

As a reminder, you are encouraged to leave your answers in the form of un-evaluated mathematical expressions, including summations and combinations (“ n choose k ”, $\binom{n}{k}$).

1. Simple Counting

- (a) You flip a coin until you get heads, but you stop after 4 flips even if you haven't gotten any heads. How many possible outcomes are there?
- (b) You roll a 6-sided die and record the result, then flip a coin that many times. How many possible outcomes are there?
- (c) How many 3-digit numbers are there where the first and second digits are distinct and the second and third digits are distinct?

2. A “fun sized” (Halloween) bag of Skittles candy has 20 Skittles (pieces of candy) in it. Each piece of candy is one of five flavors (grape, lemon, orange, strawberry, or green).

Two bags of skittles are considered identical if the mix of pieces is the same in each. For example, a bag containing ten grape and ten lemon candies is the same as every other bag with ten grape and ten lemon, regardless of the order the grape or lemon candies were added to the bag.

- (a) How many different fun-sized bags of Skittles are possible?
- (b) How many different fun-sized bags of Skittles are possible if each must contain at least one of each flavor?
- (c) How many different fun-sized bags of skittles are possible if (i) we no longer have the restriction that at least one of each flavor must be chosen and (ii) a bag contains *at most* 20 candies (and at least one piece of candy), rather than it must contain exactly 20?

3. An elevator with 7 passengers stops at 10 floors. Each passenger picks a floor on which to disembark uniformly at random (and independently of each other). The floors are numbered 1 through 10. Determine the probabilities of the following events:

- (a) Every passenger gets off on a different floor.
- (b) The passengers only get off on even-numbered floors.
- (c) Every passenger gets off on a different floor **and** they only get off on even-numbered floors.
- (d) Every passenger gets off on a different floor **or** they only get off on even-numbered floors.

4. Suppose we have a fair die that has twelve (12) sides. That is, if we roll it, each of the first 12 positive integers are equally likely to be the result of the roll.

- (a) If we roll the die, what is the probability the result is prime? As a reminder, one is **not** a prime number.
- (b) Suppose we roll this die 1000 times. What is the probability we get a prime number exactly 200 times?
- (c) Suppose we roll this die 1000 times. What is the probability we get a prime number at least 200 times?
- (d) What is the expected value of a single roll of this die?