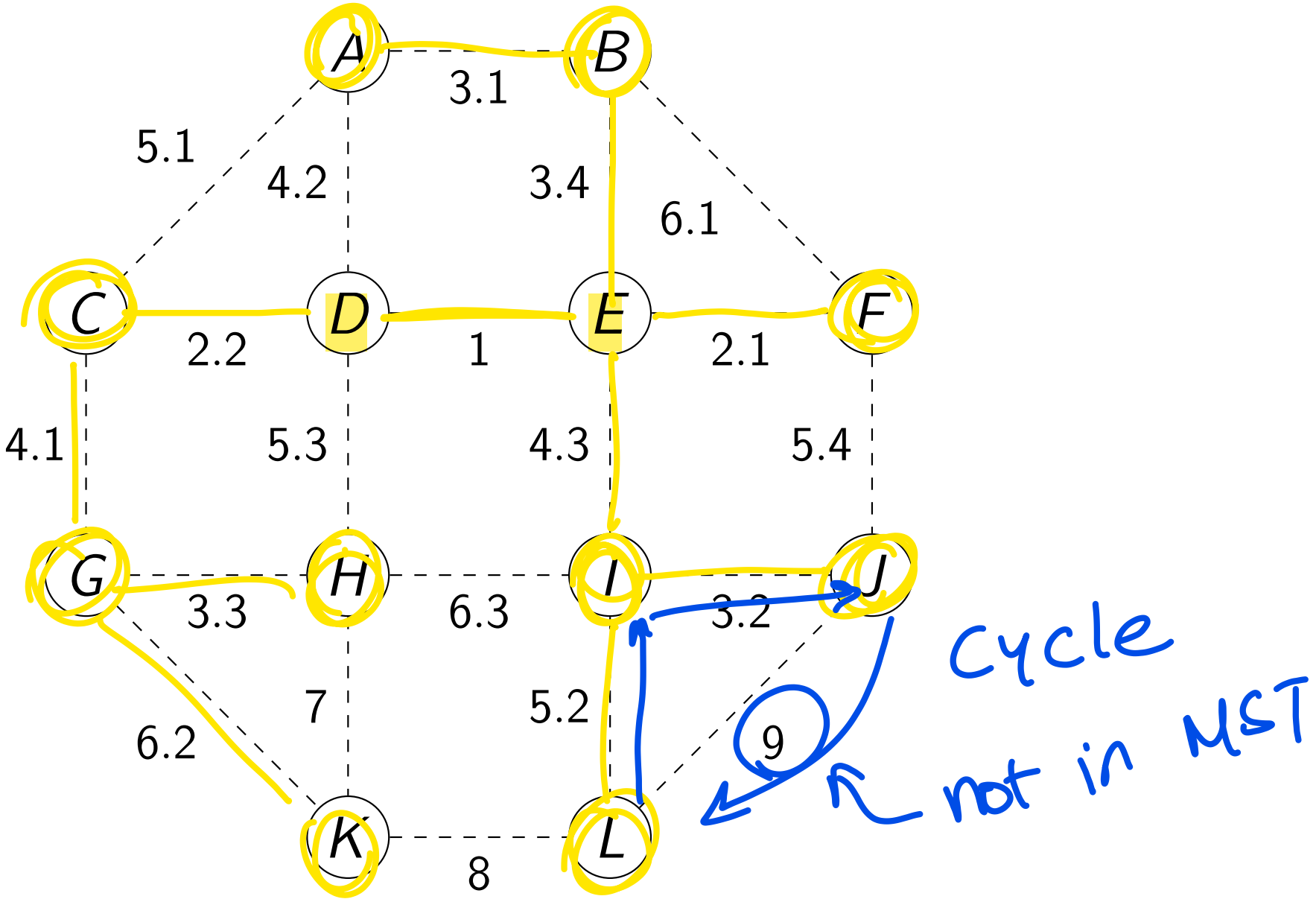
CompSci 161
Winter 2023 Lecture 16:
Greedy Algorithms:
Jarnik's Algorithm for MST

Jarnik's Algorithm



Cycle Property

- C is any cycle
- e is its heaviest edge
- Any tree T that includes e, not MST

FSOC suppose $e \in T$ and T is MST

Remove e from T. Now T_1 T_2 subtrees

Walk C from one endpt e to other

I cross $T_1 - T_2$ or vice versa at least once.

X: that edge $T_1 \cup T_2 \cup X$ = span tree, cheaper than T.

Cut Property

- Cut the vertices into X and $G - X$
- e is the lightest edge with endpoints in ~~both~~
- Any tree T that avoids e, not MST

one endpt in X, other in G-X

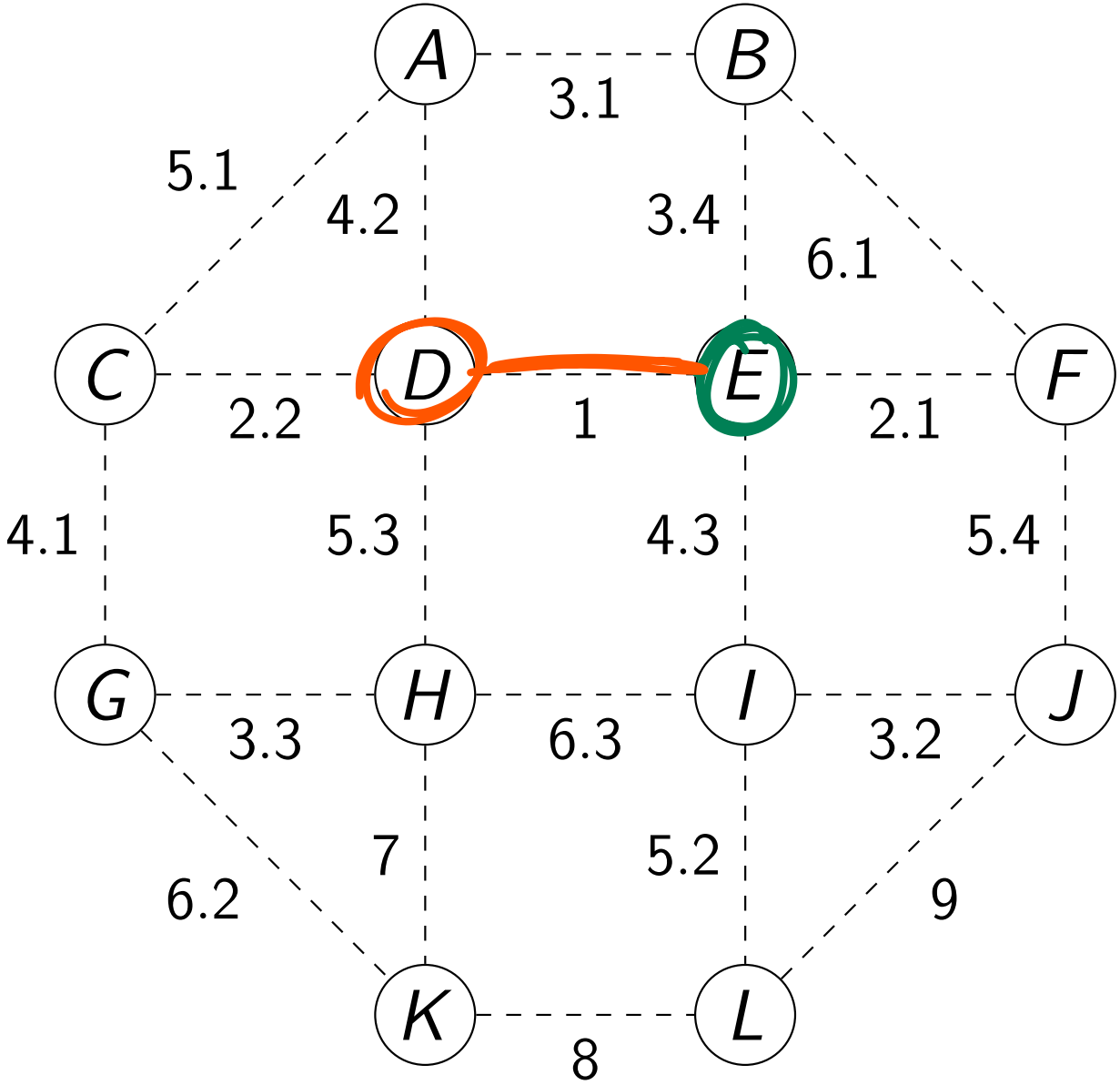
FSOC suppose T avoids e and is a MST.

∃f in T's edges s.t. one endpt in X, one in G-X.

Create T-f+e : - spanning tree

Cost(f) > cost(e) so this cheaper than T.

Jarnik's Algorithm Revisited



Path Property

- Any $x, y \in V$
- Path from x to y through the MST has the minimum possible weight for its heaviest edge.

(did not get to in lecture on Feb 22)