CompSci 161
Winter 2023 Lecture 8:
Divide and Conquer V:
Min and Max Concurrently
Binary Exponentiation

Min and Max

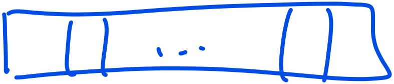
- We have an array of n distinct numbers.
- We want to find *both* – the min and max.
- Brute force method takes $2n - 3$ comparisons.
- Find a way that uses strictly fewer.

$n-1$: find min

delete min

$n-2$: find max now

Could anyone do better?



- Adversary argument:
  - All queries are made to an adversary (opponent)
  - Adversary is allowed to make up answers
  - But answers must be consistent with some input
- If we compare and find $a < b$, we say:
  - a lost the competition → a disqualified from max
  - b won the competition ← b disq. from being min
- Every non-max loses at least one ← n-1 info each
- Every non-min wins at least one ←
- This is $2n - 2$ units of information.

What should the adversary do?

We compare a and b to gain information.

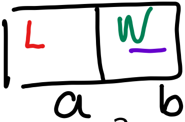
- If a, b never compared (to ANY key) before?

  Any answer same to me

  I gain 2 units info

- If exactly one of them compared before?

  I gain one unit info

- Both compared before, one won at least once?

- Both compared before, both lost before?

How many comparisons can be forced?

- We need to gain $2n - 2$ units of information.
- $c_1 = \#$ comparisons that gave us one unit.
- $c_2 = \#$ comparisons that gave us two units.
- Total units of info available is at least $2n - 2$

$$C_1 + 2C_2 \geq 2n - 2$$

- At most $n/2$ comparisons give us two units

$$-C_2 \ \cancel{\leq}\ \geq -n/2$$

$$C_1 + C_2 \geq \frac{3}{2}n - 2$$

~~Binary Exponentiation~~

(not today, sorry.):)

$$\cancel{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} F_{n-1} & F_n \\ F_n & F_{n+1} \end{bmatrix}}$$

Alg. for $\approx \frac{3}{2}n - 2$ cmp? // n even?

$\frac{n}{2}$ cmp: $\frac{n}{2}$ winners $\frac{n}{2}$ not winners

(pair up input)

$\frac{n}{2} - 1$ $\frac{n}{2} - 1$

Total: $\frac{3}{2}n - 2$