Decidable Problems Concerning Regular Languages

Question 1. Prove that $A_{DFA} = \{\langle B, w\rangle : B \text{ is a DFA that accepts input string } w\}$ is a decidable language.

Remember that $\langle B\rangle$ refers to the representation of B, which in this case is a DFA. Any reasonable representation of a DFA is fine.

This is decidable because a Turing Machine can simulate the DFA, accepting or rejecting no matter what the input.

Question 2. Prove that $A_{NFA} = \{\langle B, w\rangle : B \text{ is an NFA that accepts input string } w\}$ is a decidable language.

Question 3. Prove that $A_{REX} = \{\langle R, w\rangle : R \text{ is a regular expression that generates string } w\}$ is a decidable language.

Question 4. Prove that $E_{DFA} = \{\langle A\rangle : A \text{ is a DFA and } L(A) = \emptyset\}$ is a decidable language.

Question 5. Prove that $EQ_{DFA} = \{\langle A, B\rangle : A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$ is a decidable language.

Decidable Problems Concerning Context-Free Languages

Question 6. Prove that $E_{CFG} = \{\langle G \rangle : G \text{ is a CFG and } L(G) = \emptyset\}$ is a decidable language.

Question 7. Can we decide $EQ_{CFG} = \{\langle G, H \rangle : G, H \text{ are CFGs and } L(G) = L(H)\}$? If no, why does the strategy from Question 6 not work here?

Question 8. Are all context-free languages decidable?

Intro to Undecidability

Question 9. Is $A_{TM} = \{\langle M, w\rangle : M \text{ is a Turing Machine and } M \text{ accepts } w\}$ a Turing-recognizable language?

Question 10. How many Turing Machines exist? I don't mean how many were built in the physical world, but rather how many possible Turing Machines are there?

Question 11. How large is the set of all infinite binary sequences?

Question 12. How large is the set of all languages? For context of what I am asking, recall that the set of strings accepted by a given automaton (including Turing Machines) is a *language*.

Question 13. Is $A_{TM} = \{\langle M, w\rangle : M \text{ is a Turing Machine and } M \text{ accepts } w\}$ a Turing-**decidable** language?

We say a language is **co-Turing-recognizable** if it is the complement of a Turing-recognizable language.

Question 14. What can we say if a language is both Turing-recognizable and also co-Turing-recognizable? Prove that this is the case.

Question 15. We saw that $A_{TM} = \{\langle M, w\rangle : M \text{ is a Turing Machine and } M \text{ accepts } w\}$ a Turing-**recognizable** language. What about $\overline{A_{TM}}$?