Fundamentals

Data

Outline

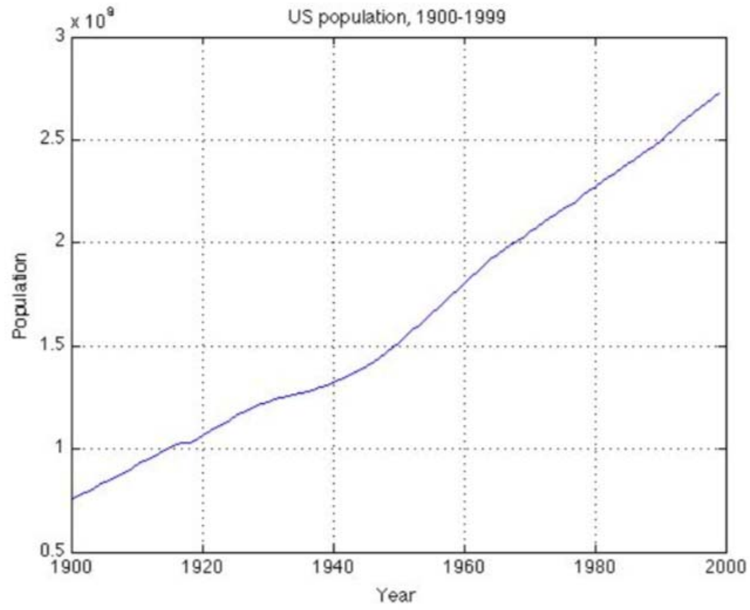
- Visualization
- Discretization
  - Sampling
  - Quantization
- Representation
  - Continuous
  - Discrete
- Noise

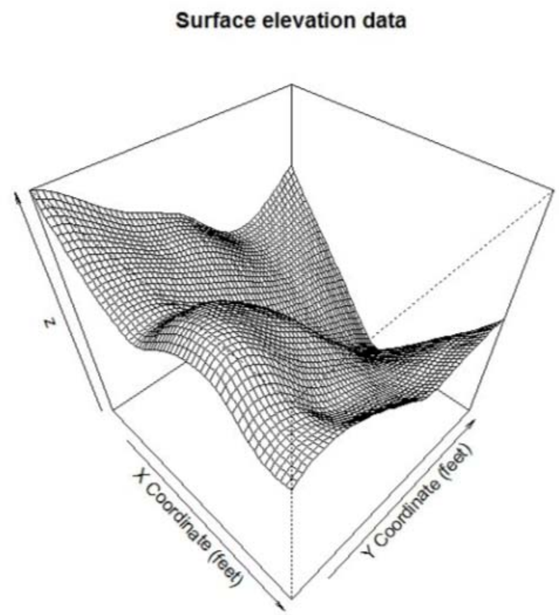
Data

- Data : Function dependent on one or more variables.
- Example
  - Audio (1D) - depends on time t - $A(t)$
  - Image (2D) - depends on spatial coordinates x and y - $I(x, y)$
  - Video (3D) - depends on spatial coordinate (x,y) and time t - $V(x, y, t)$

Visualization

- Plot of dependent variable with respect to independent ones
  - 2D plot is a *height field*





Visualization

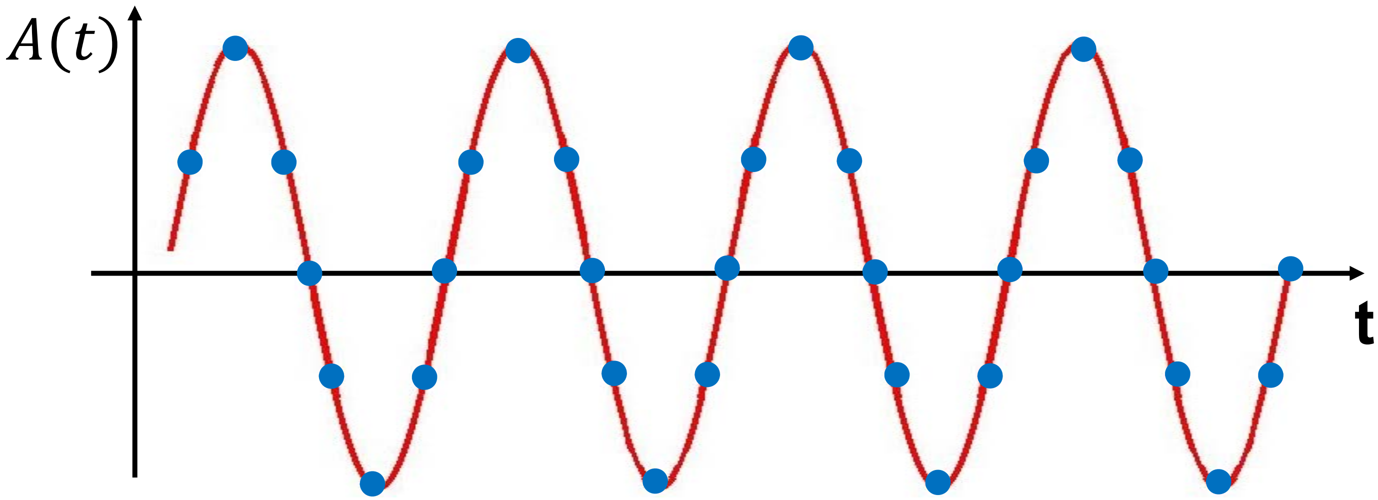
- Other kinds of visualizations
- Color image : three color channels: $R(x, y), G(x, y) and\ B(x, y)$

Discretization

- Data exists in nature as a continuous function.
- Convert to discrete function for digital representation
  - ***Discretization***
- Two concepts
  - Sampling
  - Quantization

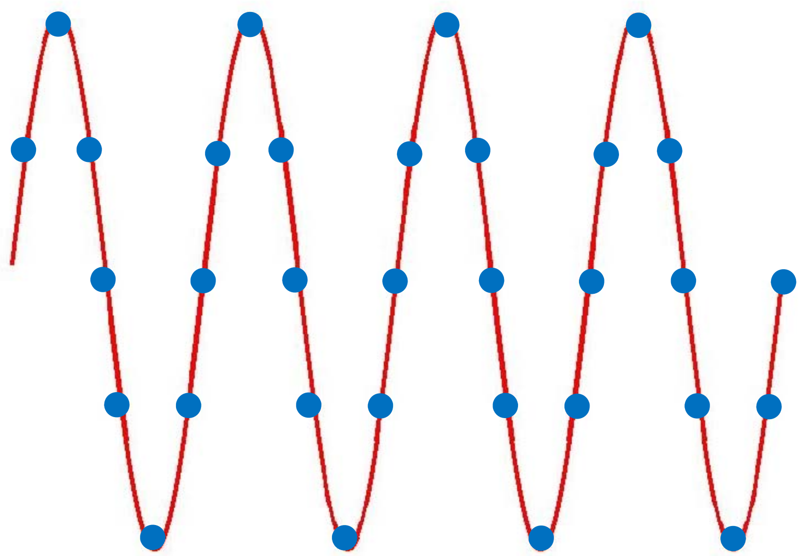
Sampling

- Set of values of continuous $f(t)$ at specific values of t.
- Reduces continuous function $f(t)$ to discrete form

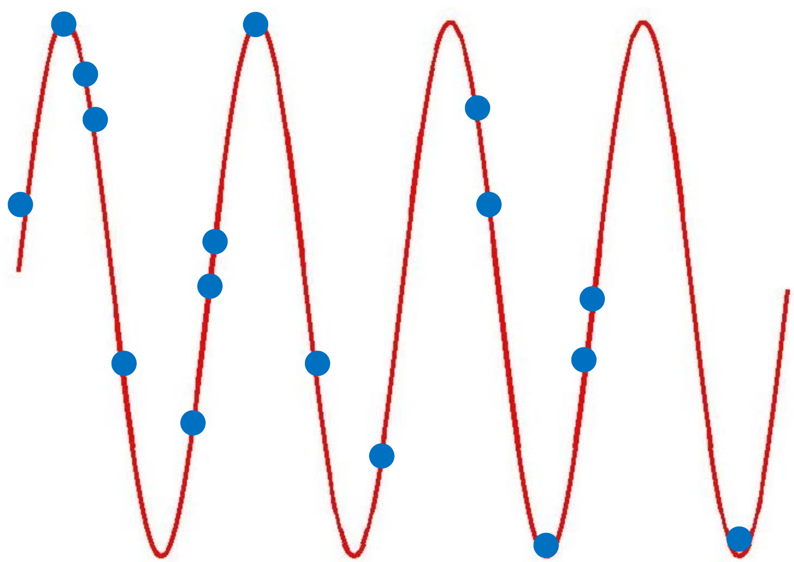


Sampling

Uniform vs Non-uniform sampling



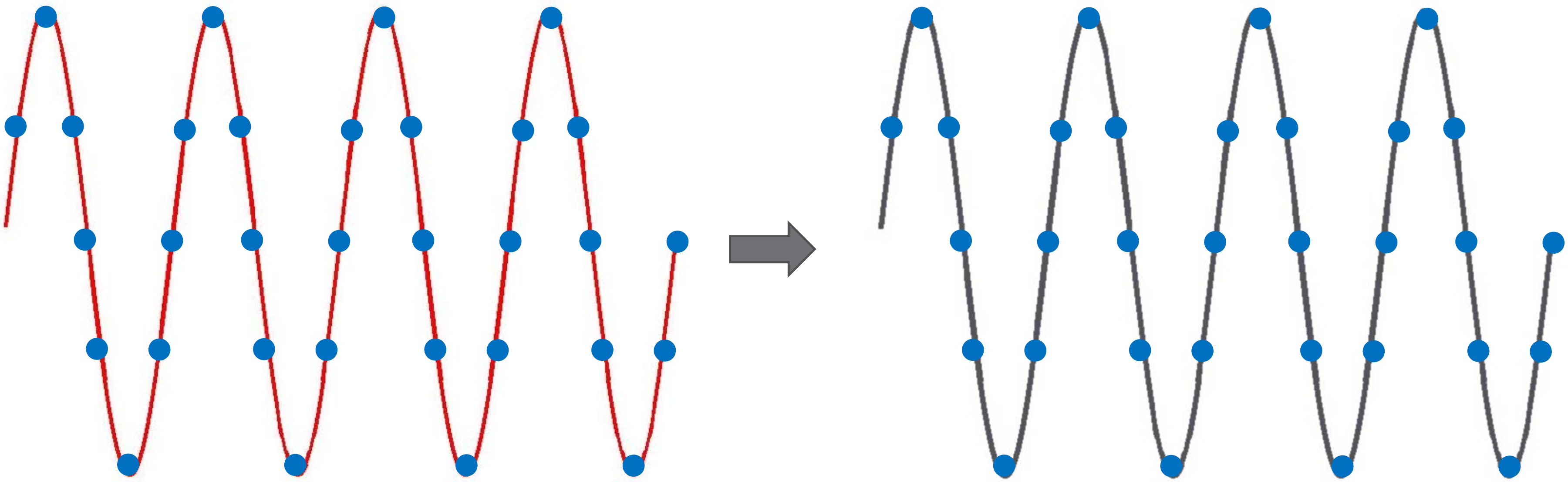
Uniform



Non-uniform

Reconstruction

- Get the continuous function from the discrete function

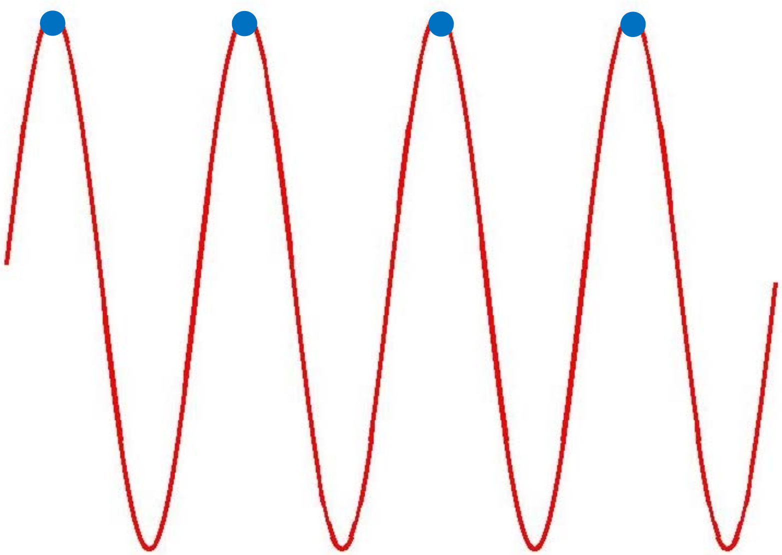


Sampling

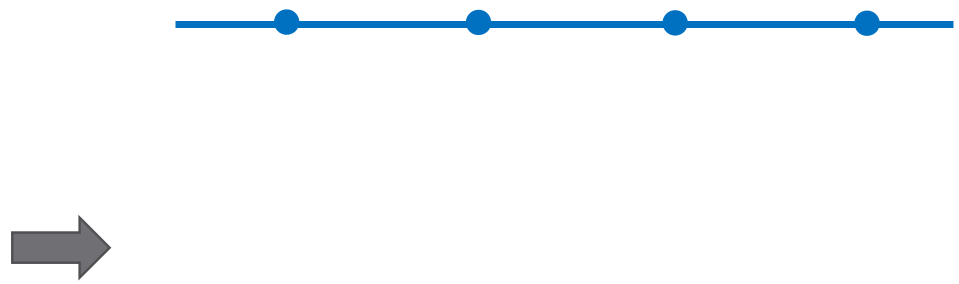
Correct Reconstruction

Reconstruction

- Accurate reconstruction needs adequate samples



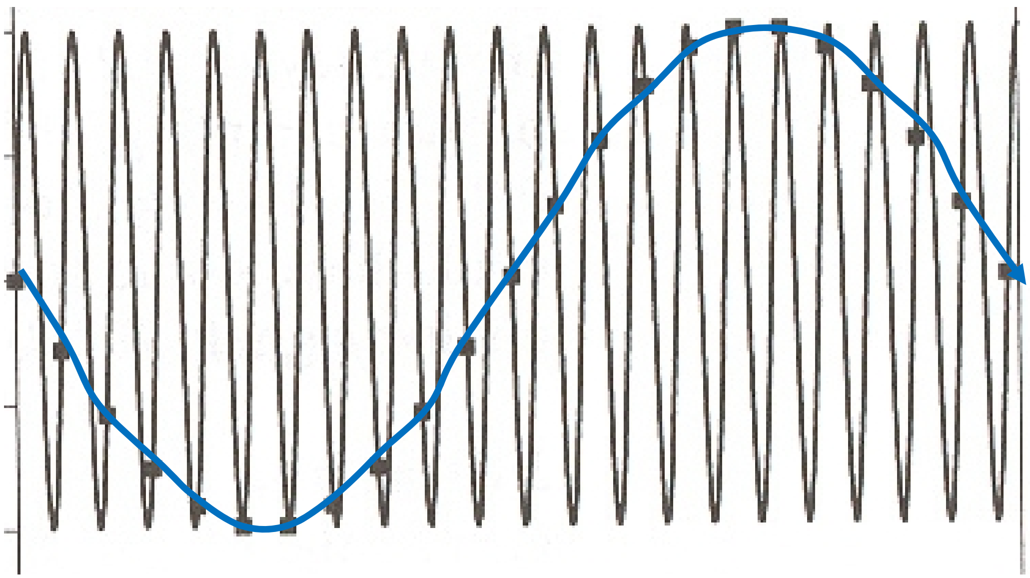
Sampling



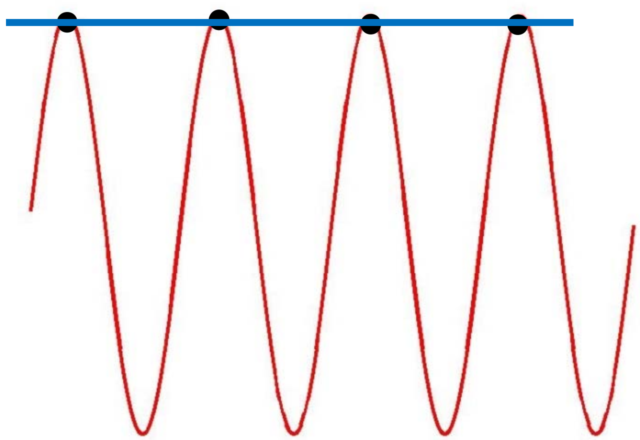
Incorrect Reconstruction

Aliasing

- Incorrect representation of some entity



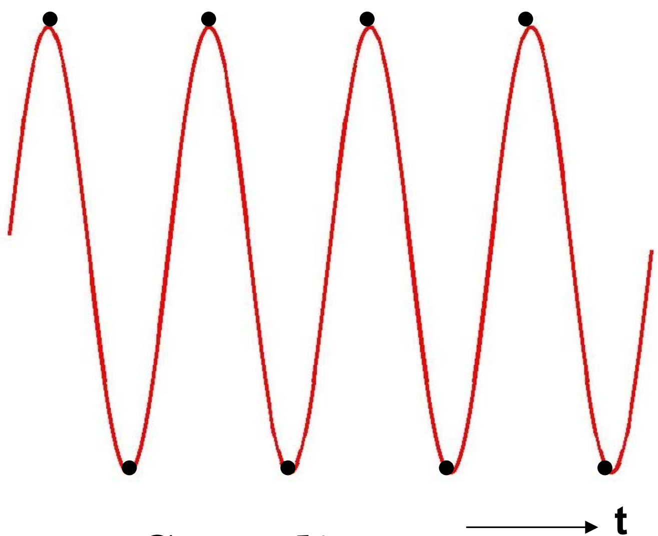
A much lower frequency



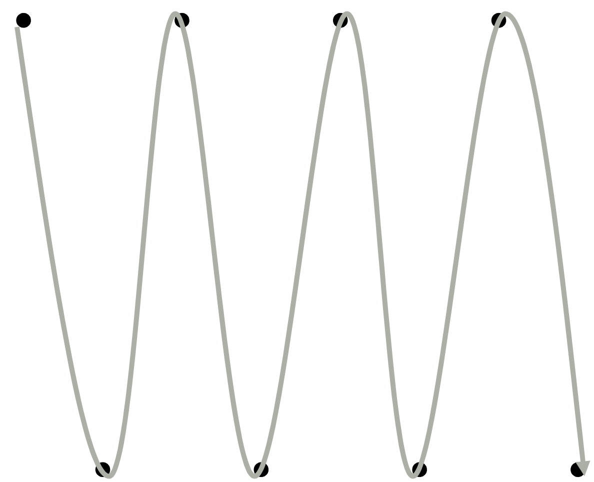
Zero frequency

Nyquist Sampling Rate

- By sampling ***at least*** twice the frequency (2 samples per cycle), signal can be reconstructed correctly.



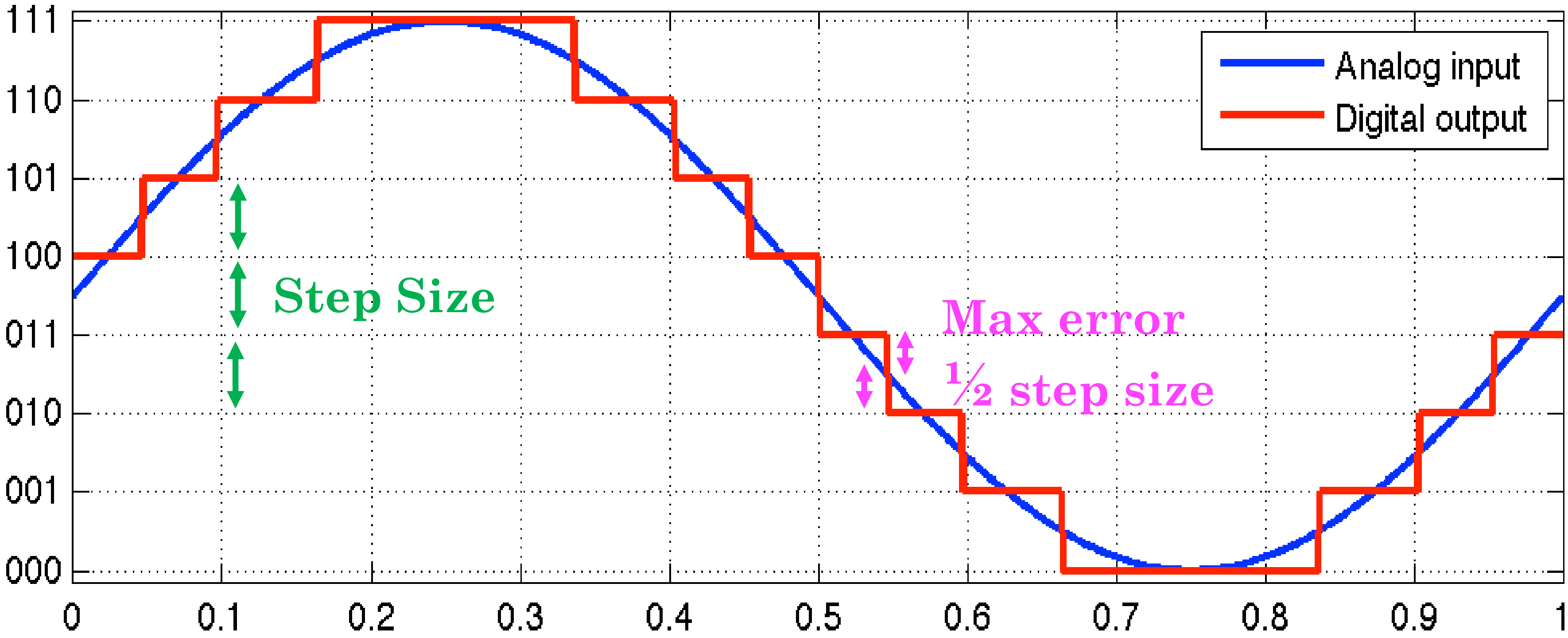
Sampling



Correct Reconstruction

Quantization

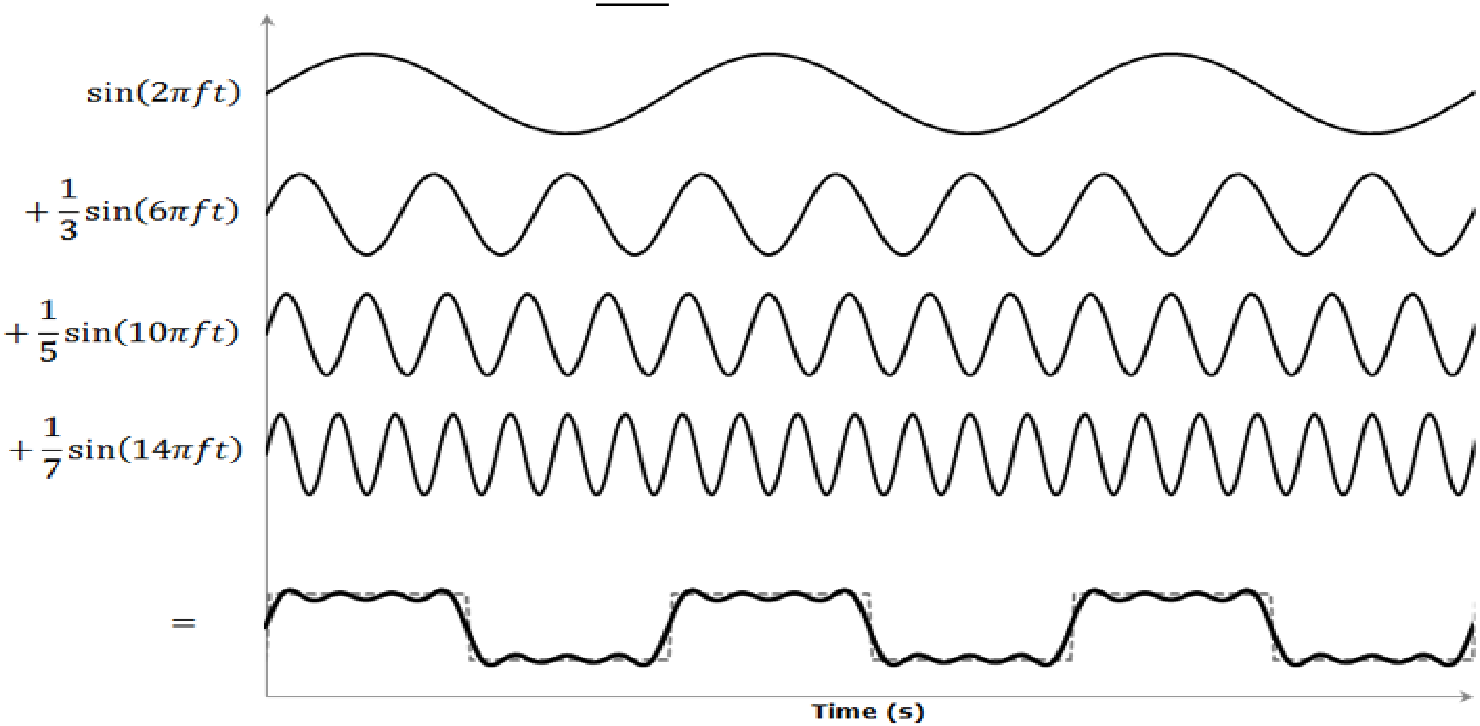
- A analog signal can have any value of infinite precision
- Digital signal can only have a limited set of value



An Alternative Representation

- ***frequency domain representation***
  - A signal is a linear combination of sine or cosine waves

∞



- Sign … or cosi…

Representation

- ***Explicit Representation***

$$y = mx + c$$

- ***Implicit Representation***

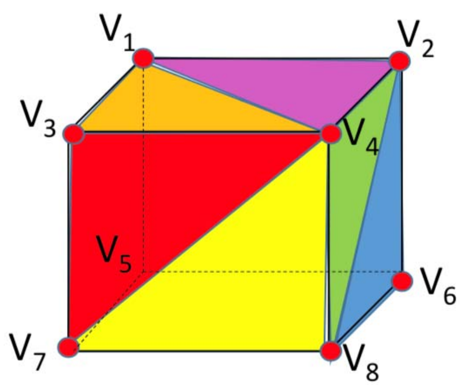
$$ax + by + c = 0$$

- ***Parametric Equation***
  - Using one or more parameters
  - ***Example:*** point p on a line segment between two points P and Q

$$p = P + t(Q - P), \qquad 0 \le t \le 1$$

Discrete Representation

- A 3D cube defined by a set of quadrilaterals or triangles
  - This is called ***Mesh***
- The entities that make up the mesh (e.g. lines, triangles or quadrilaterals) are called the ***primitives***.



Vertices

V_1 (x_1, y_1, z_1)

V_2 (x_2, y_2, z_2)

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•

•

V_8 (x_8, y_8, z_8)

Triangles

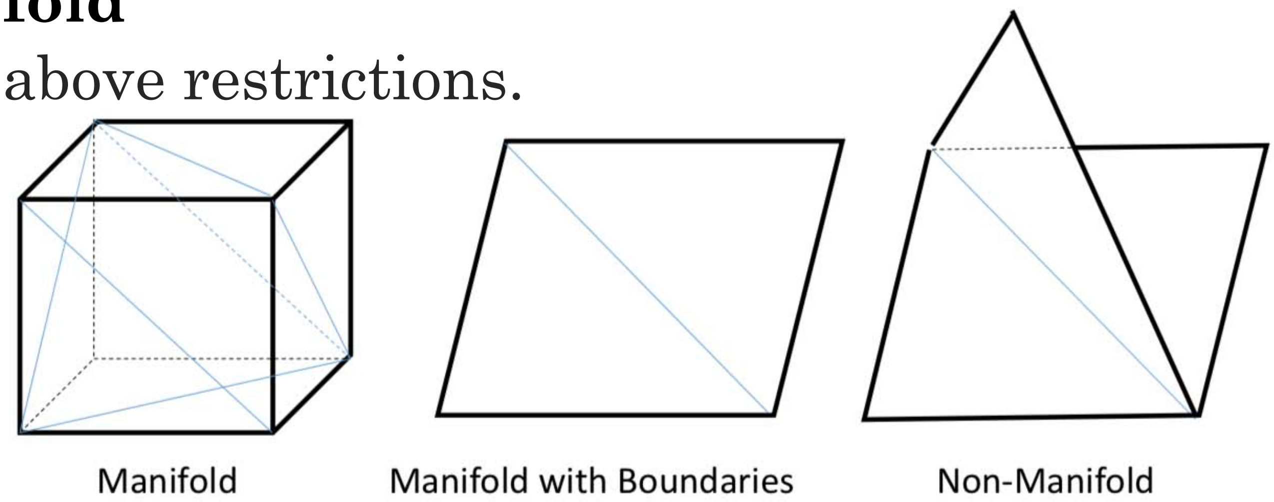
V_1, V_2, V_4; V_1, V_4, V_3; V_3, V_4, V_7;
V_4, V_8, V_7; V_4, V_2, V_8; V_2, V_6, V_8;
V_1, V_5, V_2; V_2, V_5, V_6; V_3, V_5, V_1;
V_3, V_7, V_5; V_5, V_7, V_6; V_6, V_7, V_8

Properties

- ***Geometric Properties:***
  - Position
  - Normal
  - Curvature
- ***Topological Properties:*** remains invariant to changes in geometric properties
  - Connectivity or Adjacency
  - Dimension
  - Manifold / non-manifold
  - Euler characteristic/Genus

Manifold Definitions

- **Manifold**
  - Every edge has exactly two incident triangles.
- **Manifolds with boundaries**
  - Every edge has either one or two incident triangles.
- **Non-manifold**
  - Not with above restrictions.



Euler Characteristic

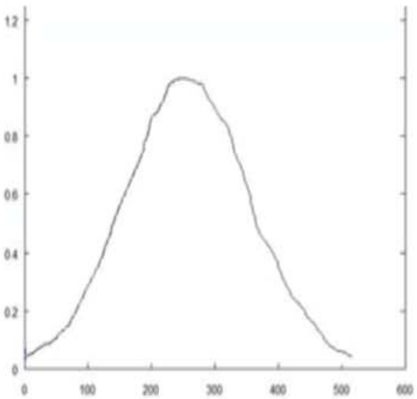
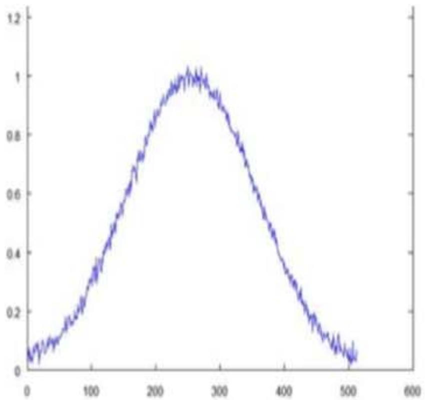
- e = V-E+F (V: Vertices, E: Edges, F: Faces).
- Cube has 8 vertices, 12 edges, 6 faces
  - e = V-E+F = 8-12+6 = 2
- Changing geometric properties keeps Euler characteristic invariant
  - Such as adding edges, vertices

Genus

- (Naïve) Number of “handles”.
- Relationship between e and g: e=2-2g
  - Sphere, cube g=0
  - torus, coffee cup g=1
- Going from coffee cup to torus
  - Changing only geometric properties

Noise

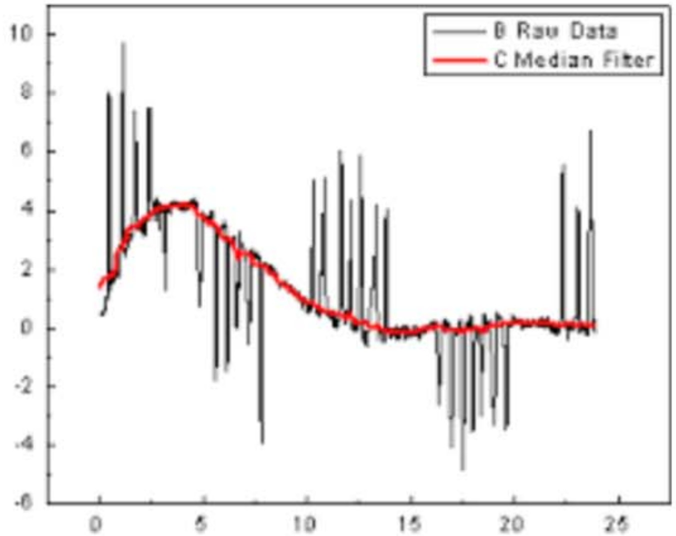
- Addition of random values at random locations in the data.
  - ***Random noise***



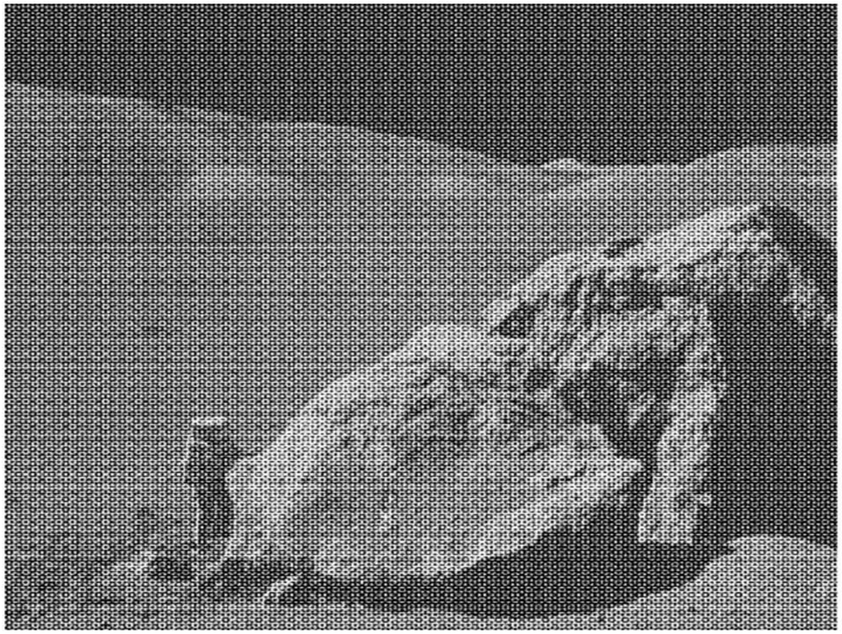
Noise

- ***Outlier Noise***
- An example of such noise is *salt and pepper* noise
- Can be solved by Median filter




Noise

- Some noise look random in spatial domain but can be isolated to a few frequencies in spectral domain.
- Can be removed by Notch filter.

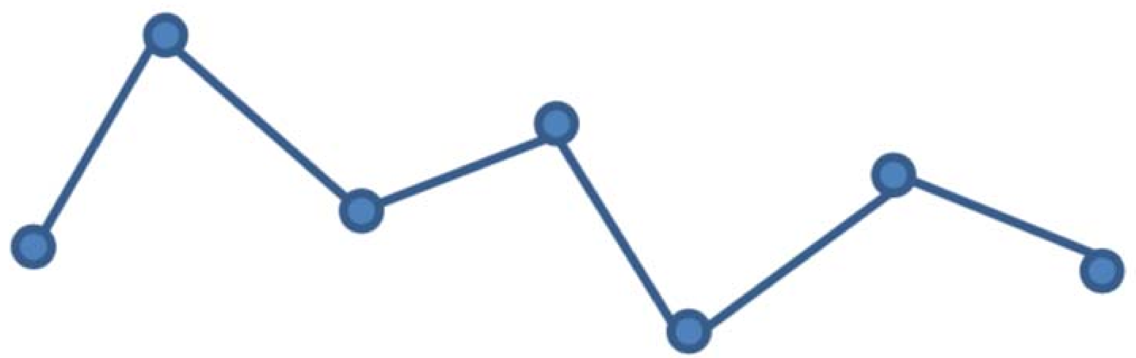


Technqiues

- Interpolation
    - Linear Interpolation
    - Bilinear Interpolation
- Geometric Intersections

Interpolation

- Estimate function for values which it has not been measured.
- **Linear interpolation:**
  - Assuming a line between samples.
  - Change abruptly at sample points
  - C^0 continuity



Interpolation

- **Non-linear interpolation:**
  - A smooth curve passes through samples
  - First derivative continuous - C^1 continuous
  - Second derivative continuous - C^2 continuous

Linear Interpolation

- Point V on the line segment V_1V_2 is given by

$$V = \alpha V_1 + (1-\alpha)V_2, \qquad 0 \leq \alpha \leq 1$$

- Example: Linear interpolation of color at point V

$$C(V) = \mathrm{C}(\alpha V_1 + (1-\alpha)V_2) = \alpha C(V_1) + (1-\alpha)C(V_2)$$

Bilinear Interpolation

- 2D Data
- Interpolating in one direction followed by interpolating in the second direction.

$$F(Q) = (1-\alpha)C + \alpha D$$

$$F(R) = (1-\alpha)A + \alpha B$$

$$F(P) \quad = \quad F(Q)\beta + F(R)(1-\beta)$$