

Sometimes you can know people for years and never really understand them. Take your friends Raj and Alanis, for example. Neither of them is a morning person, but now they're getting up at 6AM every day to visit local farmers' markets, gathering fresh fruits and vegetables for the new health-food restaurant they've opened.

In the course of trying to save money on ingredients, they've come across the following problem. There's a large set of  $n$  possible raw ingredients they could buy  $I_1, I_2, \dots, I_n$ . Ingredient  $I_j$  must be purchased in units of size  $s_j$  grams, costs  $c_j$  dollars per unit, and is safe to use for  $t_j$  days from the date of purchase.

Over the next  $k$  days, they want to make a set of  $k$  different daily specials, one each day. The order in which they schedule the specials is up to them. The  $i$ th daily special uses a subset  $S_i \subseteq \{I_1, I_2, \dots, I_n\}$  of the raw ingredients. Specifically, it requires  $a_{i,j}$  grams of ingredient  $I_j$ . Furthermore, the ingredients are partitioned into two subsets: those that must be purchased on the very day the special is offered, and those that can be used until they expire.

This is where the opportunity to save money on ingredients comes up. Often, when they buy a certain ingredient  $I_j$ , they don't need the whole thing for the special they're making that day. Thus, if they can follow up quickly with another special that uses  $I_j$ , but doesn't require it to be fresh that day, they can save money by not having to purchase  $I_j$  again. Of course, scheduling the basil recipes so close together makes it harder to schedule the goat cheese recipes close together, and so forth – that's where the complexity comes in.

So we define the DAILY SPECIAL SCHEDULING problem as follows: given data on ingredients and recipes as above, and a budget  $x$ , is there a way to schedule the  $k$  daily specials so that the total money spent on ingredients over the course of all  $k$  days is at most  $x$ ?

Prove that DAILY SPECIAL SCHEDULING is  $\mathcal{NP}$ -complete.