# CompSci 161
# Winter 2023 Lecture 11:
# Dynamic Programming II:
# Longest Common Subsequence

# Longest Common Subsequence

**Input:** Two sequences (strings etc)

**Output:** Longest common subsequence.

**Examples of common subsequences:**

| | | |
|---|---|---|
| exercise | determine | eerie |
| morning | triangle | ring |
| toward | thousand | toad |

# LCS: Recursive Solution

Cases: $O(mn)$
time each: $O(1)$

if rec. were rec lookup

Let LCS(n,m) be the length of the longest common subsequence of $X[1 \dots n]$ and $Y[1 \dots m]$.

if (0 == n or 0 == m) return 0

if (X[n] == Y[m])

return 1 + LCS(n-1, m-1)

return max( LCS(n, m-1),

LCS(n-1, m) )

# LCS: Iterative Solution

```
LCS(n,m):  // recursive for reference
```

**if** $0 == n$ or $0 == m$ **then**

  **return** 0

**else if** $X[n] = Y[m]$ **then**

  **return** $1+$ LCS$(n-1, m-1)$

**else**

  **return** max( LCS$(n-1, m)$, LCS$(n, m-1)$)

*for (j=0 ... m)
  LCS[0, j] = 0
for i=1 ... n
  LCS[i,0]=0

declare LCS[0..n, 0..m]

*
//non-base:
for i = 1 ... n
  for j = 1 ... m
    // fill in LCS[i,j] as per above

O(mn)

if X[i]==Y[j]
  LCS[i,j] = 1 + LCS[i-1, j-1]

and other case too!

# LCS Example:

| | | M | O | R | N | I | N | G |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T | 0 | | | | | | | |
| R | 0 | | | | | | | |
| I | 0 | | | | | | | |
| A | 0 | | | | | | | |
| N | 0 | | | | | | | |
| G | 0 | | | | | | | |
| L | 0 | | | | | | | |
| E | 0 | | | | | | | |

good reinforcement

# Finding the LCS itself

- We have `LCS[]` filled in.

$i \leftarrow n, j \leftarrow m, S \leftarrow$ empty stack
**while** **do**
// is $x[i]$ and $y[j]$ part of output?

good
rein forcement

# Did we do everything we need to do?

- Describe *in English* the function
  - Not *how* it works (yet)
  - Yes *what it solves*.
  - Skipping this step $=$ 0 on problem
- Give that function a meaningful variable name.
  - Not “OPT” or “DP” or “table”
  - Not a single letter either.
- Give recursive formulation.
- Describe the iterative running time.
  - Was cut-off in Friday’s slides. :(