CompSci 161
Winter 2023 Lecture 9:
Divide and Conquer VI:
Deterministic Selection

Exam Wednesday!
  - seating chart
  - logistics

Recall QuickSelect

O(n) expected running time

```
quickSelect(S, k)
    If n is small, brute force and return.
    Pick a random x ∈ S and put rest into:
        L, elements smaller than x
        G, elements greater than x
    if k ≤ |L| then
        quickSelect(L, k)
    else if k == |L| + 1 then
        return x
    else
        quickSelect(G, k − (|L| + 1))
```

} partition

<x | x | >x

L   G

worst case: $O(n^2)$
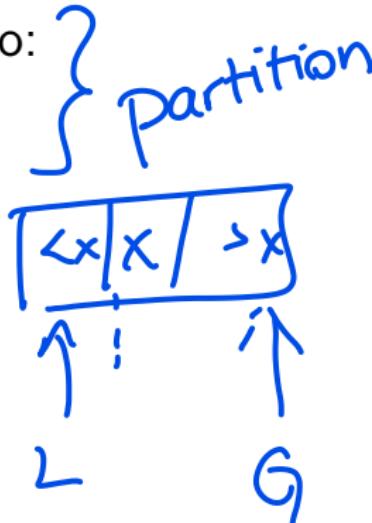
Deterministic Selection

Instead of picking x at random:

- Divide S into $g = \lceil n/5 \rceil$ groups
- Each group has 5 elements (except maybe g^{th})
- Find median of each group of 5 $\}$ O(1) each, $\Theta(n)$ total
- Find median of those medians
- Let x be that median.

$$\frac{n}{10} = \left(\frac{n}{5}\right)/2$$

Select (those, $\frac{n}{10}$)

We call this the "medians of 5" method.

Selecting Median of 5 Example

870	647	845	742	372	882	691	341	461	596
989	151	100	729	101	397	825	587	363	283
595	524	930	259	133	955	620	970	430	280
839	139	735	590	782	913	378	474	255	739
875	150	791	779	792					

742   596   151   397

524   620   735   474

791
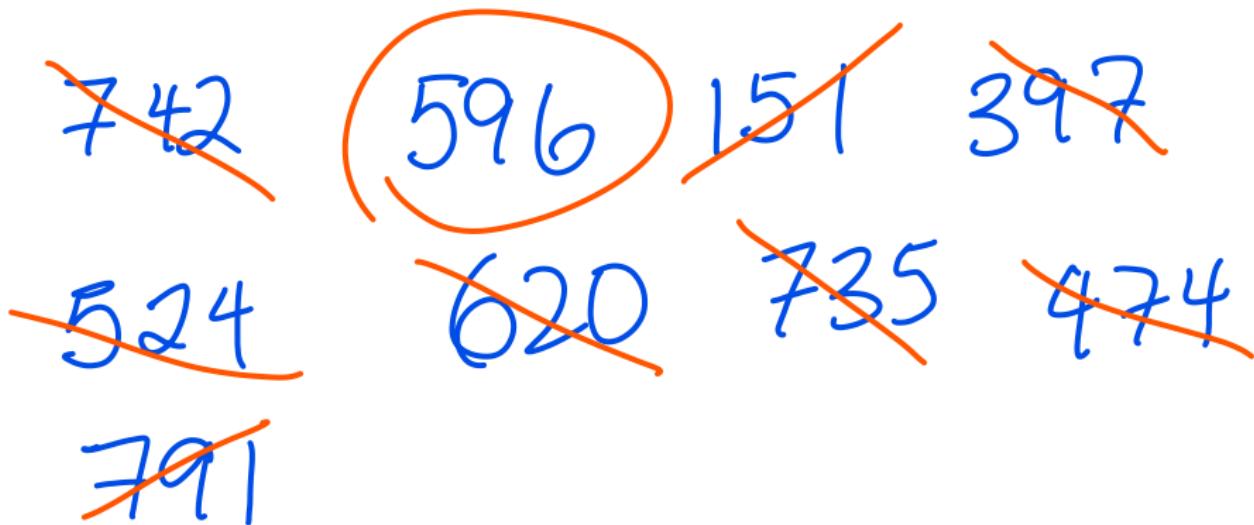
Deterministic Select

```
DeterministicSelect(S, k)
  If n is small, brute force and return.
  Pick x ∈ S via medians-of-5 and put rest into:
      L, elements smaller than x
      G, elements greater than x
  if k ≤ |L| then
    DeterministicSelect(L, k)
  else if k == |L| + 1 then
    return x
  else
    DeterministicSelect(G, k − (|L| + 1))
```

Deterministic Selection

Let's visualize: how does pivot compare to list?

M: group of "medians of 5"

$\approx \frac{n}{10}$ from M $< x$ } plus 2 friend each

$> x$ }

x equal #

$\frac{3n}{10}$ smaller than x. $\frac{3n}{10}$ larger

$\frac{4n}{10}$?

Demo Re-visualized

▶ Each column was a group of five.

▶ Each column is sorted

▶ Columns are ordered based on median-of-5

▶ Which cells are in L? G? Either?
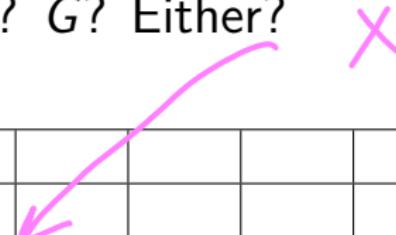
100	283	255	133	341				
101	363	378	259	461				
151	397	474	524	596	620	735	742	791
				691	955	782	845	792
				882	970	839	870	875

Deterministic Selection

▶ How few elements *must be* smaller than pivot?

▶ How few *must be* non-smaller than pivot?

▶ How many could be in either group?

Recurrence: $T(n) \leq T\left(\frac{7n}{10}\right) + T\left(\frac{1}{5}\right) + \theta(n)$

is $\theta(n)$

Topic Exam 1 Wednesday

- ▶ Seat assignments coming tonight
  - ▶ Will be released on GradeScope
  - ▶ Check GradeScope by TOMORROW NIGHT, 8PM
- ▶ Sit in assigned seat
- ▶ Test has your name on it
- ▶ Review exam rules from syllabus