

1. Order the following functions from smallest to largest asymptotic complexity. Identify any pairs of functions that have the same complexity (i.e. are  $\Theta$  of each other).

(a)  $\sqrt{n^6 \cdot \log^{-2} n}$

(b)  $n^\pi$

(c)  $(n^3 + 10n^2) \log n$

(d)  $n \cdot \sum_{i=1}^n i$

(e)  $n!$

(f)  $10^n$

(g)  $2^{4n}$

(h)  $4^{2n}$

(i)  $1000^{1000000!}$

(j)  $2^{\log(n^2)}$

(k)  $\log^n n$

(l)  $2^{(2^n)}$

2. Prove by induction (for all non-negative integers  $n$ ):

$$\sum_{i=0}^n 2^i = 2^{i+1} - 1$$

3. Consider the sequence:  $a_1 = 2$ ,  $a_2 = 4$ ,  $a_n = 5a_{n-1} - 6a_{n-2} \quad \forall n \geq 3$ . Prove using strong induction that  $a_n = 2^n$  for all  $n \geq 1$ .
4. An undirected graph with  $n$  vertices is a tree iff it has these three properties:
- (i) connected
  - (ii)  $n - 1$  edges
  - (iii) no cycles

In fact, any two of these properties implies the third. Prove by induction over  $n$  that (i) and (ii) imply (iii) for all graphs with  $n \geq 1$ .