1. In lecture, you saw the Path Property of MSTs, but we did not prove it. We will do so here. For any two vertices x and y, the path from x to y through the minimum spanning tree has the minimum possible weight for its heaviest edge. This is the **path property** of Minimum Spanning Trees.

2. Suppose we have a weighted graph G (undirected, connected, n vertices, m edges) with a unique minimum spanning tree (you may assume distinct edge costs too). Our goal is to compute the *second-best* spanning tree – that is, a spanning tree of G for which the only other spanning tree with a lower cost is the minimum spanning tree.

   (a) Let's say that two spanning trees differ by k edges if each one includes exactly k edges that the other doesn't. In other words, they have exactly $(n-1)-k$ edges in common. There must exist a second-best spanning tree that differs from the MST by only 1 edge. Try to explain (or even prove!) why this fact is true.

   (b) With that fact in mind, design an algorithm to find a second-best spanning tree in a graph.

© CompSci 161 Winter 2023– Phillip Nazarian– University of California, Irvine
This handout may not be reposted without the express written permission of the professor teaching this course.