Convolution

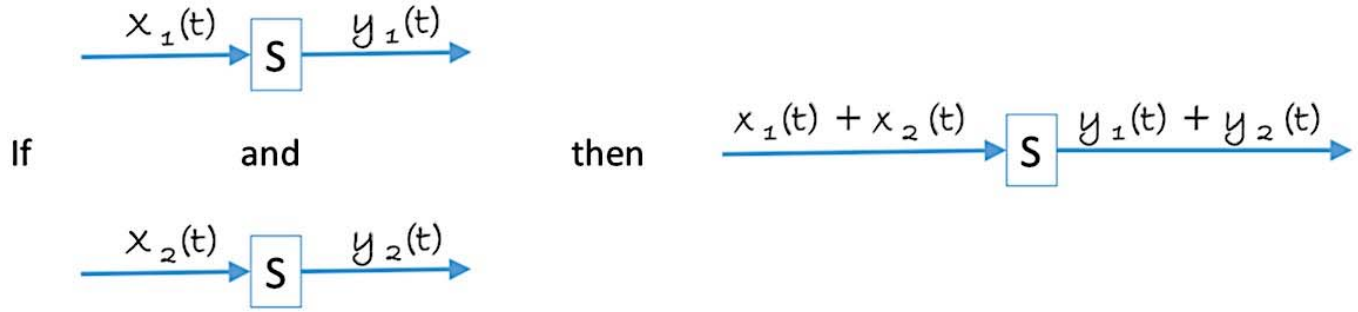
Outline

- Linear Systems
  - Properties
- Response of Linear System
- Convolution
  - Properties

Properties of Linear System

1. **Homogeneity:**

2. **Additivity:**

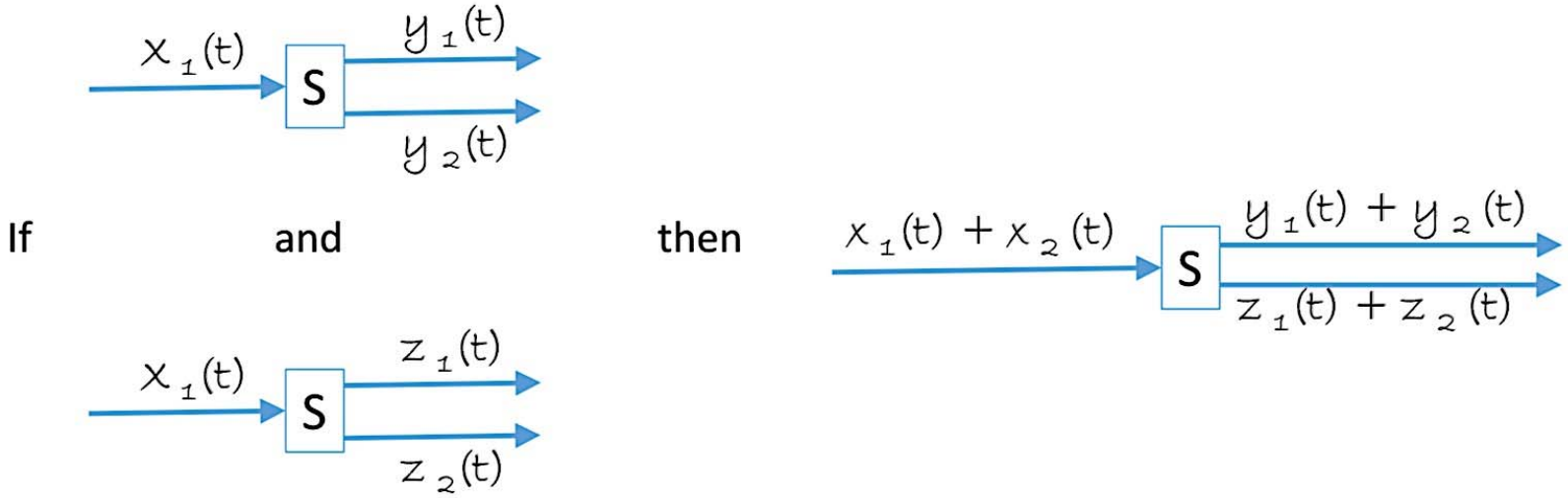


3. **Shift Invariance:**

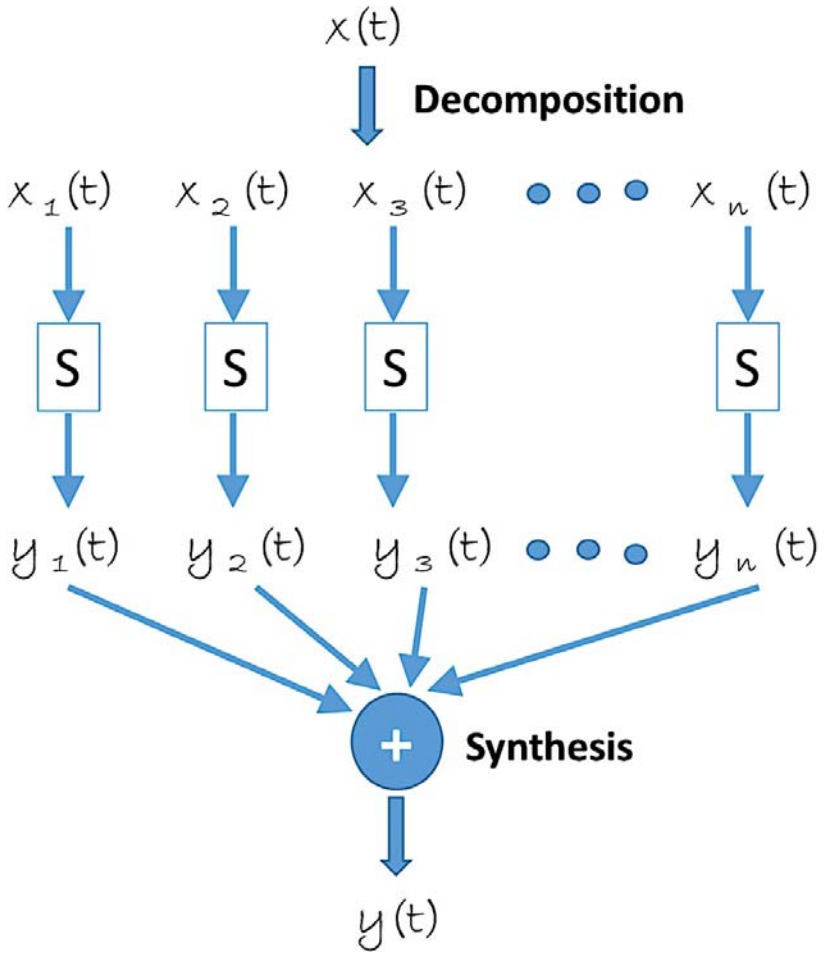
Other Properties of Linear Systems

1. **Commutative:**

2. **Superposition:** If each generates multiple outputs, Then the addition of inputs generates an addition of outputs.

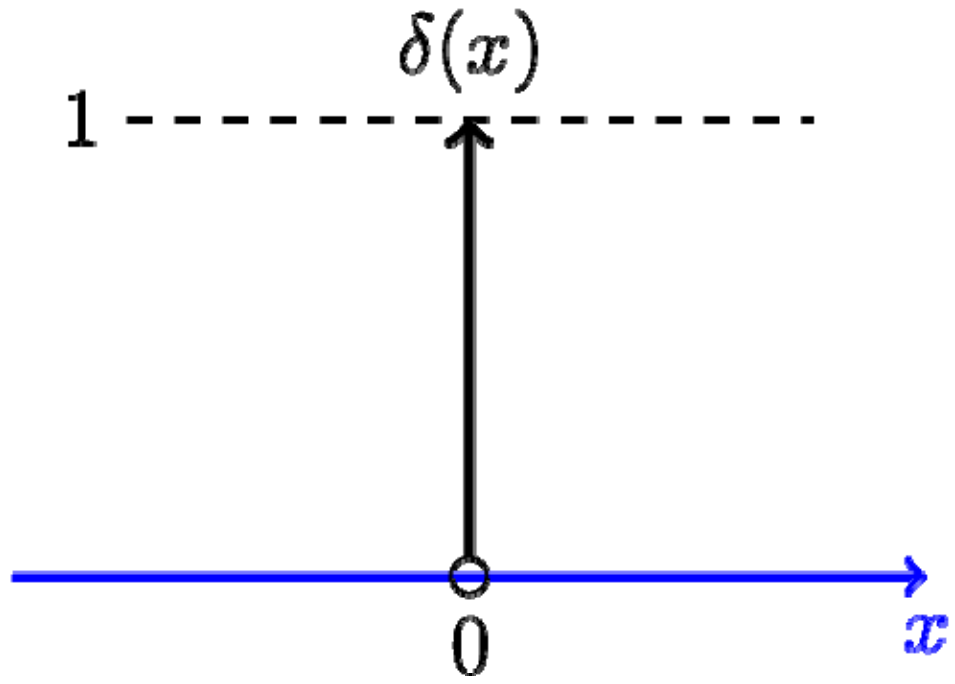


Decomposition - Synthesis



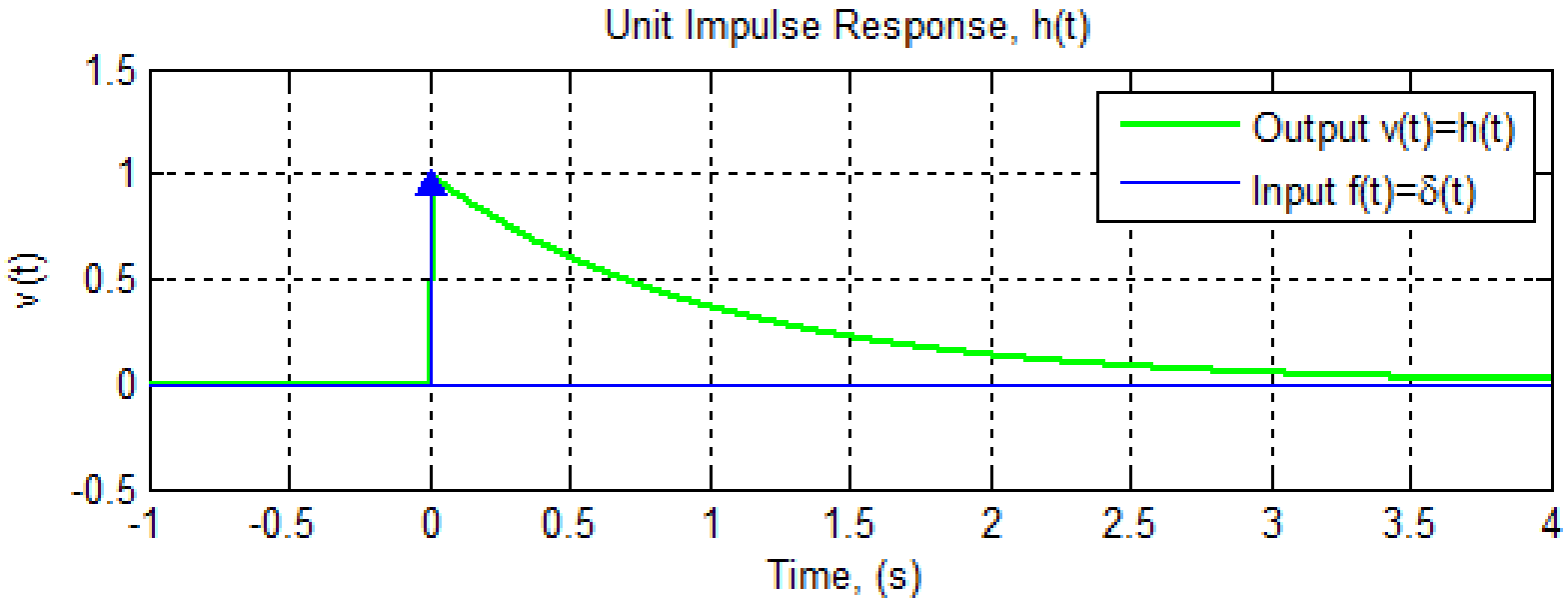
Response of Linear System

- ***Impulse:*** Signal with only one non-zero sample.
- **Delta** (δ[t]) is an impulse with non-zero sample at t = 0



Response of Linear System

- ***Impulse response*** h[t]
  - output of the system to the input δ[t].

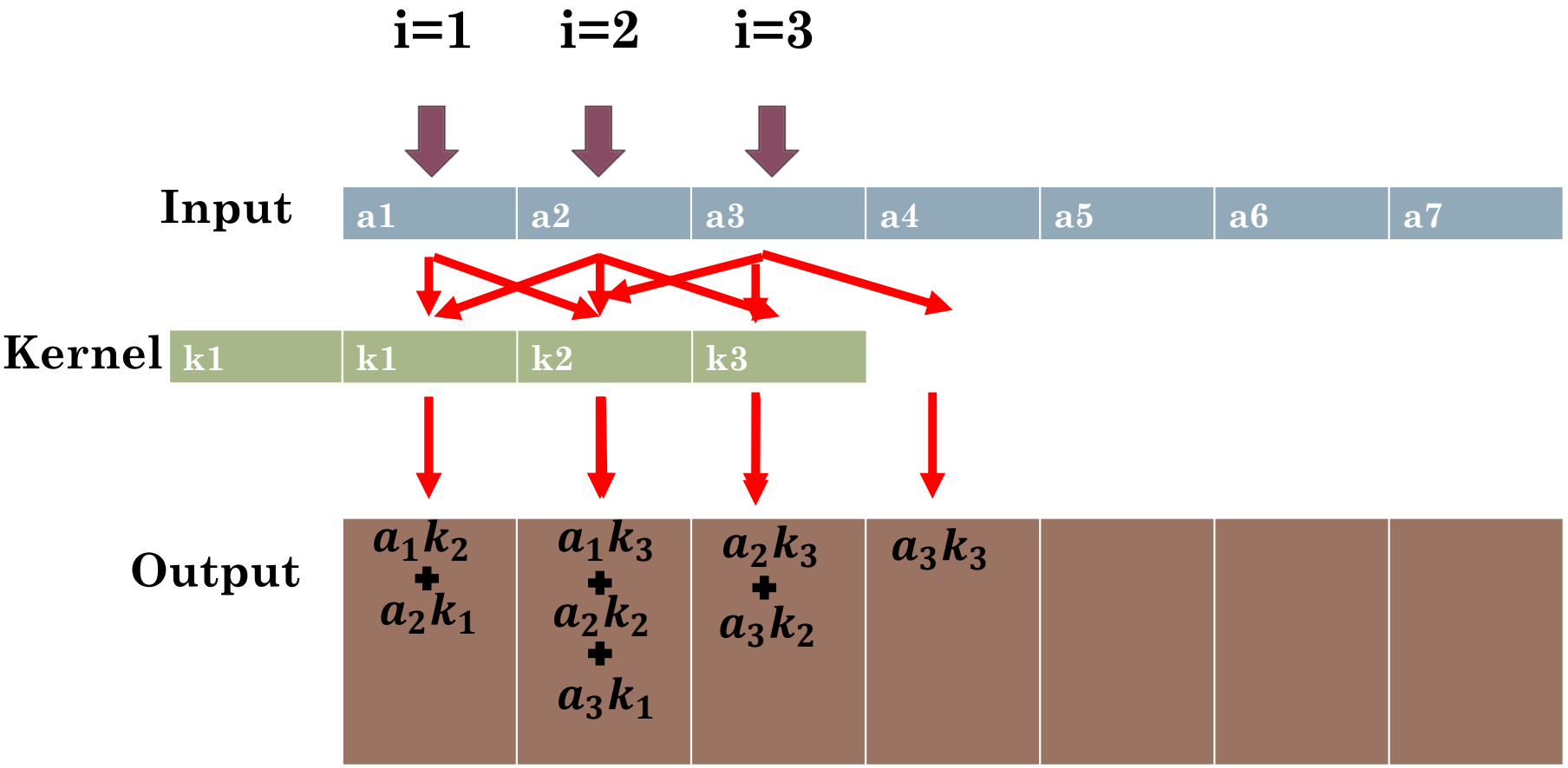


Response of Linear System

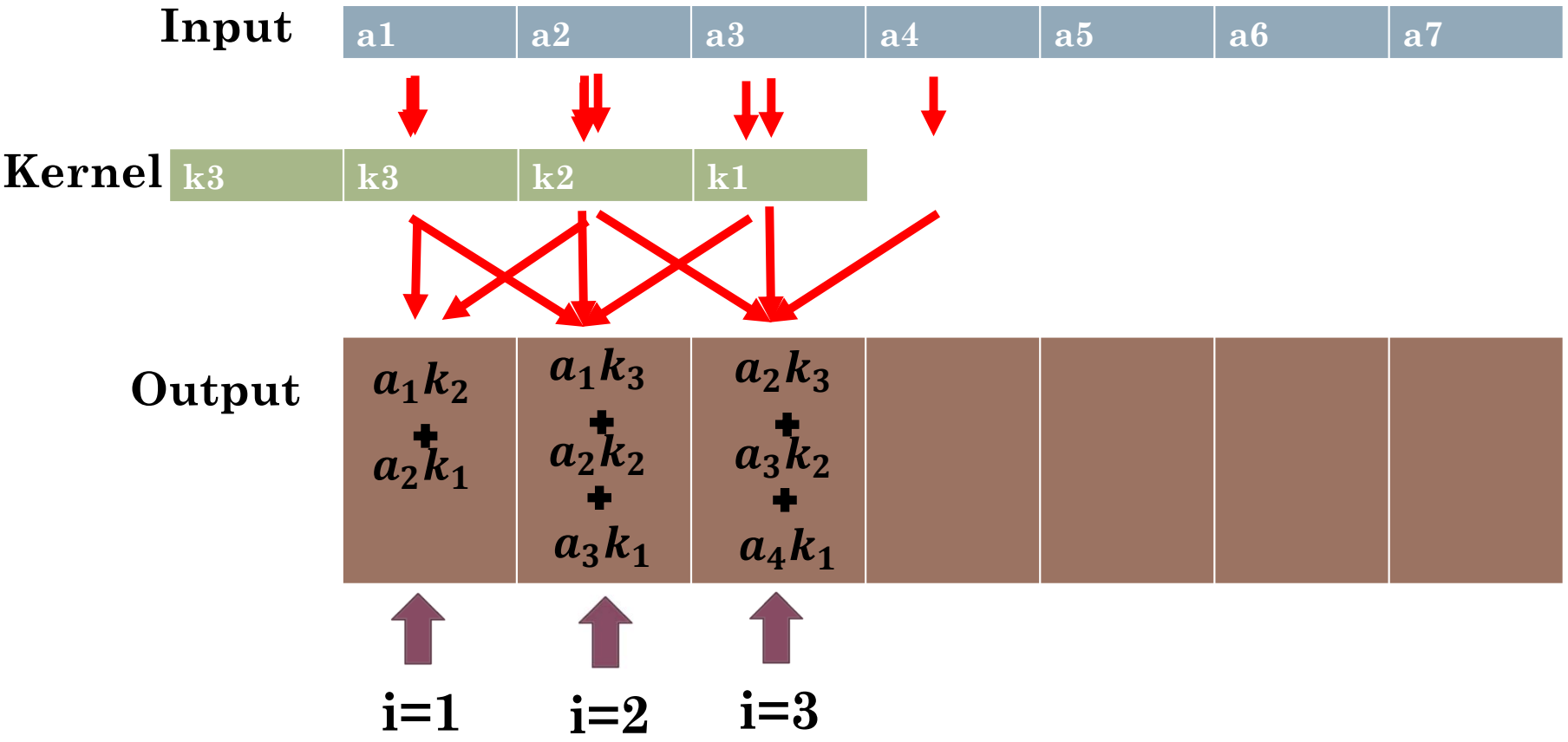
- ***Impulse response*** h[t]
  - output of the system to the input δ[t].
- ***Convolution:*** Response of a linear system with impulse response, h, to a general signal

$$R = \sum_{l=1}^{n} x[l]h[t-l] = x[t] \star h[t]$$

Convolution – Input side

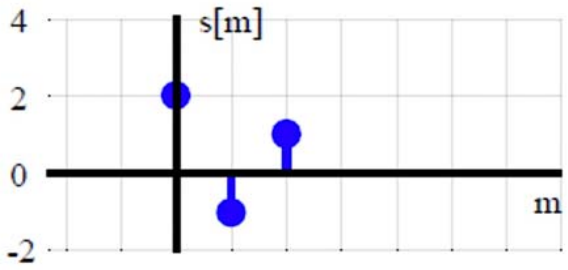


Convolution – Output side

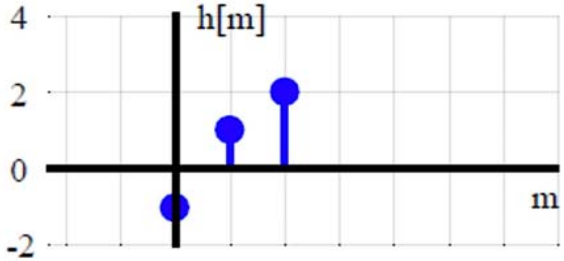


Convolution

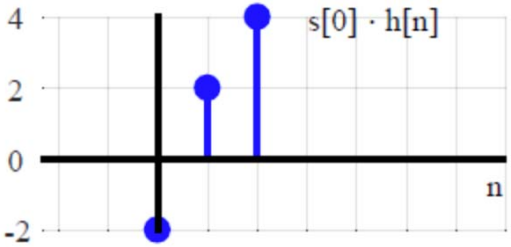
s[m]



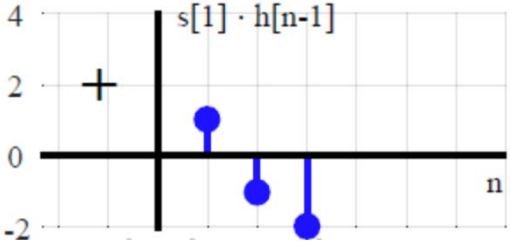
h[m]



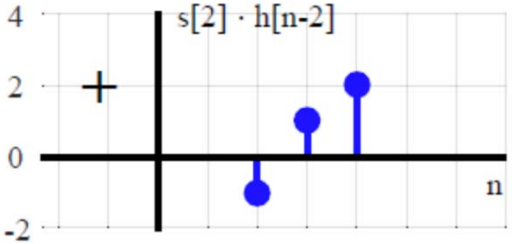
s[0].h[n]

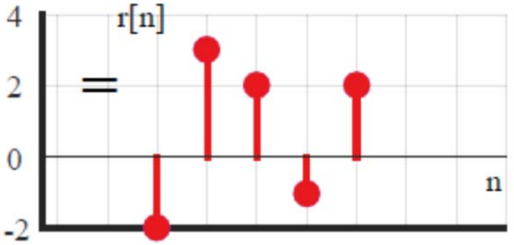


s[1].h[n-1]

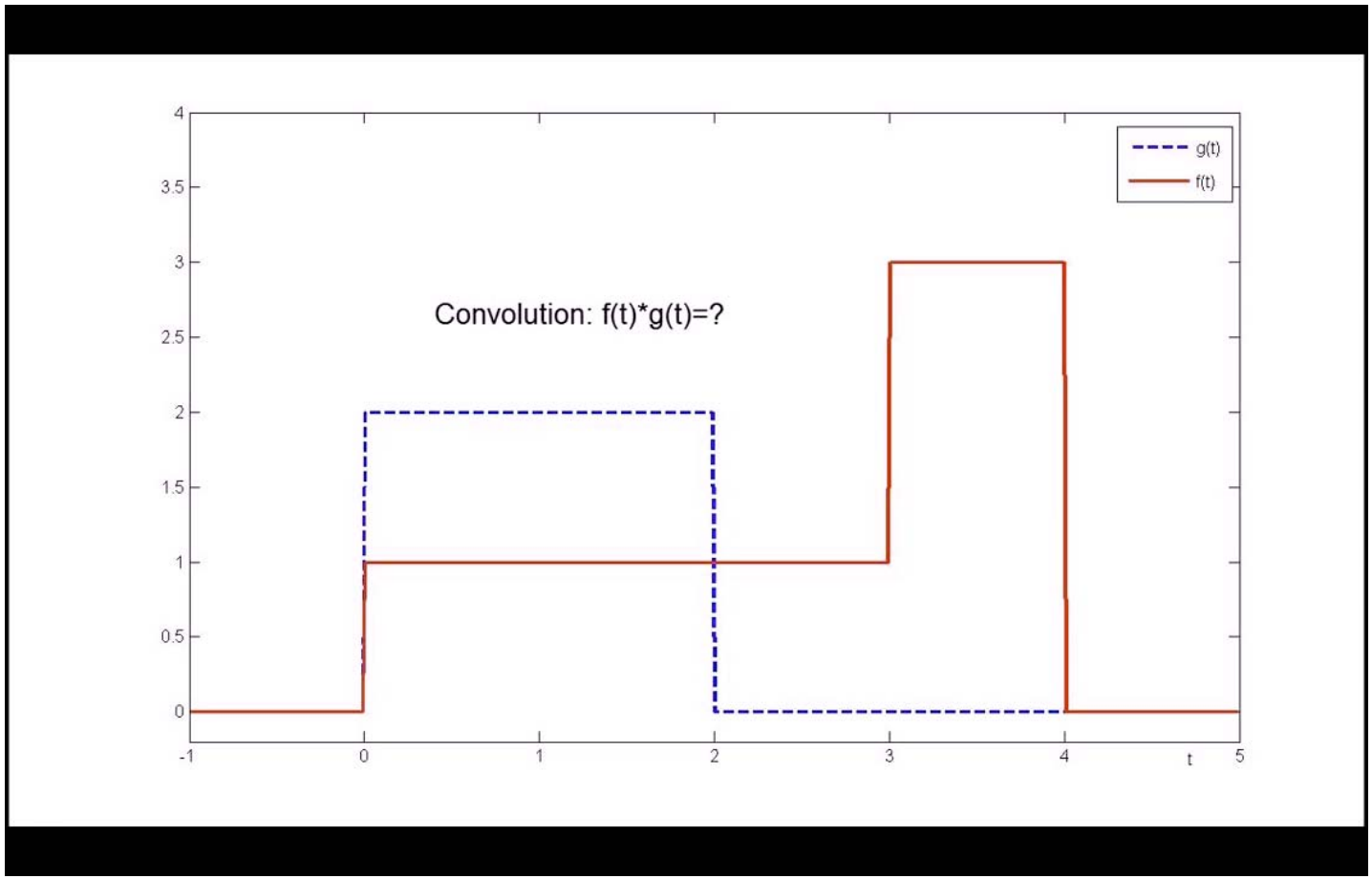


s[2].h[n-2]

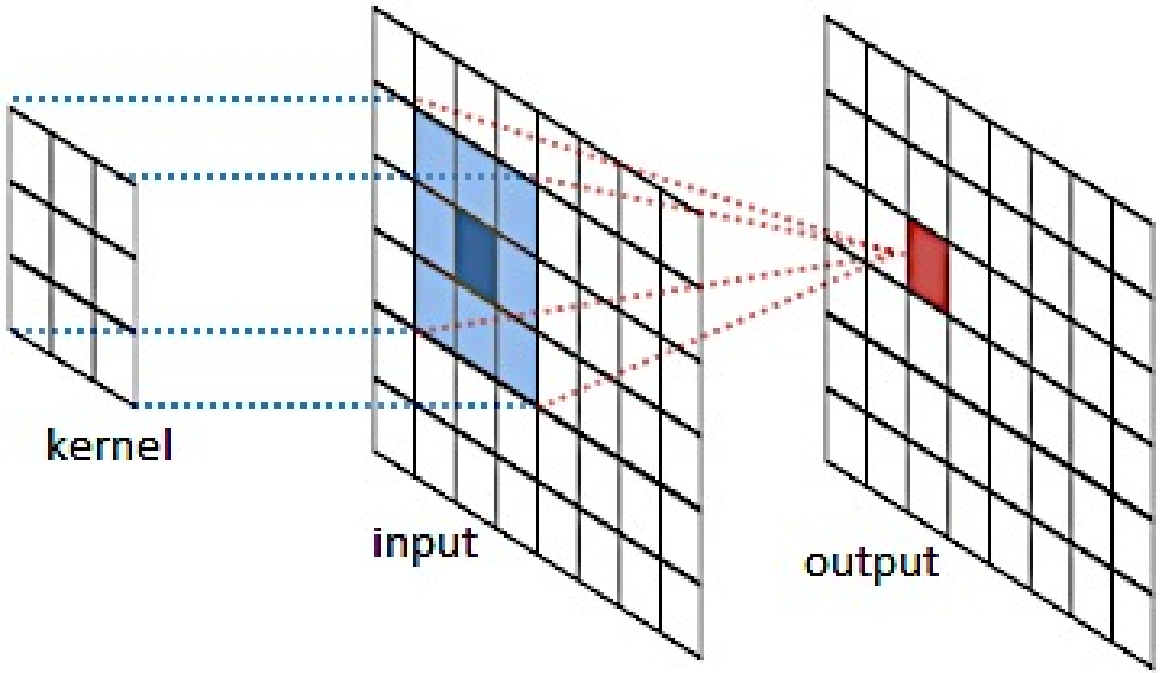




Visualization of Convolution



2D Convolution



Properties of Convolution

- *All pass system*

$$x[t] \star \delta[t] = x[t]$$

- *Amplifier* (k>0) / *attenuator* (k<0)

$$x[t] \star k\delta[t] = kx[t]$$

- *Delay*

$$x[t] \star \delta[t+s] = x[t+s]$$

Properties of Convolution

- *Conducive*

$$a[t] \star b[t] = b[t] \star a[t].$$

- *Associative*

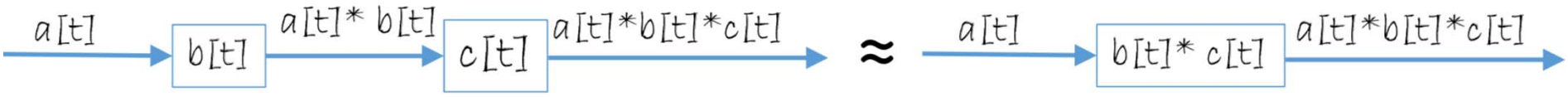
$$(a[t] \star b[t]) \star c[t] = a[t] \star (b[t] \star c[t]).$$

- *Distributive*

$$a[t] \star b[t] + a[t] \star c[t] = a[t] \star (b[t] + c[t])$$

Properties of Convolution

- Cascading convolutions



- Combination of parallel convolutions