

Pop Quiz (Week 4) Solution

1) 4 by 4

2) Fundamental rigid body transformations are translation and rotation. Translation has 3 degree of freedom which are the amount of translation in x, y and z direction.

Rotation around an arbitrary vector has 7 degree of freedom which are 3 degree of freedom for the direction of the vector and 3 degree of freedom for translation of the vector and 1 degree of freedom for amount of rotation around this vector.

3) A)

$$\begin{bmatrix} \cos(45) & -\sin(45) & 0 & 0 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 2 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(45) & -\sin(45) & 0 & 0 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 2\frac{\sqrt{2}}{2} & 2\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4)

We already know the up vector and the normal of the image plane. So by calculating the cross product of these two we can find x axis

$$x = (\text{image plane normal}) \times (\text{up vector}) = (0,1,1) \times (1,1,0) = (-1,1,-1)$$

now we have x axis and normal of image plane is parallel with z axis so by calculating the cross product of these two we can find y axis

$$y = (-1,1,-1) \times (0,1,1) = (1,-1,2)$$

now we have x, y and z axis. Now first we should normalize them . so we have:

$$x = \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right)$$

$$y = \left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right)$$

$$z = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$

So the view transformation is:

$$\begin{bmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & 0 & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$