CompSci 161
Winter 2023 Lecture 3:
Prerequisite Review: Proofs II

Primitive Pythagorean Triplets

- Primitive Pythagorean Triplet a, b, c
- Partial credit: a, b cannot both be even

FSOC suppose a, b both even

$a = 2x$ $b = 2y$ $c = 2z$

$(2x)^2 + (2y)^2 = 4x^2 + 4y^2 = (2z)^2$

even even $= 4z^2$

2 Primitive Pythagorean Triplets

- Primitive Pythagorean Triplet a, b, c
- Full credit: a, b cannot both be odd

FSOC a^2 odd b^2 odd c^2 even : c even

$a = 2x+1$ $b = 2y+1$ $c = 2z$

$(2x+1)^2 + (2y+1)^2 = (2z)^2$

$4x^2 + 4x + 1 + 4y^2 + 4y + 1 = 4z^2$

$4x^2 + 4x + 4y^2 + 4y + 2 = 4z^2$

$2x^2 + 2x + 2y^2 + 2y + 1 = 2z^2$

$2(x^2 + x + y^2 + y) + 1 =$ even

3 Induction 1

- Fibonacci : $F_n = F_{n-1} + F_{n-2}$
  $F_0 = 0$ and $F_1 = 1$
- $n \geq 0$, F_n is even iff n is divisible by 3.

if and only if

F_0 : 0 div by 3, F_0 is even

F_1, F_2 : i not $\div 3$, F_i odd

} base case / basis

Suppose true for $0 \leq n \leq K$ } Strong Inductive Hypothesis

// demonstrate for $k+1$

is $k+1$ multiple of three? Maybe. Yes or No

If Yes . $F_{k+1} = F_k + F_{k-1}$ use of Ind. Hyp

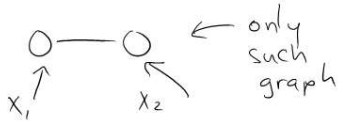
even odd odd

If no odd = one odd / one even

Induction 2

- G is connected, $|V| \geq 2$
- Two distinct vertices x_1 and x_2
  Removing either does not disconnect graph.

Basis: $|V|=2$.



Ind Hyp: true for $2 \leq |V| \leq k$

Ind Step: have G w/ $|V|=k+1$

$\exists$ vertex x s.t. $G-\{x\}$ disconnected? If no...

Else: Look at 2 ConnComp in $G-\{x\}$: A, B

$A \cup \{x\}$