

Unless otherwise stated, or otherwise clear from context, all languages are over  $\Sigma = \{a, b\}$ . Use the pumping lemma to prove that each of the following are non-regular. On the exam,  $\Sigma$  will be clear.

Recall the pumping lemma for regular languages:

If  $L$  is a regular language, then there is a number  $p$  (the pumping length) where if  $w$  is any string in  $L$  of length at least  $p$ , then  $w$  may be partitioned into three pieces,  $w = xyz$ , satisfying the following conditions:

- $|xy| \leq p$
- $|y| > 0$
- for each  $i \geq 0$ ,  $xy^iz \in L$

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1. Let  $L$  be the language  $\{ww \mid w \in \{a, b\}^*\}$
2. Let  $L$  be the language  $\{a^{n^2} \mid n \geq 0\}$  – that is, the set of strings whose length is a perfect square.
3. Let  $L$  be the language  $\{a^ib^k \mid i > k\}$
4.  $\{a^{10^n} \mid n \geq 0\}$
5.  $\{a^n b^n c^n \mid n \geq 0\}$
6. Let  $L$  be the set of odd-length strings in which the first, middle, and last symbols are the same.
7. **Challenge** : Let  $L$  be the set of odd-length strings where the middle symbol also appears elsewhere in the string.