in L_3$  and also  $|s| \geq p$

- ▶ Partition  $s = uvxyz$

IS  $vxy$  entirely <sup>contained</sup> <sub>1st within</sub> half?

$uv^2x^2y^2z^2$  begins a,  
2<sup>nd</sup> half begins b

entirely <sup>contained</sup> <sub>2nd within</sub> half?

$uv^2x^2y^2z^2$  ends b,  
1<sup>st</sup> half ends a

if  $vxy$  is "in middle"?

$uxz$  is  $a^p b^i a^j b^p$   
 $i \neq p$  or  $j \neq p$  (or both)

CompSci 162  
Spring 2023 Lecture 13:  
Closures of Languages  
Regular and Context Free

## Regular Languages: Closed under Union

If  $L_1$  and  $L_2$  are regular, so is  $L_3 = L_1 \cup L_2$

- ▶ Take NFAs  $N_1$  and  $N_2$
- ▶ Create a new start state
- ▶  $\epsilon$  transition this to  $N_1.start$  and  $N_2.start$

*We will build an  
NFA for  $N_3$*

## Regular Languages: Closed under Concatenation

If  $L_1$  and  $L_2$  are regular, so is

$$L_4 = \{w_1 w_2 : w_1 \in L_1, w_2 \in L_2\}$$

- ▶ Take NFAs  $N_1$  and  $N_2$
- ▶  $N_1$ 's accept states stop being accept states
- ▶  $N_1$ 's accept states gain  $\varepsilon$  to  $N_2.\text{start}$
- ▶  $L_4.\text{start} = N_1.\text{start}$

## Regular Languages: Closed under Kleene Star

If  $L_1$  is regular, so is  $L_5 = L_1^*$

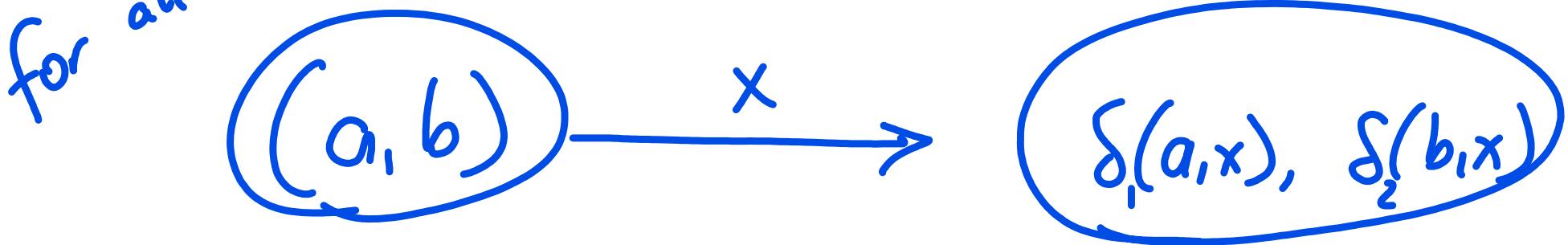
- ▶ Take NFAs  $N_1$
- ▶ Create a new start state
  - ▶ This is an accept state
- ▶  $\varepsilon$  transition this to  $N_1.start$
- ▶ For each  $a \in N_1.accept$ 
  - ▶ Add  $\varepsilon$  to  $N_1.start$

## Regular Languages: Closed under Complement

If  $L_1$  is regular, so is  $L_6 = \overline{L_1}$

- ▶  $L_1$  has a DFA  $D_1$
- ▶ Invert all accept/non-accept states of  $D_1$

## Regular Languages: Closed under Intersection

 $D_1$     $D_2$ : DFAsIf  $L_1$  and  $L_2$  are regular, so is  $L_7 = L_1 \cap L_2$  for  $L_1, L_2$ Build DFA w/  $|D_1 \text{ states}| \times |D_2 \text{ states}|$  statesfor all  $x \in \Sigma$  Each is  $(s_1 \in D_1, s_2 \in D_2)$ 

accept in  $(p, q)$  iff  $p, q$  both accept

2<sup>nd</sup> proof:  $L_7 = \overline{L_1} \cup \overline{L_2}$