1. Convert the following CFG to a PDA

$$S \rightarrow SS|aSb|bSa|\epsilon$$

2. Prove, without using the Pumping Lemma, that the language

$$L = \{w \in \{a, b\}^*| \text{ the number of a's in } w \neq \text{ the number of b's in } w\}$$

is not regular. You may state and use, without proof, any result from previous practice quizzes, real quizzes, lectures, homework, or reinforcement exercises.

3. As seen earlier in the quarter, NFA/DFAs are unable to accept a certain class of languages. This accepted class of languages expands when we introduce a stack (as seen in the PDA). In your own words, explain why this additional unit (stack) allows us to accept more languages than before.

4. Show that the following languages are not context free by using the pumping lemma:
   (a) $L = \{a^i b^i a^i b^i | i \geq 0\}$
   (b) $L = \{a^i | i \text{ is prime}\}$
   (c) $L = \{a^i \# a^{2i} \# a^{3i} | i \geq 0\}$
   (d) $\{babaabaaab \dots ba^{n-1}ba^n b : n \geq 1$

5. Show that the following languages **are** context-free by using closure under union.
   (a) $a^m b^n : m \neq n$
   (b) $\{a, b\}^* - \{a^n b^n : n \geq 0\}$
   (c) $\{a^m b^n c^p d^q : n = q \text{ or } m \leq p \text{ or } m + n = p + q$
   (d) $\{a, b\}^* - L$, where L is the language $\{babaabaaab \dots ba^{n-1}ba^n b : n \geq 1$
   (e) $\{w \in \{a, b\}^* : w = w^R\}$

6. If $L_1, L_2 \subseteq \Sigma^*$ are languages, the **right quotient of** L_1 **by** L_2 is defined as follows.

$$L1/L2 = \{w \in \Sigma^* : \text{there is a } u \in L_2 \text{ such that } wu \in L_1\}$$

   (a) Show that if L_1 is context free and R is regular, then L_1/R is context-free
   (b) Prove that $\{a^p b^n : p \text{ is a prime number and } n > p\}$ is not context free.