

CompSci 161  
Winter 2023 Lecture 17:  
Greedy Algorithms:  
Interval Scheduling

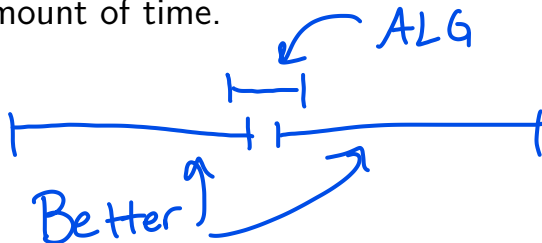
## Unweighted Interval Scheduling Problem

Two possible algorithms (four on handout):

- ▶ Sign up for the class that begins earliest.



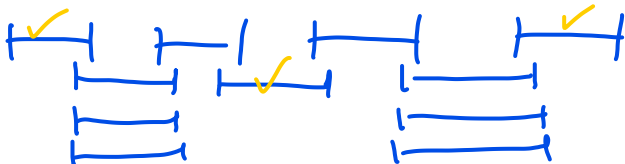
- ▶ Sign up for the class that meets for the least amount of time.



## Unweighted Interval Scheduling Problem

Two more algorithms (four on handout):

- ▶ Sign up for the class that conflicts with the fewest other classes.



ALG  
better:  
top 4

- ▶ Sign up for the class that ends earliest.

# Interval Scheduling Problem (proof)

Correct Algorithm:

- ▶ Sign up for the class that ends earliest.
- ▶ Remove it and all overlapping classes from the set of available classes.
- ▶ Repeat this process until no classes remain.

**Claim:** There is an optimal solution that includes the first-ending class.

**Proof of Claim:** Suppose all optimal solutions do not. Select an arbitrary optimal solution OPT.

OPT': OPT, remove the first ending from OPT  
add first ending from input

# Proof of Correctness

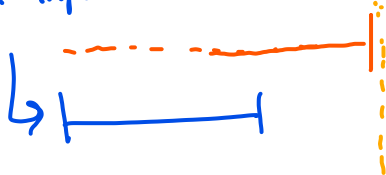
- ▶ We began with an arbitrary optimal set  $OPT$ 
  - ▶ Its first element was not first-ending.
  - ▶ We removed that one
  - ▶ We added our first one: the first-ending.
  - ▶ This forms a set we'll call  $OPT'$
- ▶ **Claim:**  $OPT'$  is an optimal solution.
  - ▶ Is it the same size as every optimal solution?

first  
ending  
of input

Yes (maybe say why)

- ▶ Is it a valid solution?

first ending from  $OPT$



Rest of  $OPT$   
is after dotted line

Additional Slide (if needed)

"How To Prove It"

by Daniel Velleman (spelling?)

# Proof of Correctness

- ▶ We proved that an optimal solution exists that includes the first-ending class.
- ▶ What does the full proof look like?