Fourier Transform in Graphics and Vision

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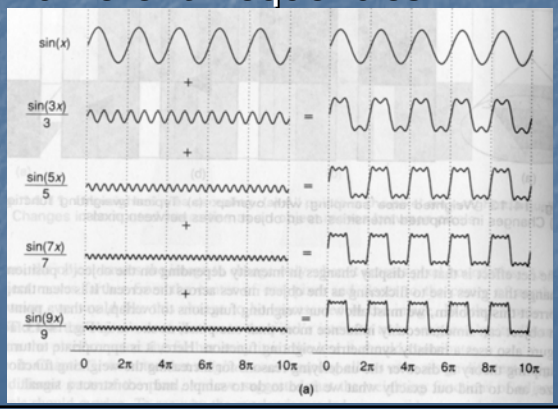
ICS 288

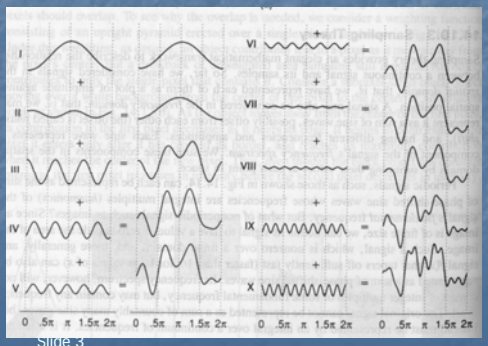
Frequency Based Representation of a Spatial Signal

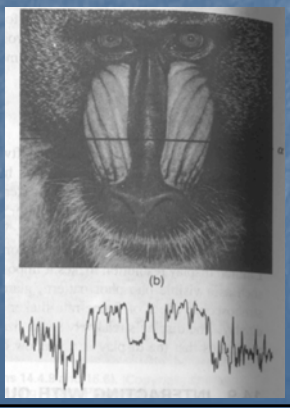
- Any one dimensional function can be represented as a linear combination of sine waves of different frequencies



1D Signal

- Example: Once scan line of an image
- Amount of each wave defined by its amplitude and phase

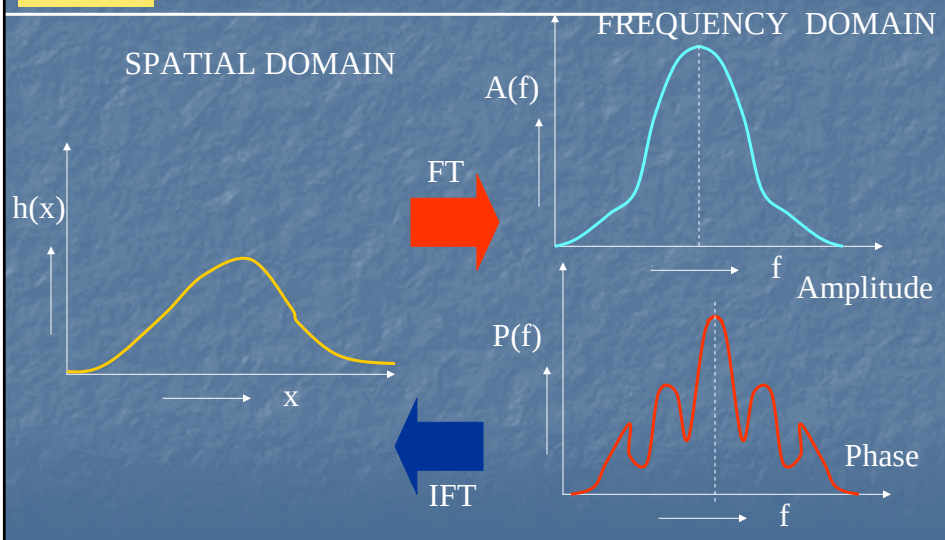




Sine Waves form a Basis

- Forms a basis for the set of functions
  - Sine wave of one frequency is linearly independent of another
  - Any function is a linear combination of this linearly independent basis function
- Infinite Basis
  - When we do it in digital domain, we can find a finite basis
  - Matlab

Fourier Transform (FT)



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Extending it to 2D

- Any 2D function (an image) can be represented as a linear combination of sine waves of different frequencies and orientation

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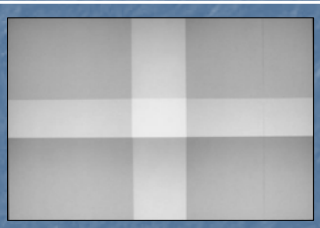
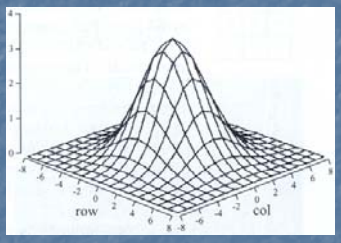
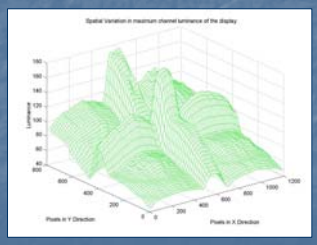
Extending it to 2D



FT

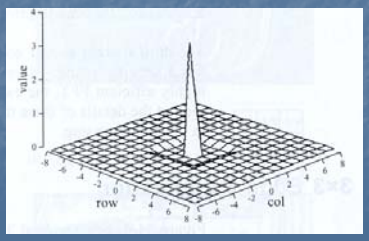
Amplitude





IFT

Phase



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Amplitude

- Amplitude
  - **How** much details?
  - Sharper details signify higher frequencies
  - Will deal with this mostly

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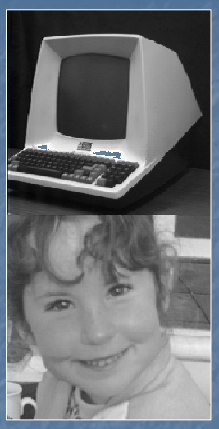
Phase

- **Where** are the details?
- Though we do not use it much, it is important, especially for perception



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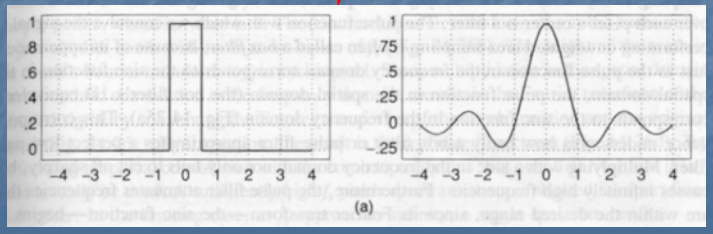
Properties of Fourier Transform

- Symmetric

h(x) → H(f)

H(x) → h(f)



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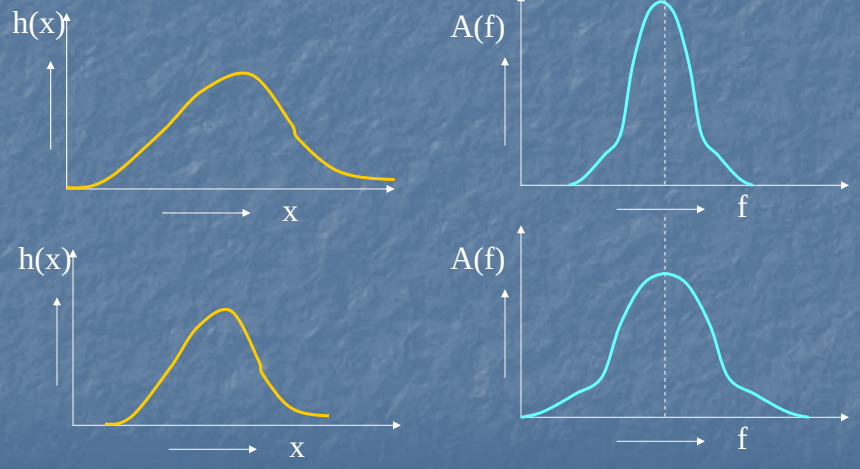
Properties of Fourier Transform



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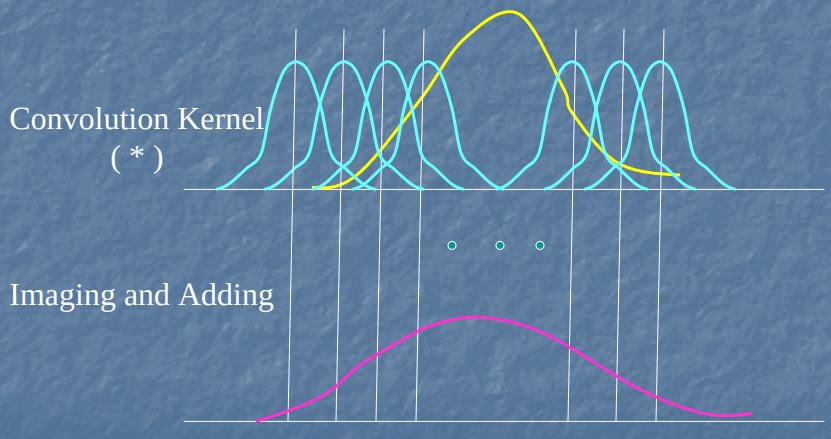
Convolution



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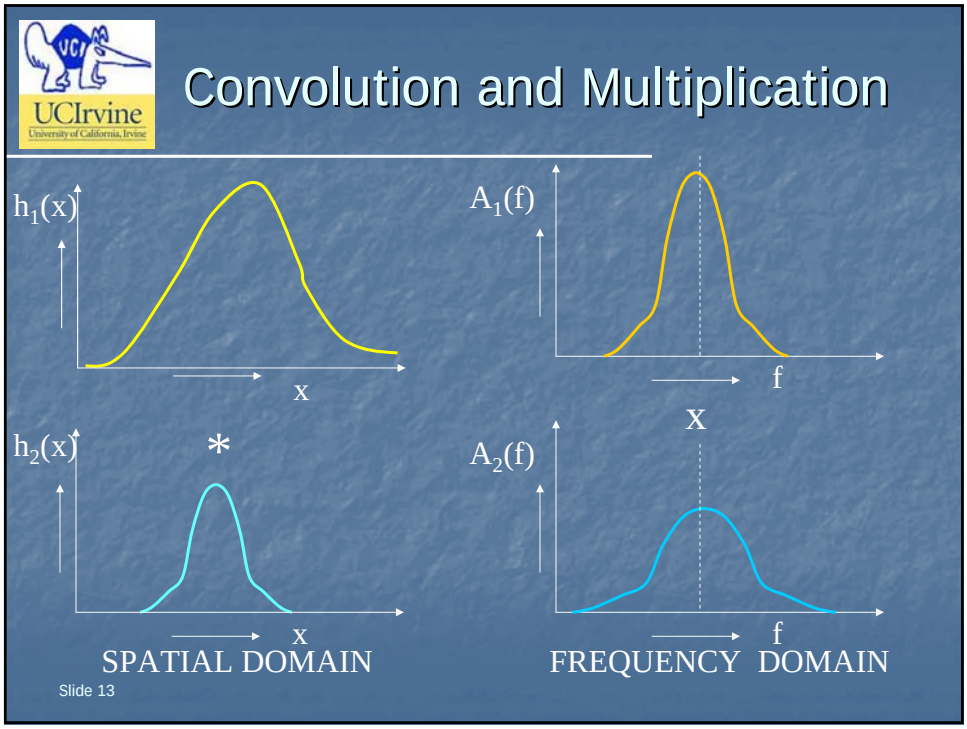
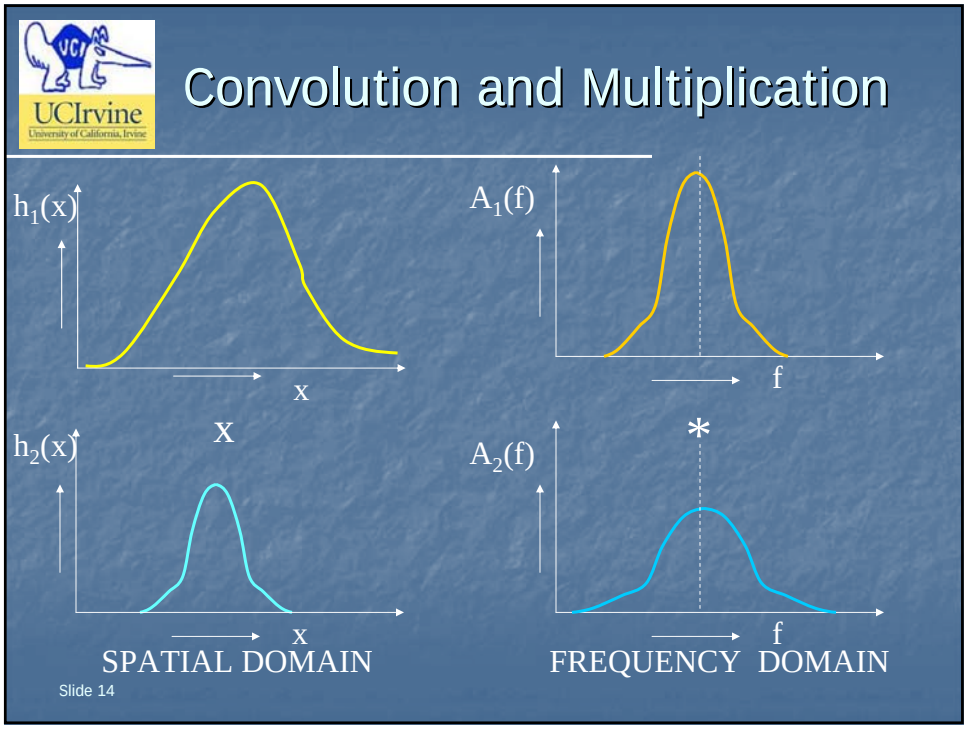
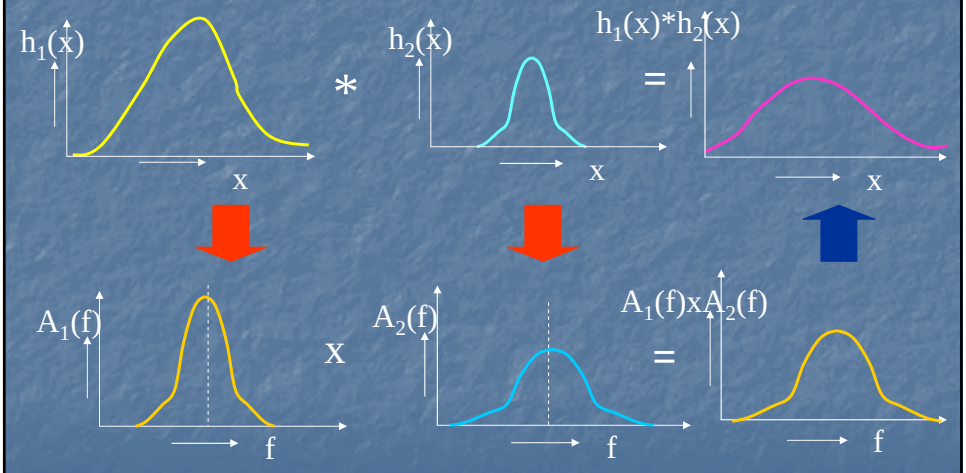

Convolution and Multiplication
UCIrvine
University of California, Irvine
h1(x)
x
A1(f)
f
h2(x)
*
x
x
A2(f)
f
SPATIAL DOMAIN
FREQUENCY DOMAIN
Slide 13


Convolution and Multiplication
UCIrvine
University of California, Irvine
h1(x)
x
A1(f)
f
h2(x)
x
x
A2(f)
*
f
SPATIAL DOMAIN
FREQUENCY DOMAIN
Slide 14

Convolution and Multiplication

SPATIAL DOMAIN

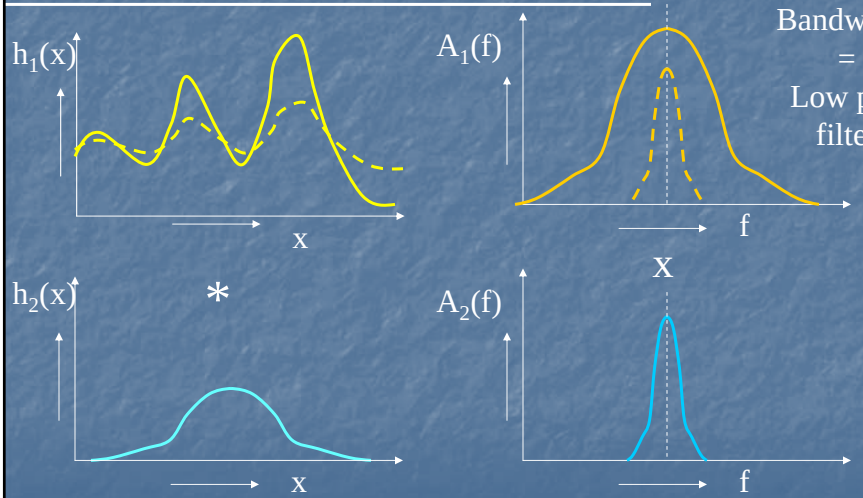


FREQUENCY DOMAIN

Slide 15

Image Filtering

Reduced the Bandwidth = Low pass filter



SPATIAL DOMAIN

FREQUENCY DOMAIN

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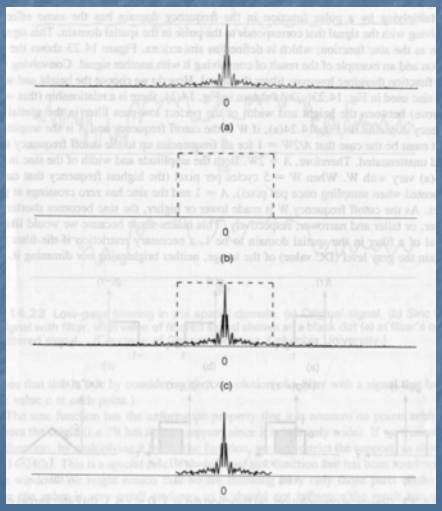
Ideal Low Pass Filter



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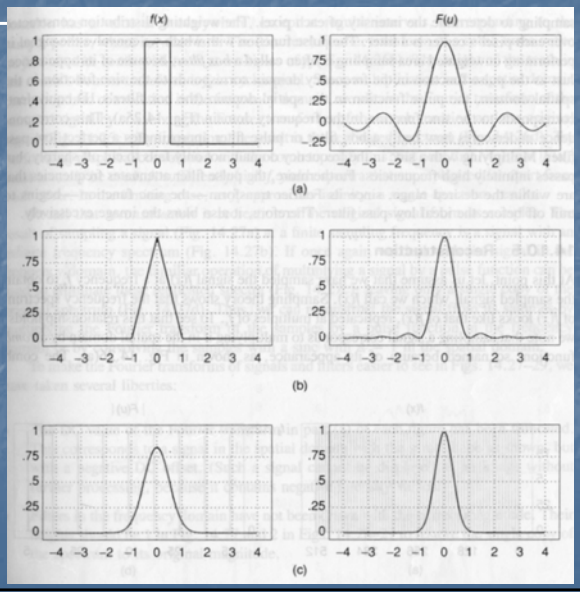
Types of Filters



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Choosing Right Filters

- Should not pass high frequency
- Should be limited in spatial domain
- Box Filter in spatial domain
  - Not good since passes high frequencies
- Synch Filter in spatial domain
  - Infinitely long in spatial domain
- Gaussian is a good compromise

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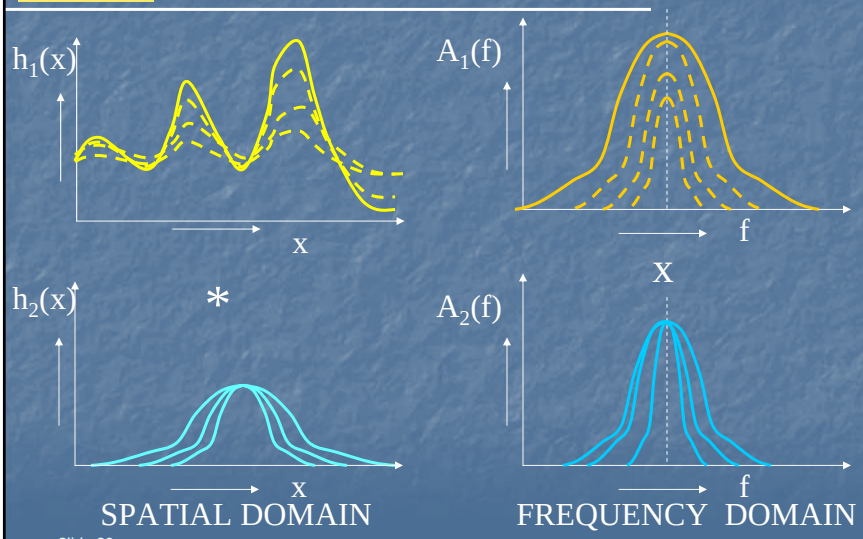
Hierarchical Image Filtering



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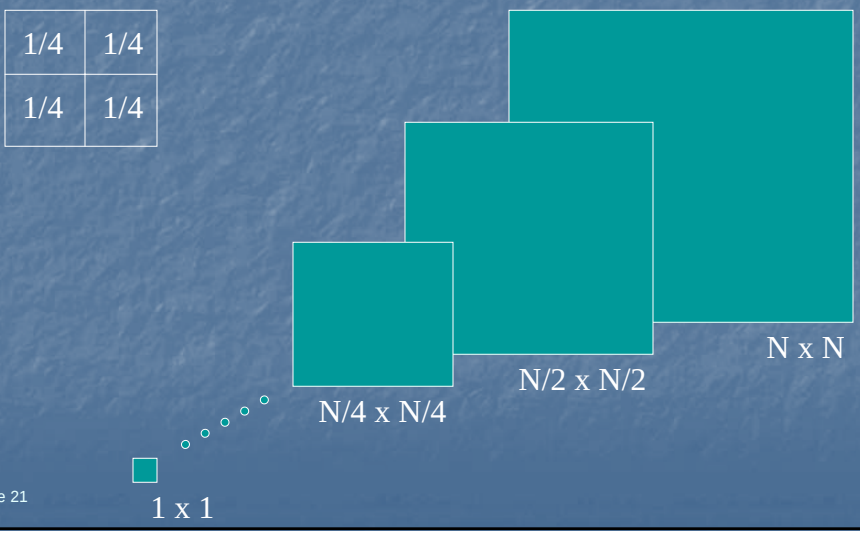
Hierarchical Filtering

1/4	1/4
1/4	1/4



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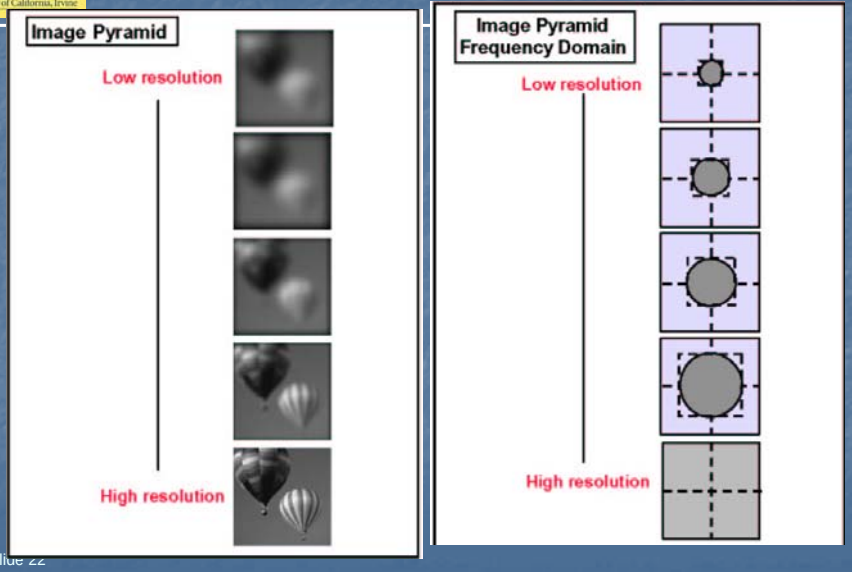
Gaussian Pyramid



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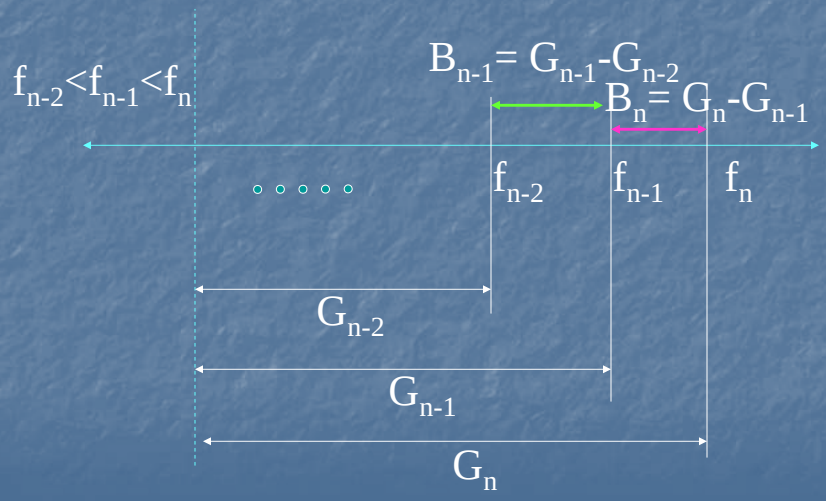
Band-limited Images (Laplacian Pyramid)



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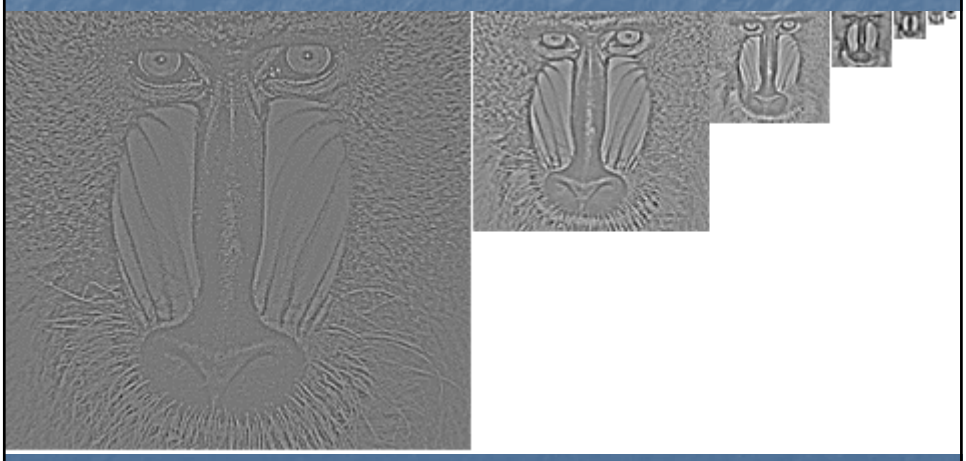
Band-limited Images (Laplacian Pyramid)



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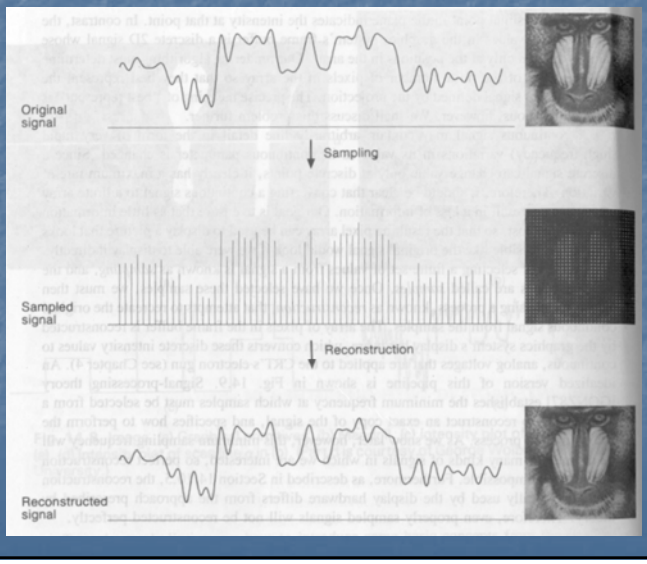
Sampling and Reconstruction



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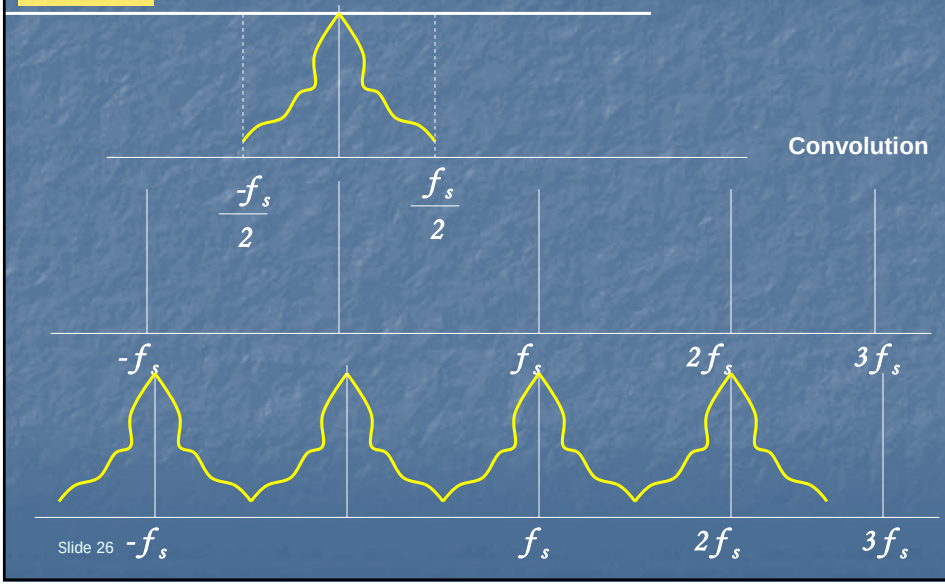
Sampling



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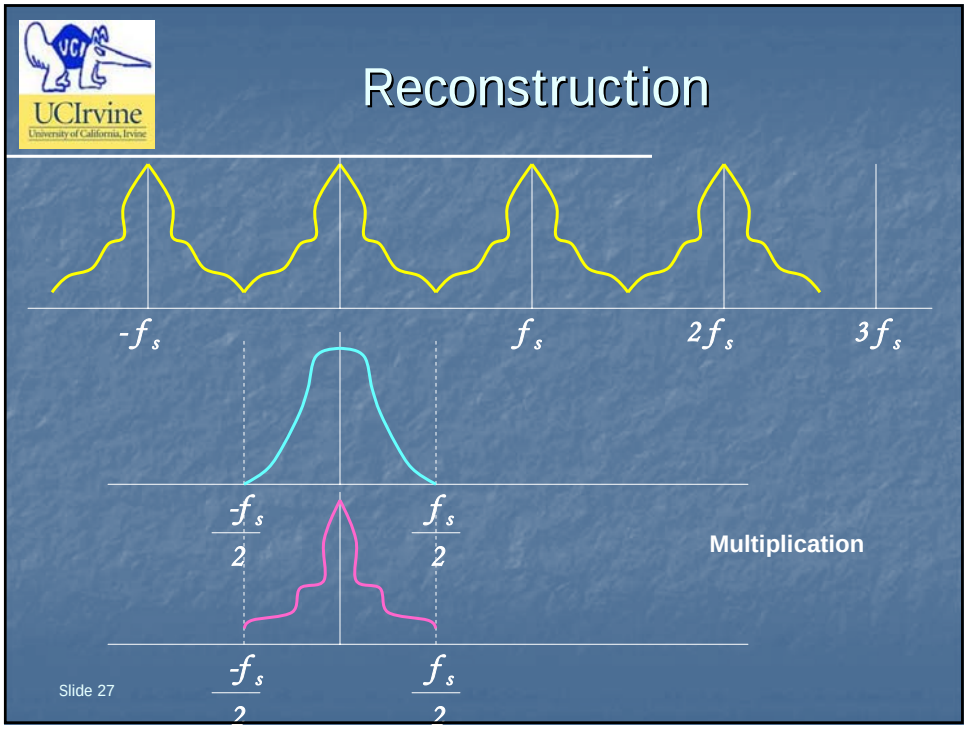
UCIrvine
University of California, Irvine
Reconstruction
$-f_s$
f_s
$2f_s$
$3f_s$
$\frac{-f_s}{2}$
$\frac{f_s}{2}$
Multiplication
$\frac{-f_s}{2}$
$\frac{f_s}{2}$
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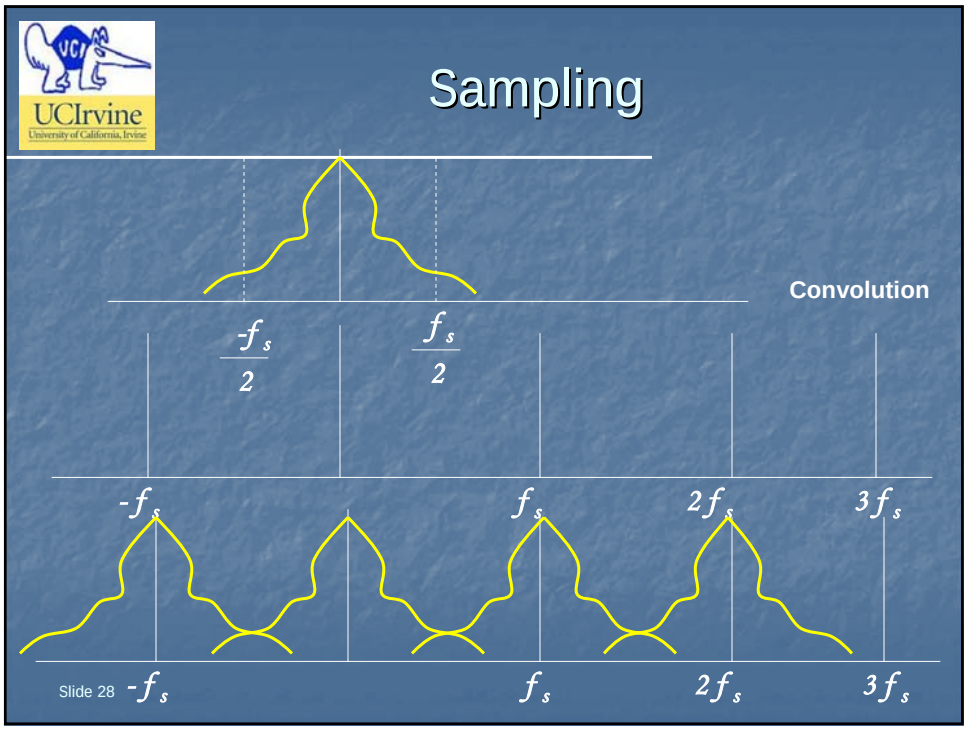
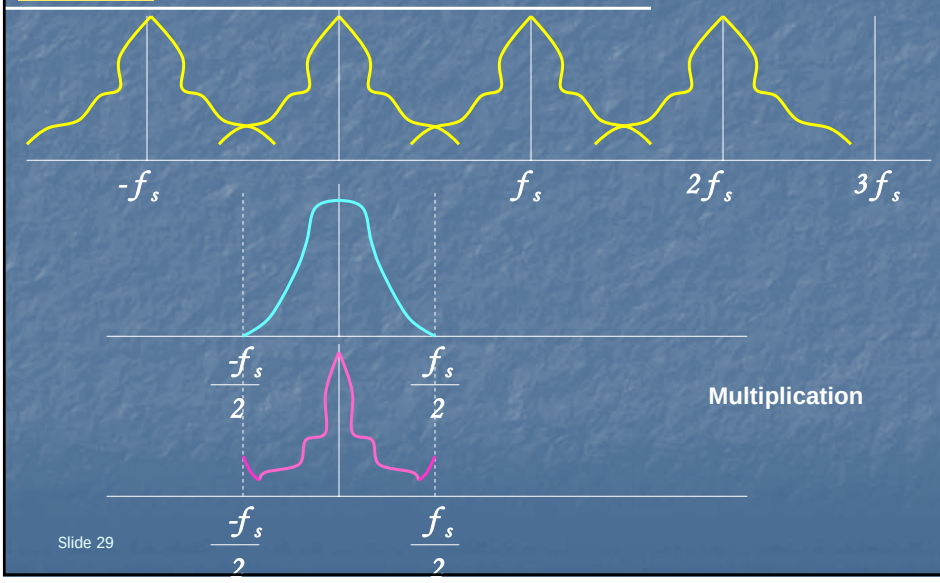

UCIrvine
University of California, Irvine
Sampling
Convolution
$\frac{-f_s}{2}$
$\frac{f_s}{2}$
$-f_s$
f_s
$2f_s$
$3f_s$
Slide 28
$-f_s$
f_s
$2f_s$
$3f_s$

Reconstruction



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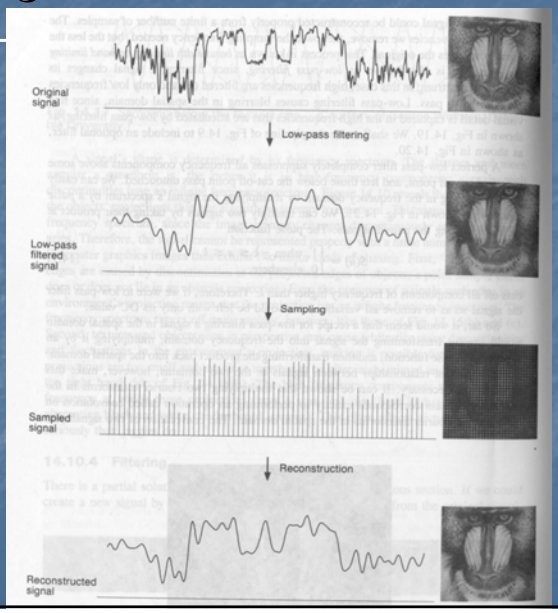
Aliasing

Slide 30

Aliasing

Slide 31

Signal is band-limited first



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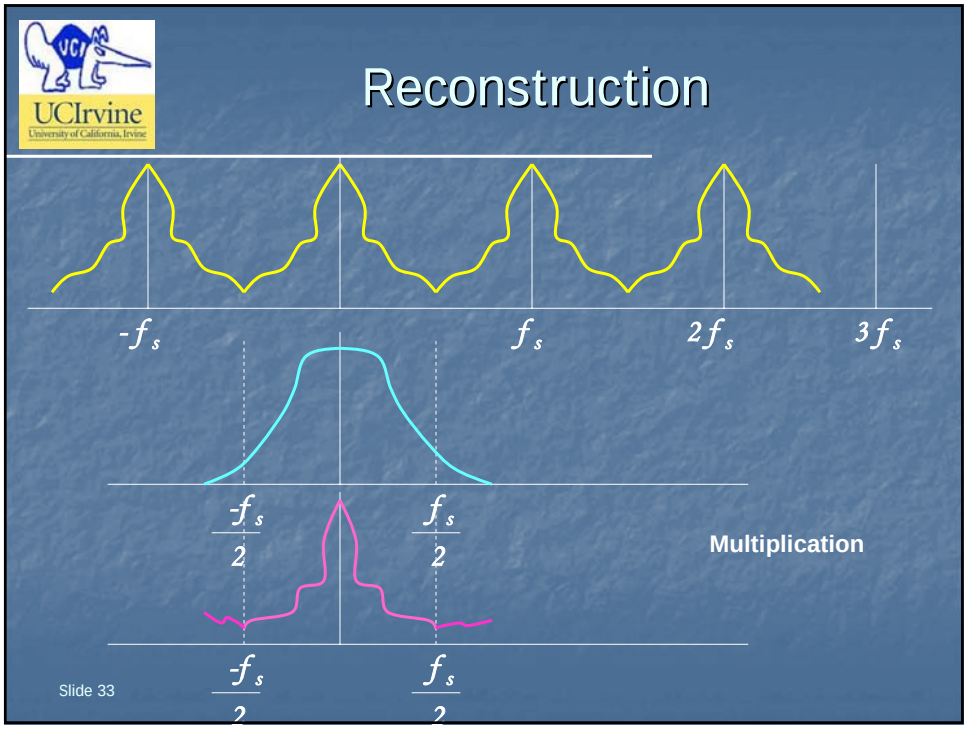
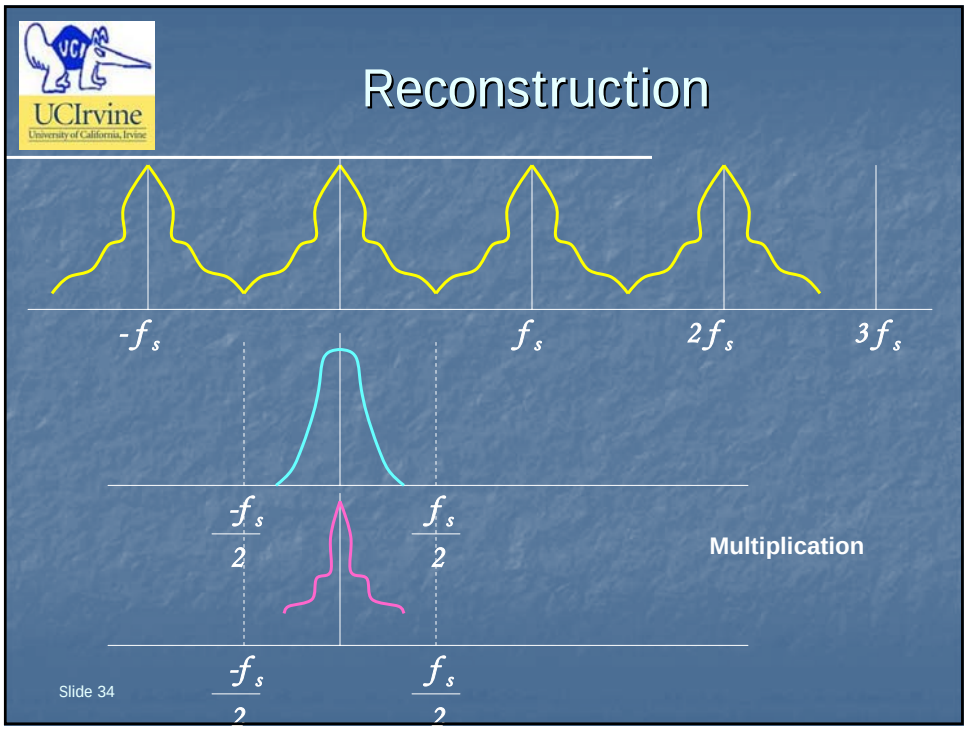

Reconstruction
UCIrvine
University of California, Irvine
$-f_s$
f_s
$2f_s$
$3f_s$
$\frac{-f_s}{2}$
$\frac{f_s}{2}$
Multiplication
Slide 33


Reconstruction
UCIrvine
University of California, Irvine
$-f_s$
f_s
$2f_s$
$3f_s$
$\frac{-f_s}{2}$
$\frac{f_s}{2}$
Multiplication
Slide 34

Blurring

Slide 35

Blurring

Slide 36