We have continued to expand our Algorithmic Pizza Parlor stores, and now are going to try to gain new customers by advertising. We have a budget of B dollars to spend on advertising, and n possible ways to spend it (billboards, television spots, radio ads, etc). For each possible spending method i, we can allocate anywhere between 0 and B dollars, but must do so in even increments of one dollar (we cannot allocate partial dollars). Our marketing department has come up with a series of functions $\{f_i\}$ to determine how effective (in terms of customers gained) each advertising method will be. If we spend d dollars on advertising method i, we will gain $f_i(d)$ customers. These functions are all non-decreasing: if $d < d'$, then $f_i(d) \leq f_i(d')$. You may assume that $f_i(0) = 0$ for all i, if you would like.

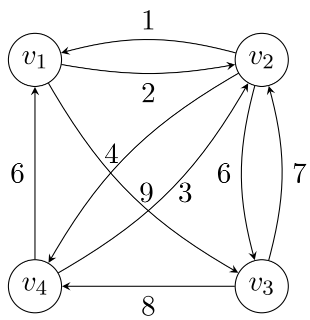
Given these functions, use **dynamic programming** to design an algorithm that will determine the maximum number of new customers we can gain within our budget. Provide the recursive solution (including the base case), explain *briefly* why it is correct, and state what the running time of the iterative algorithm would be.
You *do not* need to give the iterative algorithm, nor do you need to produce a listing of where to spend the money – simply have your algorithm find the best possible number of customers gained within the budget.

Hint: The correct solution is not to figure out where to put the last marginal dollar.

Traveling Salesperson Problem

Consider the TRAVELING SALESPERSON problem. We are given a simple (not necessarily complete) directed graph. Our goal is to find the Hamiltonian Cycle of lowest total weight.

Example:



Tour	Length
v_1, v_2, v_3, v_4, v_1	22
v_1, v_3, v_2, v_4, v_1	26
v_1, v_3, v_4, v_2, v_1	21

The last tour listed is the optimal one for this input.

A dynamic programming algorithm

In general, we could enumerate every possible tour in time $\mathcal{O}(n!)$, although this is probably a poor idea. Use dynamic programming to produce a better algorithm for this problem. You should not expect to get a polynomial running time for this problem.

Instead, let's use dynamic programming, *allowing* for super-polynomial running time.

Hint 1: Without loss of generality, every tour "begins" and "ends" at v_1. Every such cycle can be thought of as going from v_1 to some v_j, going through zero or more vertices along the way, and then returning to v_1 going through the remaining vertices.

Hint 2: Any subset of vertices can be represented with a bit vector of n bits. The ith bit corresponds to whether or not v_i is included. However, you should focus on the concept, and instead treat this as if your algorithm can take a parameter of a subset of vertices. The detail of how to represent the set is important when you implement the algorithm.