1. In lecture, we were told that the language $L = \{a^i b^j c^k \mid i == j \text{ or } j == k\}$ is inherently ambiguous

   (a) Give a CFG for this language.
   Hint: two choices from the start: generate a string where $i == j$ and another choice where $j == k$, with the unbound variable can have any number of the relevant letter.

   (b) Give two different derivations of the string *aabbcc* in your grammar.

   (c) Give a PDA for this language. Note that this is *similar* to one from lecture, but is not the same language.

2. In lecture, for a PDA, we have to finish reading the input tape and end up in an accept state, but the stack does not need to be empty at that time. Suppose the rule instead were that the stack needed to be empty at the end. Show that these definitions are equivalent; that is, the set of languages acceptable in the first rule is the same as the set accepted in the latter.

3. As seen earlier in the quarter, NFA/DFAs are unable to accept a certain class of languages. This accepted class of languages expands when we introduce a stack (as seen in the PDA). In your own words, explain why this additional unit (stack) allows us to accept more languages than before.

4. Give context free grammars for the following languages. In all cases, $\Sigma = \{a, b\}$. Do not do this by creating a PDA and then converting it to a CFG. Separately, either give a PDA for each (not by converting the CFG to a PDA; write one from the beginning), either explicitly or by describing how you would build it.

   (a) $\{w|w$ contains at least three as$\}$

   (b) $\{w|w$ starts and ends with the same symbol$\}$

   (c) $\{w|w$ has odd length$\}$

   (d) $\{w|w$ has odd length and a middle symbol of $a\}$

   (e) $\{w|w$ is a palindrome$\}$

   (f) The empty set

   (g) $\{w|w$ has more as than bs$\}$

   (h) The complement of the language $\{a^n b^n | n \geq 0\}$

   (i) $\{w\#x|w^R$ is a substring of x $\}$

   (j) $\{x_1\#x_2\#\ldots\#x_k|$ each $x_i \in \{a, b\}^*$, and for some i and $j, x_i = x_j^R\}$