

1. Order the following functions from smallest to largest asymptotic complexity. Identify any pairs of functions that have the same complexity (i.e. are Θ of each other).

- (a) $\sqrt{n^6 \cdot \log^{-2} n}$
- (b) n^π
- (c) $(n^3 + 10n^2) \log n$
- (d) $n \cdot \sum_{i=1}^n i$
- (e) $n!$
- (f) 10^n
- (g) 2^{4n}
- (h) 4^{2n}
- (i) $1000^{1000000!}$
- (j) $2^{\log(n^2)}$
- (k) $\log^n n$
- (l) $2^{(2^n)}$

2. Prove by induction (for all non-negative integers n):

$$\sum_{i=0}^n 2^i = 2^{i+1} - 1$$

3. Consider the sequence: $a_1 = 2$, $a_2 = 4$, $a_n = 5a_{n-1} - 6a_{n-2}$ $\forall n \geq 3$. Prove using strong induction that $a_n = 2^n$ for all $n \geq 1$.

4. An undirected graph with n vertices is a tree iff it has these three properties:

- (i) connected
- (ii) $n - 1$ edges
- (iii) no cycles

In fact, any two of these properties implies the third. Prove by induction over n that (i) and (ii) imply (iii) for all graphs with $n \geq 1$.