Linear Filters

Blurring filters

- More blurring implies widening the base and shortening the height of the spike further.
- What does it look like?
- Box filters are not best blurring filters but the easiest to implement.

1/3	1/3	1/3

1D box filter
(size=3)

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

2D box filter
(size=3x3)

1/5	1/5	1/5	1/5	1/5

1D box filter
(size=5)

1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25

2D box filter
(size=5x5)

Frequency domain representation

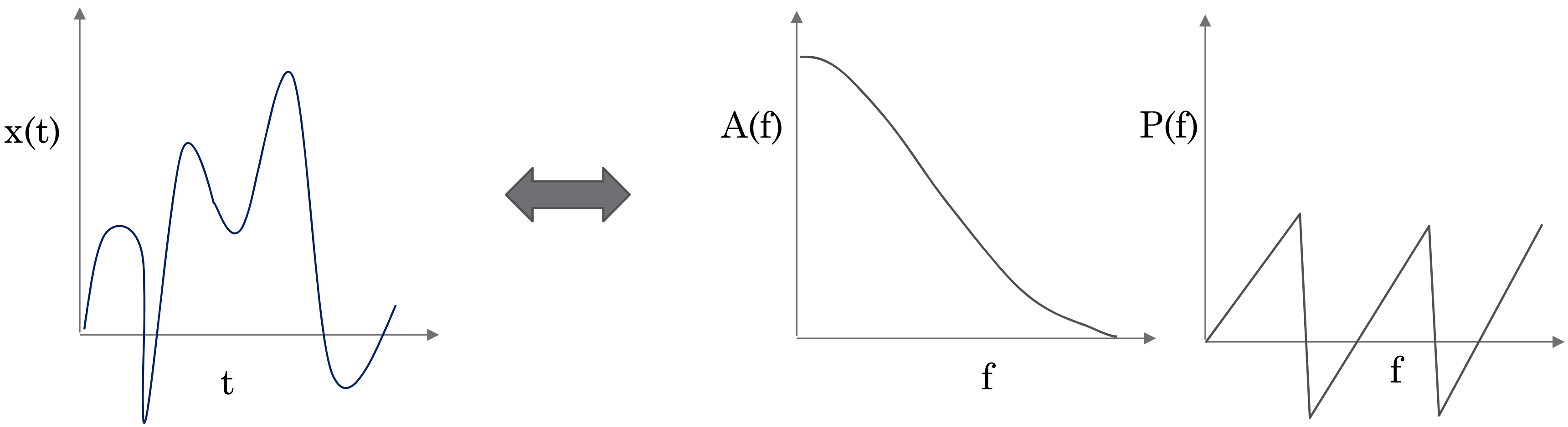
- ***frequency domain representation***
  - A signal is a linear combination of sine or cosine waves

$$c(t) = \sum_{i=1}^{\infty} a_i \, Cos(f_i + p_i)$$

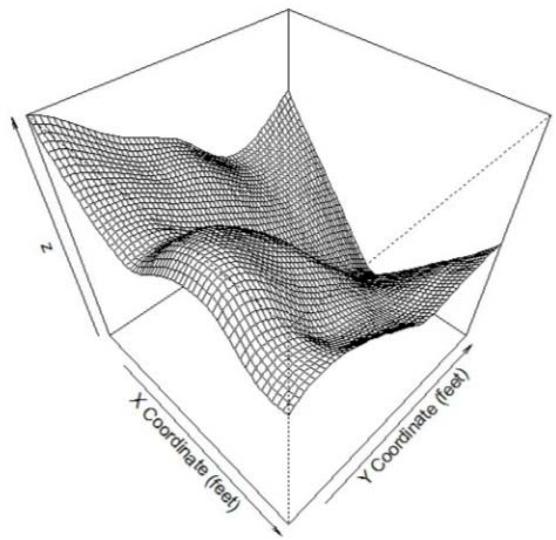
- Signal can be represented by the coefficients of these sine or cosine waves

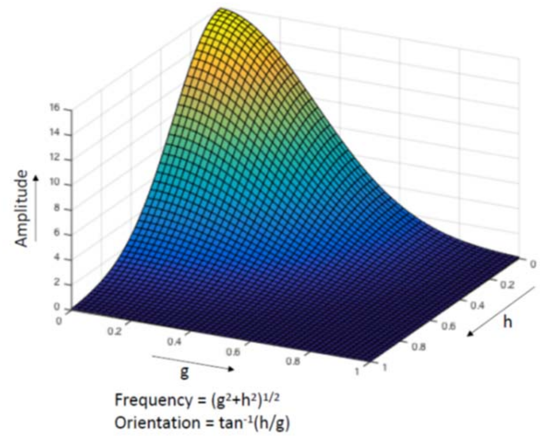
Time/Frequency or Primal/Dual

- Lower energy in higher frequencies
- Amplitude is more important
- Phase information is better studied in time domain



What happens in 2D?

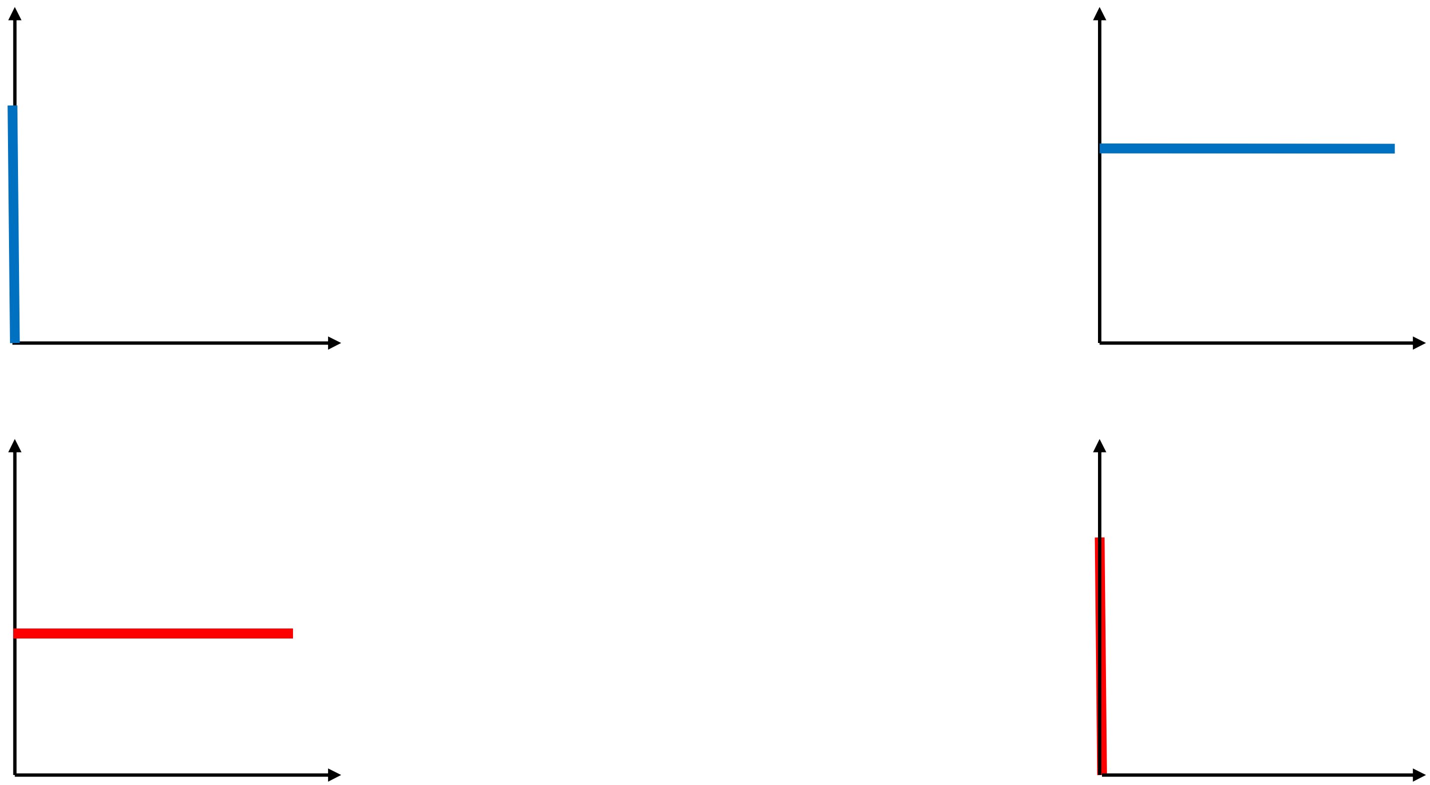




Duality

Spatial Domain

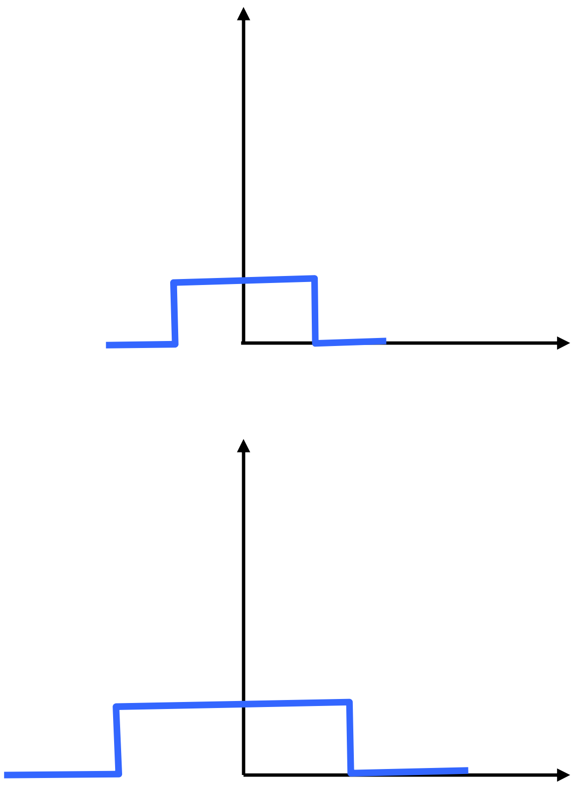
Frequency Domain



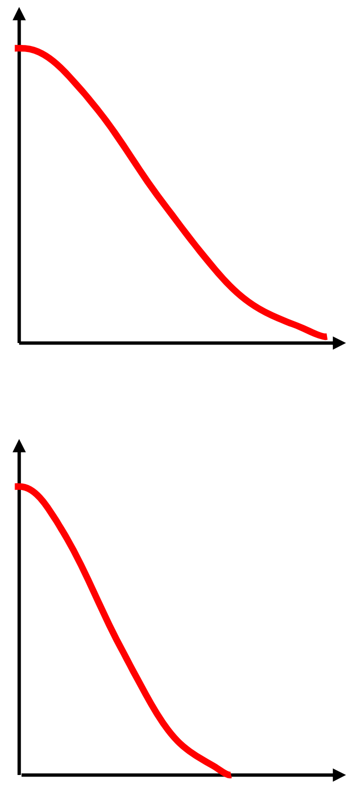
Duality

Spatial Domain

Frequency Domain



Widening in one domain is narrowing in another and vice-versa.

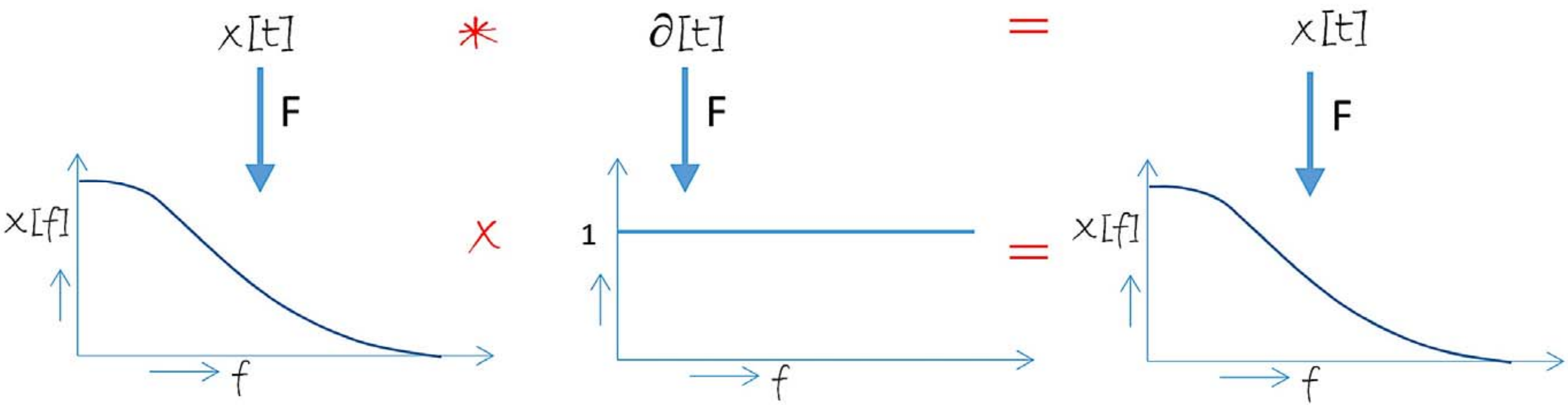


Duality

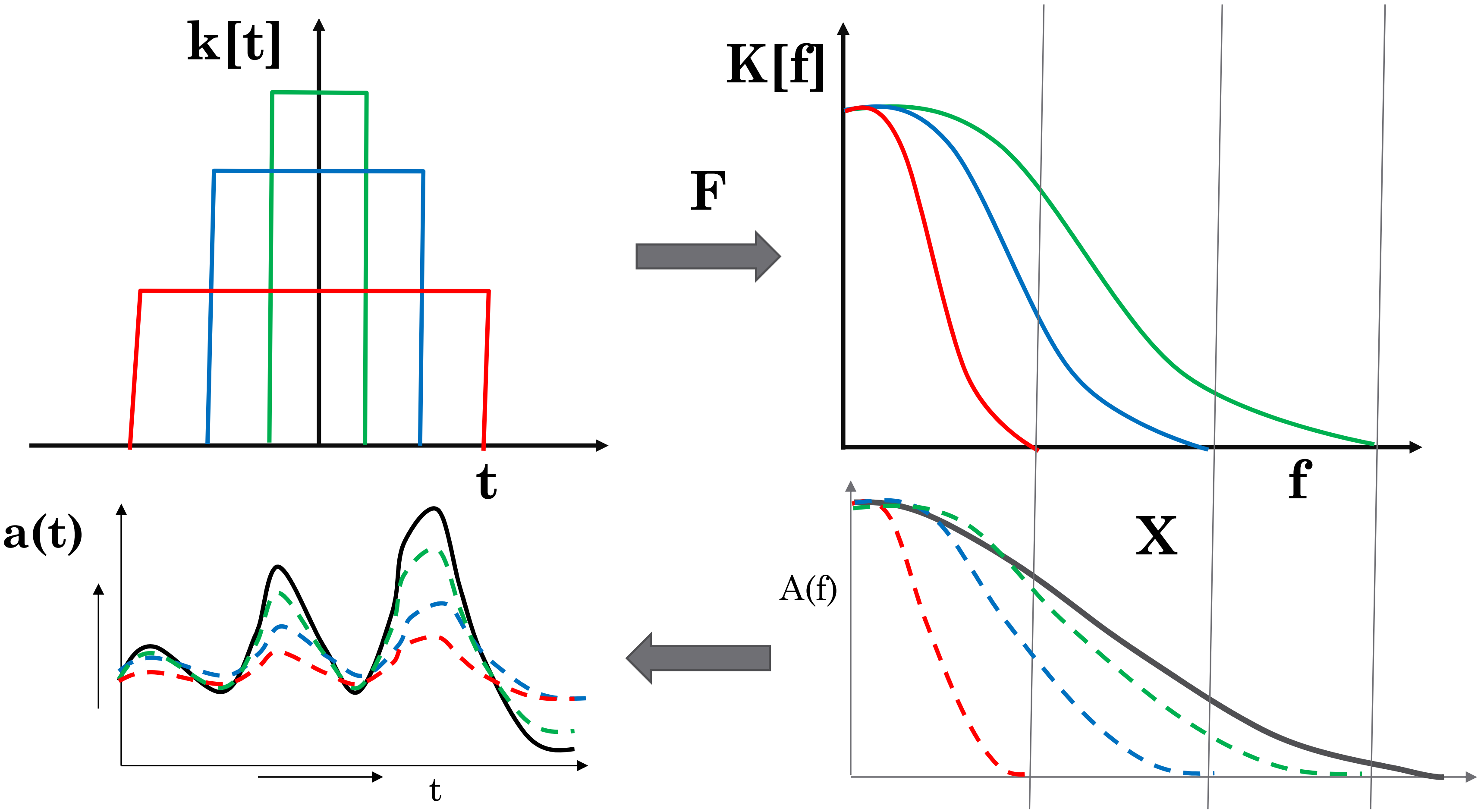
- Convolution of two functions in time/spatial domain is a multiplication in frequency domain
- Vice Versa

All Pass Filter

$$x[t] * \partial[t] = x[t]$$



Low Pass Filter



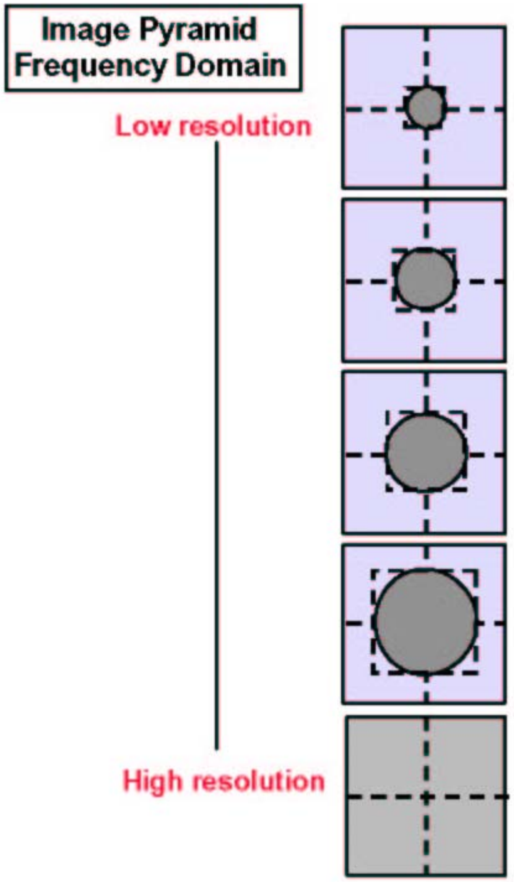
Low Pass Filtering

- Box filter is known as *low pass filter*.

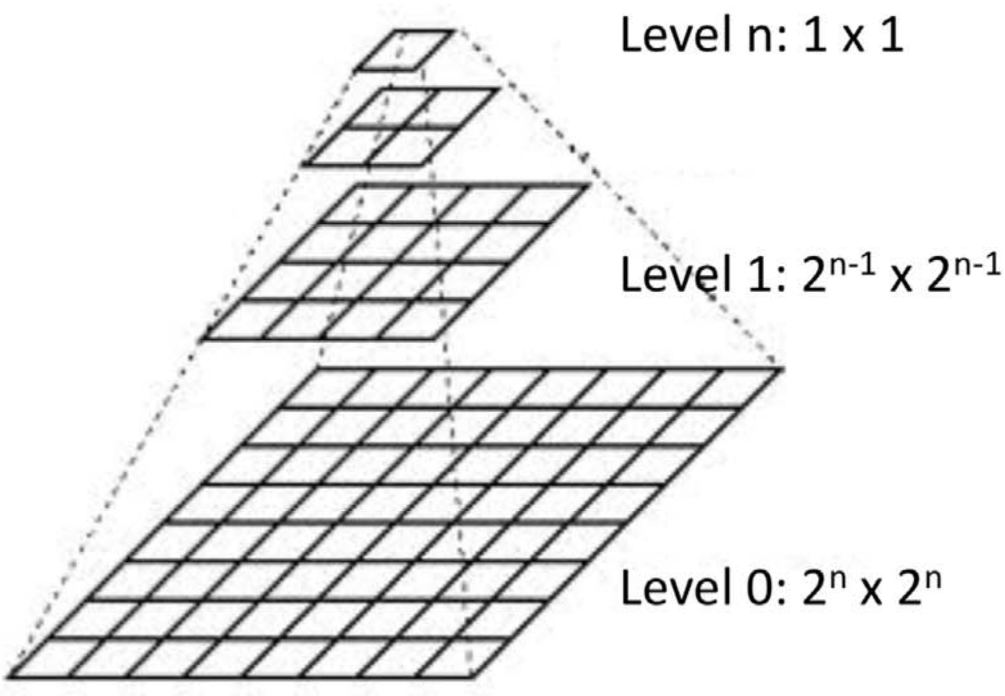
Box Filter

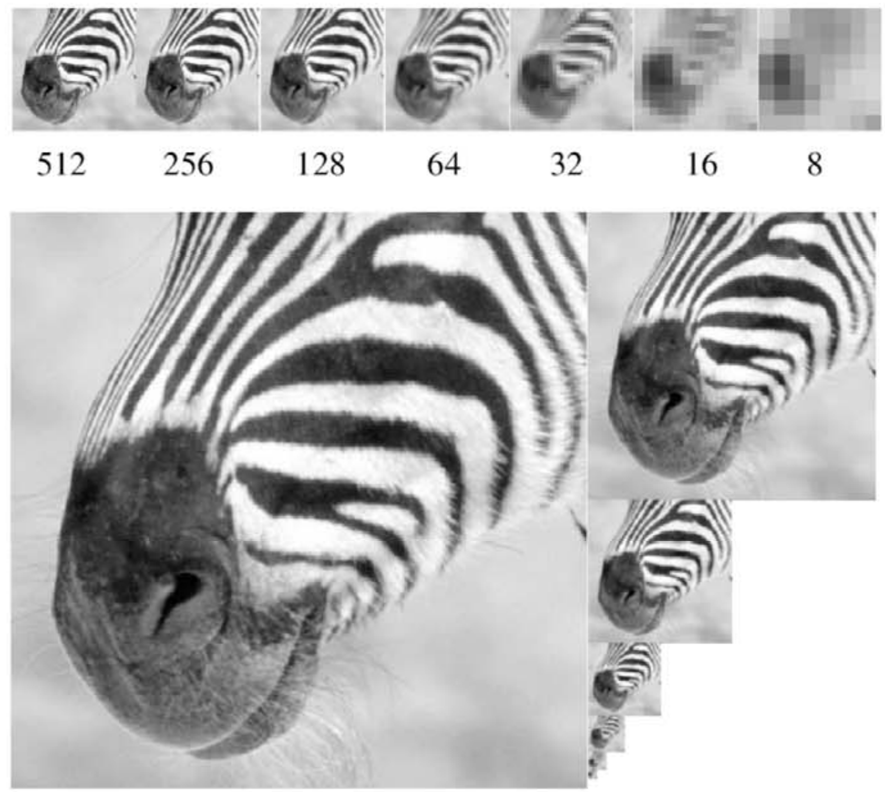
- Effect of increasing the size of the box filter

Gaussian Pyramid



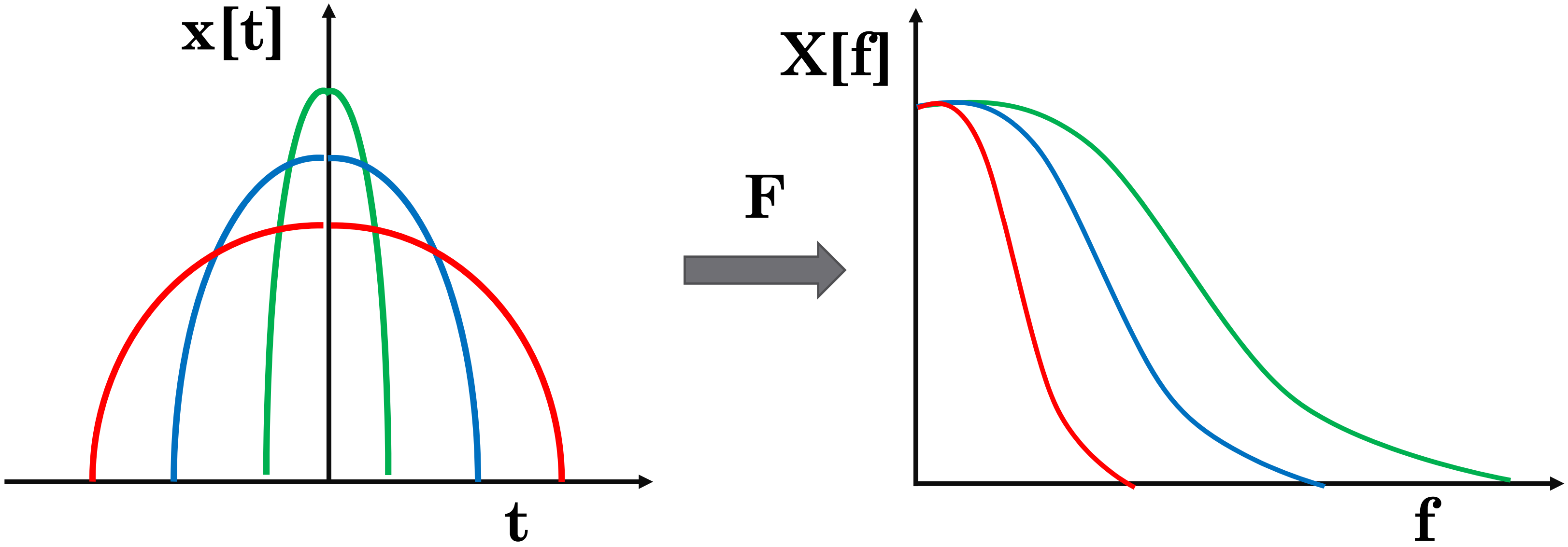
Gaussian Pyramid





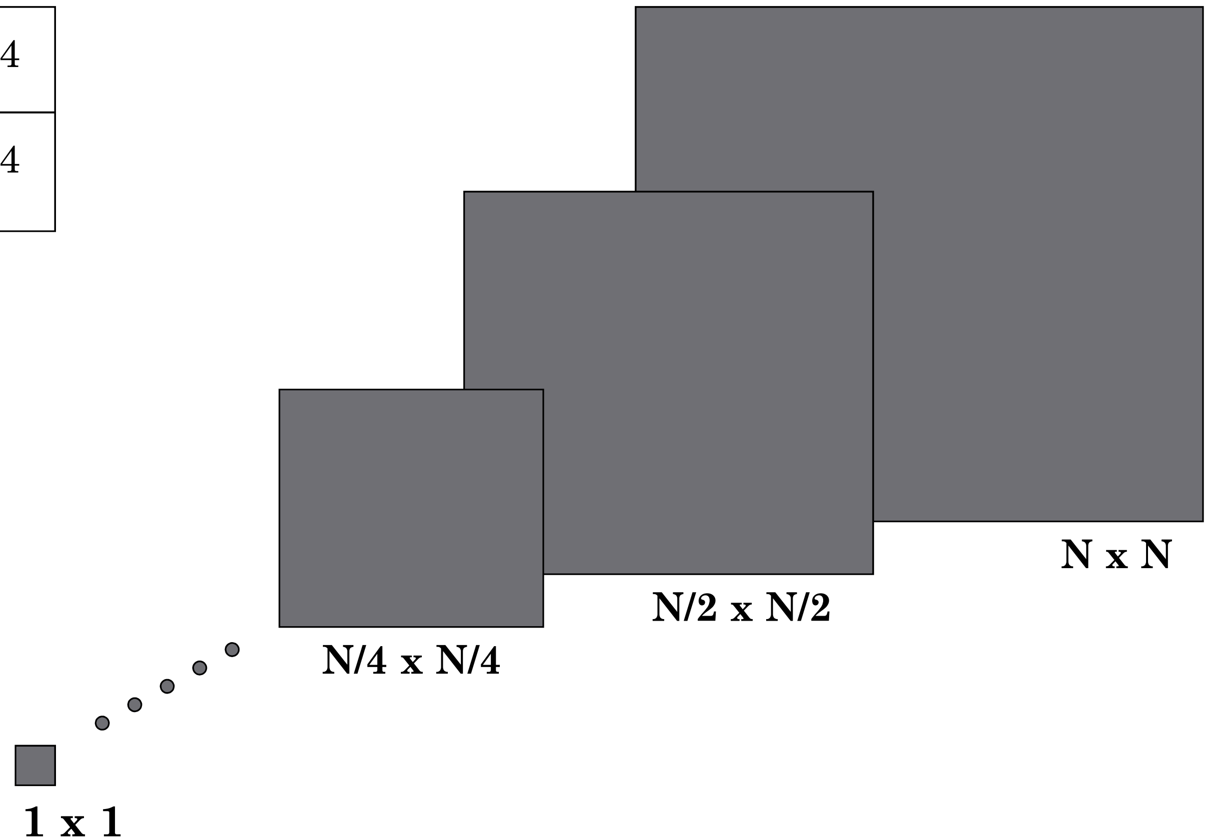
Box is not the only shape

- Gaussian is a better shape
- Any thing more smooth is better



Hierarchical Filtering

1/4	1/4
1/4	1/4



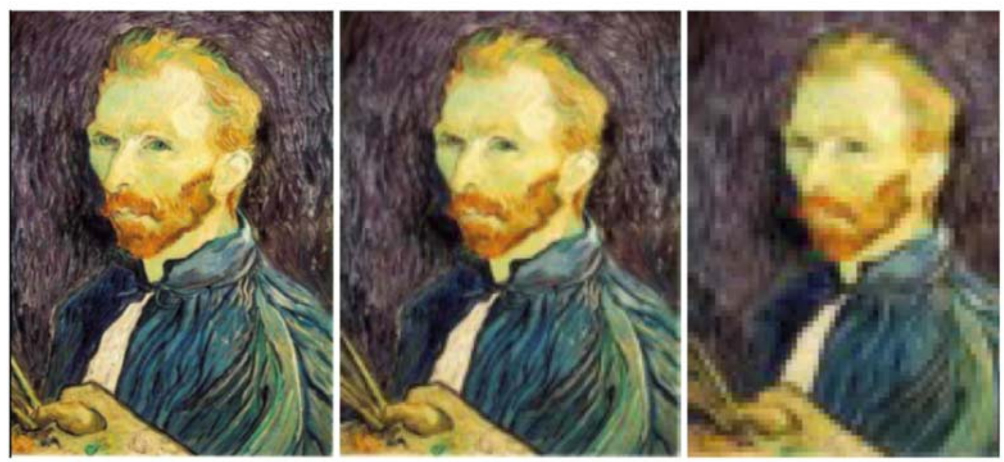
Issue of Sampling

- As an image undergoes low pass filtering, its frequency content decreases
- Minimum number of samples required to adequately sample the low pass filtered image is less.
- Low pass filtered image can be at a smaller size than the original image.

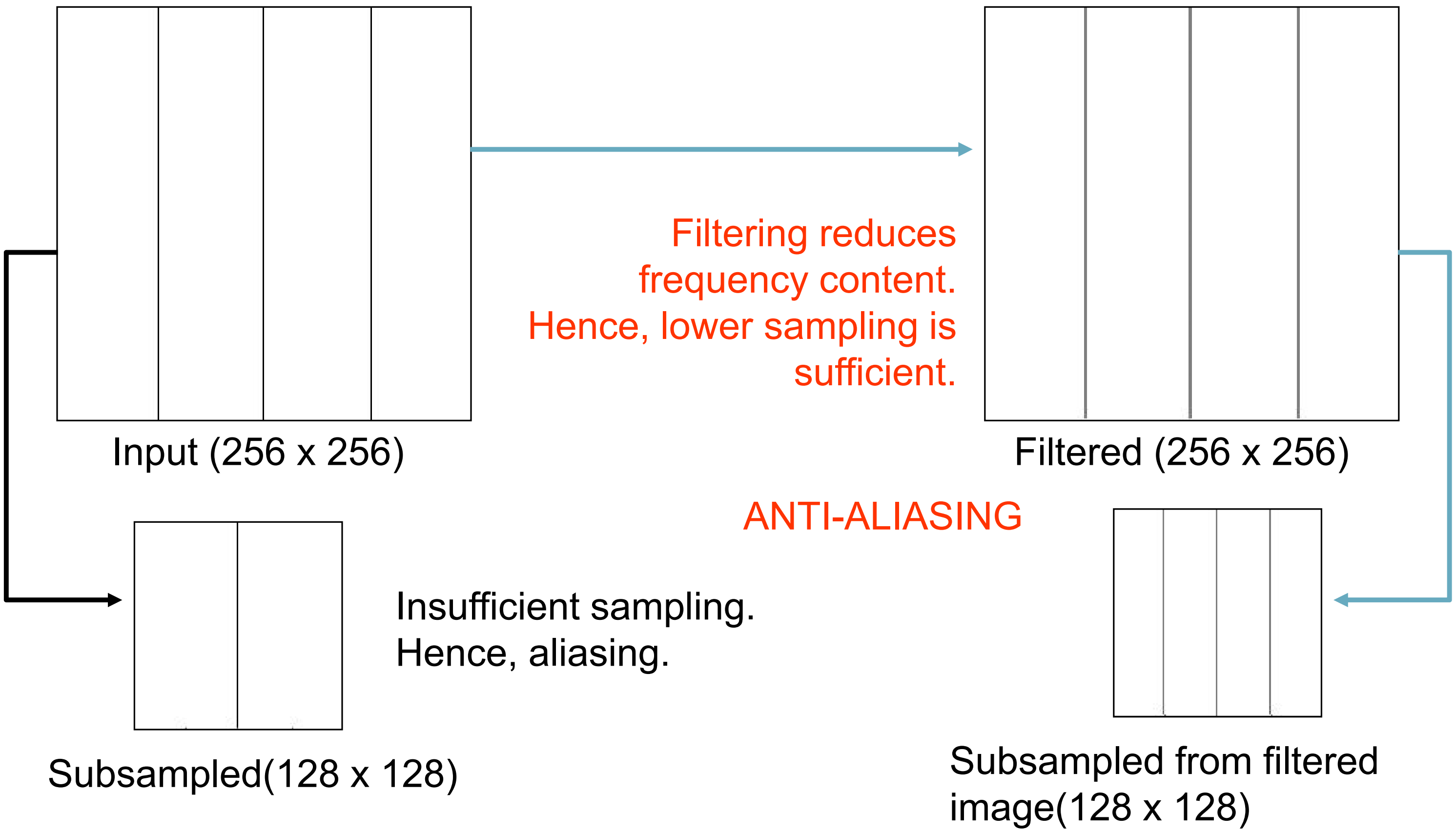
Subsampling

simple subsampling

pre-filtering and subsampling



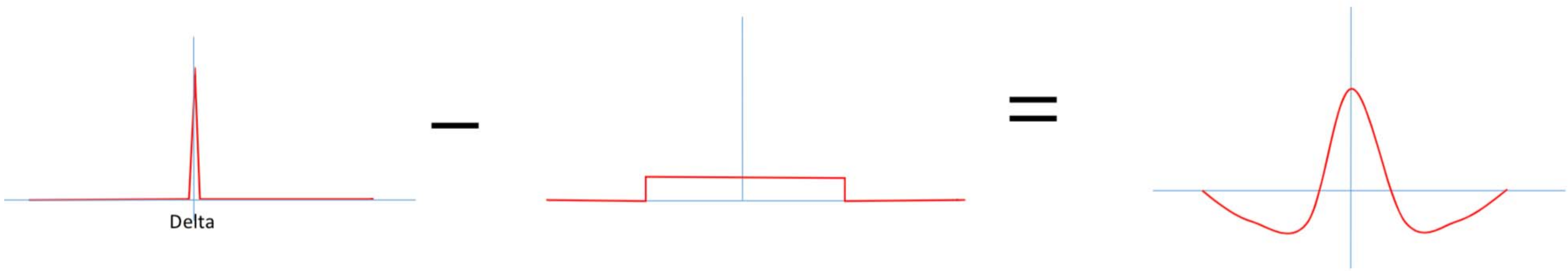
Aliasing Artifact



High Pass Filter

- subtract the low pass filtered image from the original image

$$I_h = I - I * l$$
$$= I * \partial - I * l = I * (\partial - l)$$



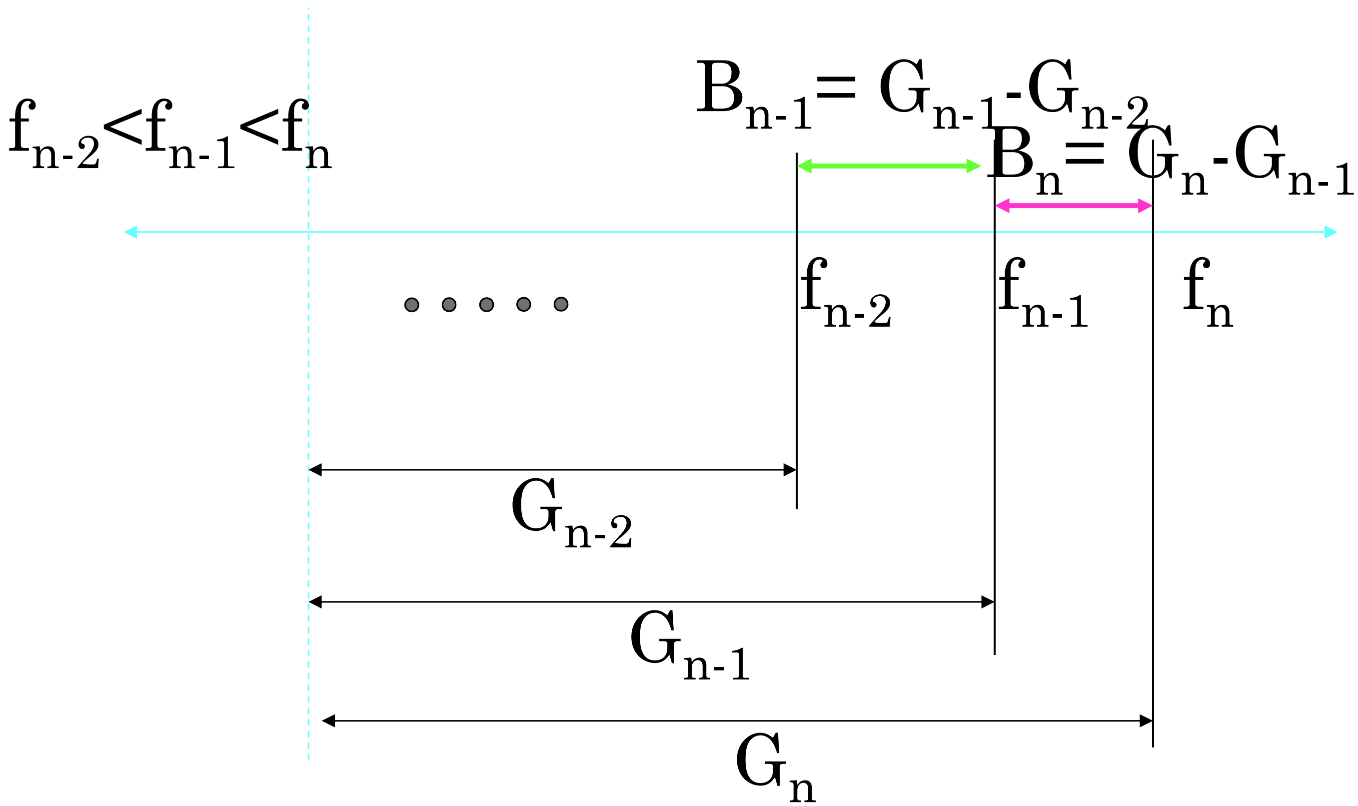
High Pass Filter

Original Image

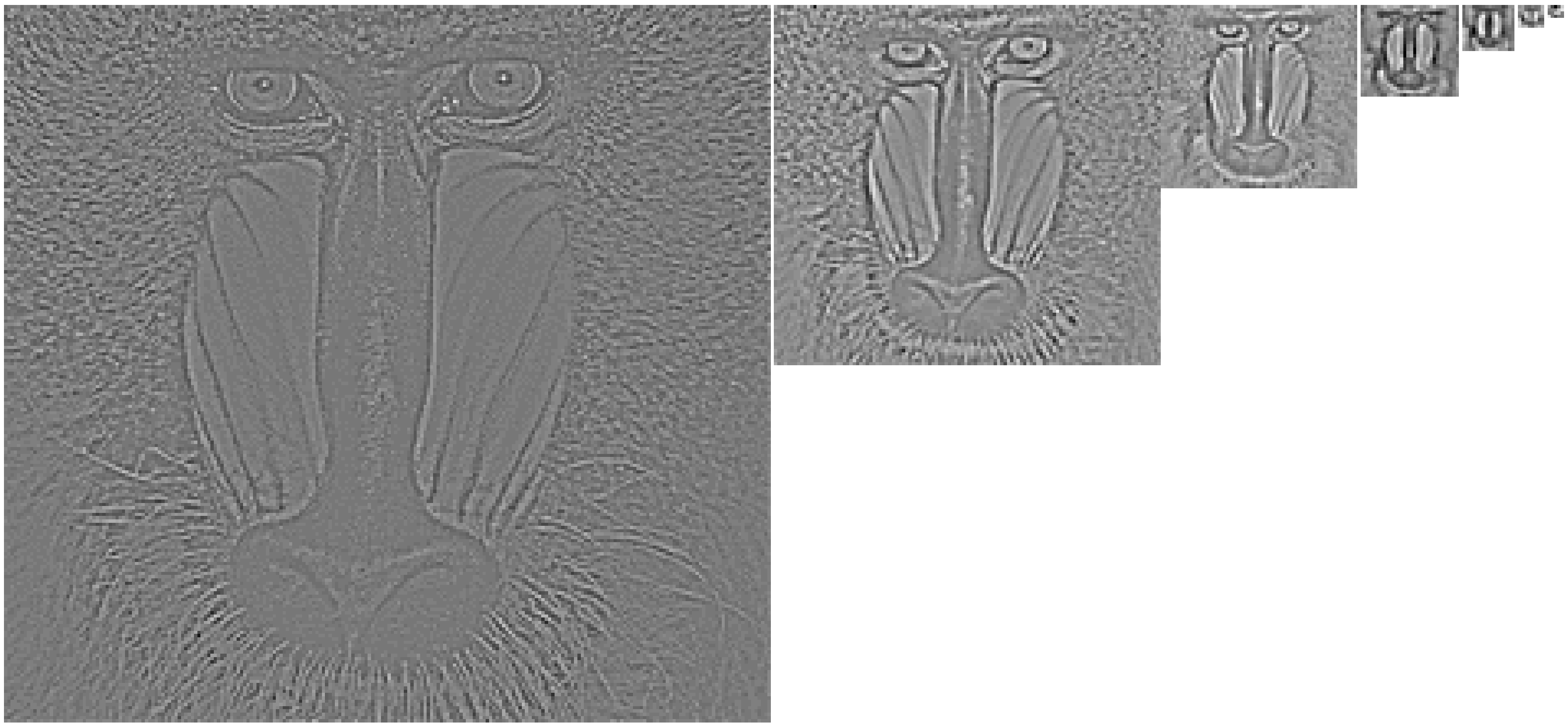
Low pass filtered

High pass filtered

Band-limited Images (Laplacian Pyramid)

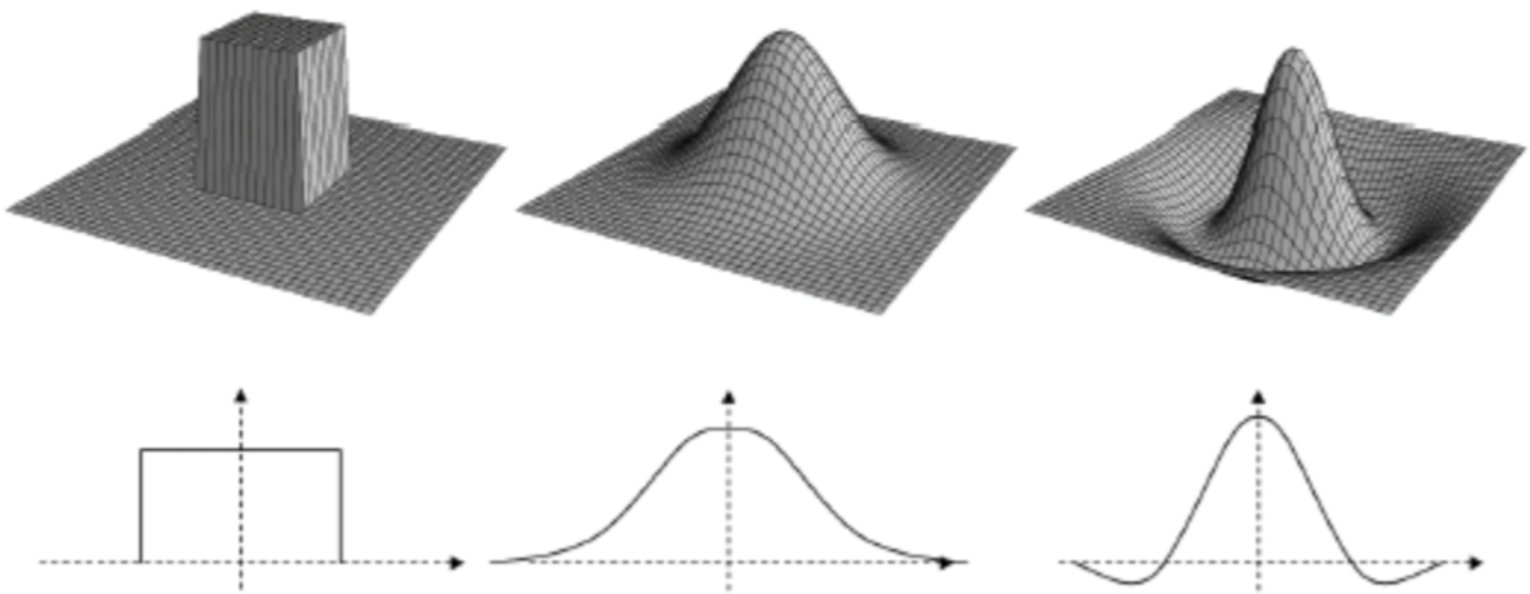


Band-limited Images (Laplacian Pyramid)



2D Filter Separability

- Visualizing 2D filters from their 1D counter part



Box Filter Gaussian Filter High Pass Filter

2D Filter Separability

- **Separability**
  - 2D filter h ($p \times q$) is separable if h can be separated into two 1D filters a and b such that

$$h[i][j] = a[i] \times b[j]$$

  - Convolving image with h is same as convolving its rows with a and then its columns with b

2D Filter Separability

- **Advantage**
  - Separable filters can be implemented more efficiently
  - Convolving with h
    - *Number of multiplications* $= 2pqN$
  - Convolving with a and b
    - *Number of multiplications* $= 2(p+q)N$