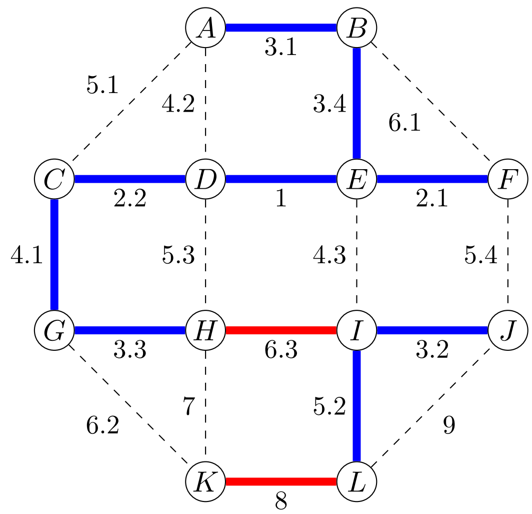
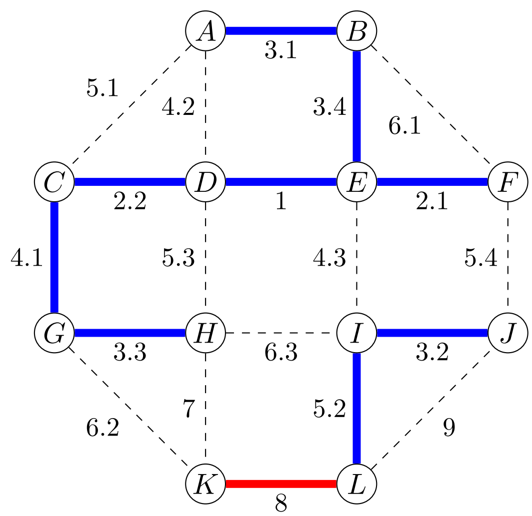
This is an attempt to illustrate part of the proof of the Cycle Property as a back-and-forth conversation, showing someone that what they thought was an MST is not, in fact, one, due to its inclusion of edges that are the heaviest in a cycle.

Suppose your friend is trying to find the MST of the graph we saw in lecture and finds the following *spanning tree*. Note that it is a spanning tree (it is connected and spans the vertex set), but it is not of minimum cost. I have highlighted in red two edges that are not in the optimal tree; of course, your friend probably didn't highlight their own wrong answers when they wrote this.



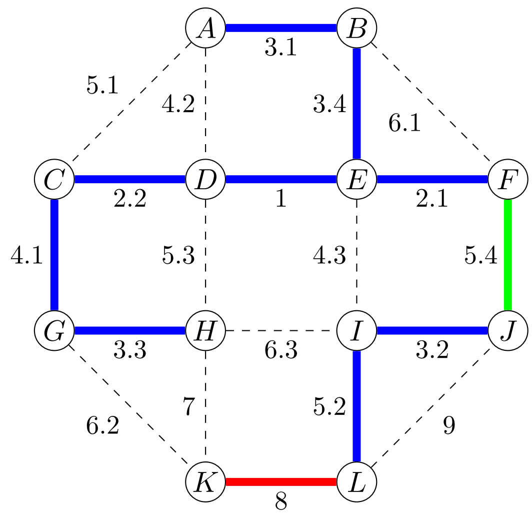
Oh no!, you say; edge HI can't be in the optimal spanning tree. We know this is because of the cycle property, but that would be a circular argument at this point. Instead, let's use the ideas behind it to convince our friend to drop the edge and reconsider.

First, drop the edge, at least tentatively. We can always put it back. We now have this forest of two trees:



Note that H and I are in different sub-trees now. There are many cycles of which HI is the heaviest edge; let's walk around one in particular: $HDEFJI$. I start at vertex H. Even though HD is not in the tree, I can still cross the edge: after all, I'm walking around the cycle in the original graph. But H and D are in the same sub-tree, so I haven't crossed to the other tree (yet – I have to at some point, because my destination of I is in the other tree, and every vertex is in one or the other, so *somewhere* I will). I walk to E, then F, remaining in the same subtree, before crossing edge FJ. Now I'm in the other sub-tree.

And note: if I add edge FJ to the collected edges, I have re-merged the sub-trees, forming a single spanning tree again. And this one is of lower cost than the one we had before:



Note that the cycle property is constructive: it gives us both a way to demonstrate that an edge *does not* belong in a MST and a way to improve any spanning tree that includes at least one such edge.

Note that FJ **is also not** in the optimal minimum spanning tree – but that doesn't matter. The point I wanted to show is that because HI is the heaviest in a cycle, it does not belong in the MST. When choosing a cycle to traverse earlier, I purposefully chose one that will cause the replacement edge to be one that is not in the optimal minimum spanning tree. And when someone claims that the first diagram's tree is a MST, we can show it isn't by showing a *lower-cost spanning tree* than it is, regardless of whether or not that one is optimal.