Additional Divide and Conquer Problems Problems
CompSci 260—Winter, 2020—Dillencourt

1. Suppose you are given an unsorted array $A[1..n]$, which contains all but one of the $n+1$ integers in the range $0, \dots, n$ (so exactly one of these elements is missing from A). To simplify the problem somewhat, we will assume that $n = 2^k - 1$ for some integer k. Hence each array element has a binary representation using k bits.

   You want to determine the missing integer. You are not allowed to access an entire integer in A with a single operation. The only way to access the elements of A is by calling the function `bitvalue`(i, j), which returns the value of the jth bit of $A[i]$. Give a divide-and-conquer algorithm that finds the missing integer and makes only $O(n)$ calls to the function `bitvalue`().

   Note: There are $(n-1)\log n$ bits, so you cannot afford to look at every bit.

2. Give a asymptotic solutions (using $\Theta()$ notation) for each of the following recurrence equations.

   (a) $T(n) = 6T(n/4) + n \lg n$
   (b) $T(n) = 2T(n/4) + \sqrt{n}$
   (c) $T(n) = 6T(n/3) + n^2$