

This is a few counting problems from when I taught discrete math that I hope can help you if you want to review/refresh that topic.

1. Suppose that a password for a computer system must have at least 8, but no more than 12, characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of six special characters.
 - (a) How many different passwords are available for this computer system?
 - (b) Using your answer to part (a), determine how long it takes a hacker to try every possible password, if it takes a nanosecond to check any given password.
(Students who find this question interesting may want to check what realistic assumptions are here.)
2. How many bit strings contain exactly eight 0s and 10 1s if every 0 must be immediately followed by a 1?
3. Explain why $\sum_{k=1}^n k \binom{n}{k} = n \cdot 2^{n-1}$ *Hint: Count in two ways the number of ways to select a committee and to then select a leader of the committee.*
4. A bagel shop has eight varieties of bagels. How many ways are there to choose
 - (a) six bagels?
 - (b) a dozen bagels?
 - (c) a dozen bagels with at least one of each kind?
5. How many solutions to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ are there where $x_i \geq 0$ and also (each of these parts is independent of the others)
 - (a) $x_1 \geq 1$
 - (b) $x_i \geq 2$ for $i = 1, 2, 3, 4, 5$?
6. How many solutions are there to the inequality $x_1 + x_2 + x_3 \leq 11$, where x_1, x_2 , and x_3 are all non-negative integers?
7. A “fun sized” (Halloween) bag of Skittles candy has 20 Skittles (pieces of candy) in it. Each piece of candy is one of five flavors (grape, lemon, orange, strawberry, or green).

Two bags of skittles are considered identical if the mix of pieces is the same in each. For example, a bag containing ten grape and ten lemon candies is the same as every other bag with ten grape and ten lemon, regardless of the order the grape or lemon candies were added to the bag.

 - (a) How many different fun-sized bags of Skittles are possible?
 - (b) How many different fun-sized bags of Skittles are possible if each must contain at least one of each flavor?
 - (c) How many different fun-sized bags of skittles are possible if (i) we no longer have the restriction that at least one of each flavor must be chosen and (ii) a bag contains *at most* 20 candies (and at least one piece of candy), rather than it must contain exactly 20?