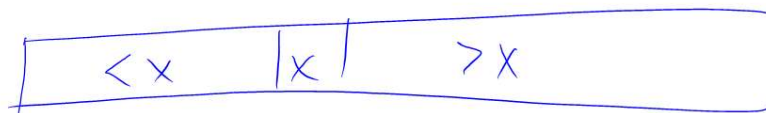


CompSci 161
Winter 2023 Lecture 05:
Divide and Conquer II:
QuickSort and Order Statistics

² QuickSort Step 1: Partition

85	24	63	45	17	31	96	50
----	----	----	----	----	----	----	----

1. Choose a pivot.
2. Place that pivot^x in the right spot.
3. Pivot the rest of the array.



3

QuickSort

85	24	63	45	17	31	96	50
17	24	31	45	50	63	96	85

4

How fast is QuickSort?

- ▶ $T(n) = T(\text{lower}) + T(\text{upper}) + \Theta(n)$
- ▶ If lower and upper are both size $n/2$?

$$T(n) = 2T(n/2) + n = \Theta(n \lg n)$$

- ▶ What if we select a pivot uniformly at random?
- ▶ What if we could find a median in $\Theta(n)$...

5

Average Case Analysis of QuickSort

Suppose

- ▶ All permutations equally likely
- ▶ All n values are distinct (for simplicity)
- ▶ Define S_1, S_2, \dots, S_n as sorted order.

Let $P_{i,j}$ be probability we compare S_i and S_j .

$$P_{i,j} = \frac{\# \text{ yes}}{\# \text{ total}} = \frac{2}{j-i+1}$$

6

Expected number of comparisons

$X_{i,j}$: IRV

$$\begin{aligned} E\left(\sum_{i=1}^n \sum_{j=i+1}^n X_{i,j}\right) &= \sum_{i=1}^n \sum_{j=i+1}^n E(X_{i,j}) \\ &= \sum_{i=1}^n \sum_{j=i+1}^n \frac{2}{j-i+1} \\ &= \sum_{i=1}^n \sum_{k=2}^{n-i+1} \frac{2}{k} \\ &< \sum_{i=1}^n \left(\sum_{k=1}^n \frac{2}{k} \right) \text{ harmonic} \\ &\text{is } \Theta(n \lg n) \end{aligned}$$

7

The Selection Problem

- ▶ Given a list S and numeric k
- ▶ Want: if we sorted S , what is S_k ?
- ▶ Brute force:
 - ▶ Sort S in $\Theta(n \log n)$
 - ▶ Return S_k
- ▶ Can we do better?

8

Randomized Selection

`quickSelect(S, k)`

If n is small, brute force and return.

Pick a random $x \in S$ and put rest into:

L , elements smaller than x

G , elements greater than x

} aka
partition

if $k \leq |L|$ **then**

return quickSelect(L, k)

else if $k == |L| + 1$ **then**

return x

else

return quickSelect($G, k - |L| - 1$)

Randomized Selection

- ▶ What is the worst-case running time?
- ▶ What would cause that bad time?
- ▶ Estimate the *expected* running time?
Hint: on average, the pivot is the median.

$$T(n) = T(n/2) + n$$

is $\Theta(n)$