Scan Converting Line

Equation of a line :-

Explicit :- $y = mx + c$

How to find this given two points?

$A = (5, 8)$. $B = (9, 11)$

$8 = 5m + c$ --- (1)

$11 = 9m + c$ --- (2)

(2) - (1) gives

$4m = 3$

$\therefore m = 3/4$

Plugging $m = 3/4$ in (1)

$8 = 5 \cdot \frac{3}{4} + c$

$\therefore c = 17/4$

$\therefore y = \frac{3}{4}x + \frac{17}{4}$, $3x - 4y + 17 = 0$

(Implicit)

DDA Method

Case I

$0 \leq m \leq 1 \Rightarrow \underset{(m)}{\text{Angle}} \leq 45°$

Let us consider two pts (x_1, y_1) and (x_2, y_2) lying on the line.

$y_2 = mx_2 + c$
$y_1 = mx_1 + c$

$$\therefore \frac{y_2 - y_1}{x_2 - x_1} = m = \frac{\Delta y}{\Delta x} \quad \text{———} \quad (3)$$

DDA method essentially calculates Δy for $\Delta x = 1$

$\therefore$ Everytime we step 1 unit in x direction, how much should we advance in y direction.

From (3) we know that if $\Delta x = 1$

$\therefore \Delta y = m \Delta x$

Algorithm

$A = (5, 8) \qquad B = (9, 11)$.

Mark A. , $x = A_x$, $y = A_y$

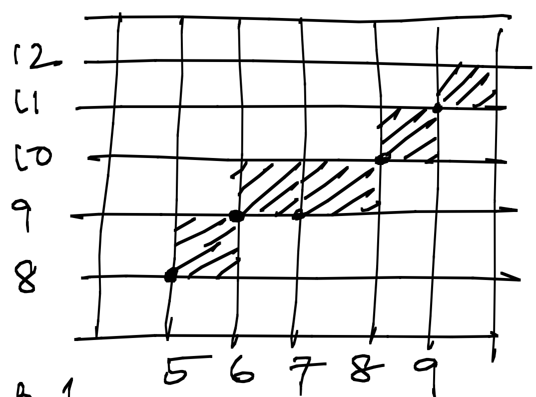
$x = x + \Delta x$

$y = y + m$

Mark $(x, \text{round}(y))$

Do this till the y of B is reached.

* Pixels accessed by their bottom left corner



<u>Step 1</u>

$x = 5, \quad x + 1 = 6$

$y = 8 + \frac{3}{4} = 8.75$

Mark $(6, 9)$

<u>Step 2</u>

$x = 7$

$y = 8.75 + 0.75 = 9.5$

Mark $(7, 9)$

<u>Step 3</u>

$x = 8$

$y = 9.5 + 0.75 = 10.25$

mark $(8, 10)$

<u>Step 4</u>

$x = 9$

$y = 10.25 + 0.75 = 11$

mark $(9, 11)$

$x = 9, \quad y = 11$

$\therefore$ Reached B & hence algorithm terminates.

Problems

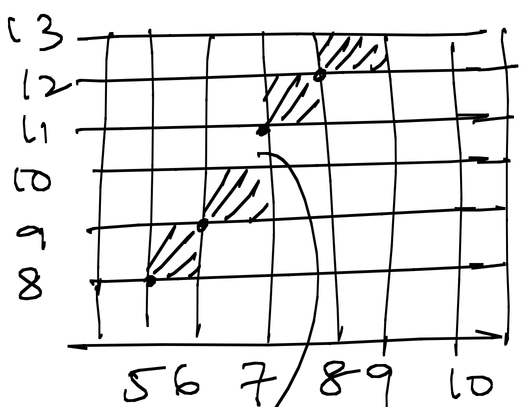
What if $m > 1$.

Say $A = (5, 8)$, $\quad B = (8, 12)$

$\therefore \; 8 = 5m + c$

$12 = 8m + c$

$\therefore m = \frac{4}{3} = 1.33$



Break or Gap in the line.

Step 1

$x = 6$

$y = 8 + \frac{4}{3} = 9.33$

Mark $(6, 9)$

Step 2

$x = 7$

$y = 9.33 + 4/3 = 10.66$

Mark $(7, 11)$

Step 3

$x = 8$

$y = 10.66 + 4/3 = 12$

Mark $(8, 12)$

What are acceptable lines?

a) No gaps between adjacent pixels → CONTINUOUS

b) Pixels close to ideal line → ACCURATE

c) Smooth looking → AESTHETIC

d) Even brightness in all orientation → UNIFORMITY IN THICKNESS

e) Same line for AB & BA → CONSISTENT.

How to handle different m in DDA?

<u>Case I</u> $m > 1$

a) Flip line about $y = x$
   ↳ Achieved by flipping x & y coordinates

b) Rasterize

c) Flip it back
   ↳ switch x & y coordinates again

<u>Case II</u> $-1 \leq m \leq 0$

a) Flip about x axis
   ↳ Negate y

b) Rasterize

c) Flip it back

Disadvantages of DDA

Rounding & finding y are Floating point operations, hence very expensive, especially since it needs to be done for every pixel.

Bresenham Method (Using Integer Arithmetic only)

Again, consider $0 \leq m \leq 1$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}$$

$$\therefore \quad y = mx + c$$

$$= \frac{dy}{dx} \cdot x + c$$

or $dx \cdot y = dy \cdot x + dx \cdot c$

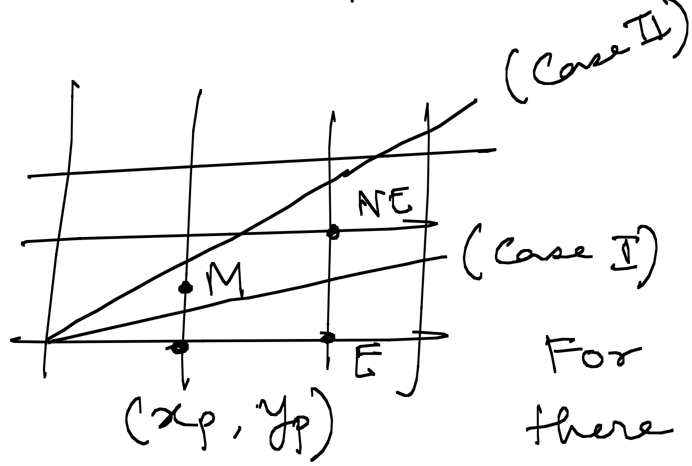
or $dy \cdot x + (-dx) \cdot y + dx \cdot c = 0$

or $Ax + By + C = 0 = f(x, y)$

where $A = dy \quad B = -dx \quad C = dx \cdot c$

Note: Essentially the implicit form of line equation.

Idea is simple.



Say at step n, we have marked (x_p, y_p)

For the line to be continuous there are only two choices for $(n+1)$ th step.

(a) $E \rightarrow (x_p+1, y_p)$

(b) $NE \rightarrow (x_p+1, y_p+1)$

Question is which one should we choose?

$$M = (x_p + 1, y_p + 1/2)$$

<u>Case I</u>:

If the line passes below M, choose E.

$\therefore$ M should be above line.

given by $F(M) < 0$

i.e. If $F(x_p + 1, y_p + 1/2) < 0$ choose E.

Case II

Line passes above M,

Choose NE

$\therefore$ M should be below the line.

$$F(M) \geq 0$$

$\therefore$ If $F(x_p + 1, y_p + \frac{1}{2}) \geq 0$

Choose NE.

$\therefore d = F(x_p + 1, y_p + 1/2)$ is the decisive factor.

Note: Decision made based on only comparisons.

More Optimizations

To make the method efficient, we want to compute d in step $(n+1)$ incrementally from d computed in step n.

$$d(\text{prev}) = F(x_p + 1, y_p + \frac{1}{2})$$

$$= A.(x_p + 1) + B.(y_p + \frac{1}{2}) + C$$

Case I

E is chosen.

$$\therefore d(\text{new}) = F(x_p+2,\ y_p+1/2)$$

$$= A\cdot(x_p+2) + B\cdot(y_p+1/2) + C$$

$$= A\cdot(x_p+1) + A + B\cdot(y_p+1/2) + C$$

$$= d(\text{prev}) + A$$

$$= d(\text{prev}) + dy$$

$\therefore$ Update d by adding only dy.

$\therefore$ Integer Operation.

Case II

NE is chosen.

$$d_{new} = F(x_p+2,\ y_p+3/2)$$

$$= A\cdot(x_p+2) + B\cdot(y_p+3/2) + C$$

$$= A(x_p+1) + B(y_p+1/2) + C + A + B$$

$$= d(\text{prev}) + A + B$$

$$= d(\text{prev}) + dy - dx$$

$\therefore$ Update d by $(dy - dx)$

Algorithm

At each step

If $d < 0$ choose E

else choose NE.

Update d.

Initialize d

Say the first pt is (x_0, y_0)

$$d = A(x_0 + 1) + B(y_0 + 1/2) + C$$

$$= Ax_0 + By_0 + C + A + B/2$$

Since (x_0, y_0) is on the line,

$$d = A + B/2 = dy - \frac{dx}{2}$$

Note that this is not integer.

But starting with $2d$ is fine too.
Since we are checking the sign of d, $2d$ is not going to make a difference since sign remains same with scaling.

$\therefore 2d = 2dy - dx \rightarrow$ Now a integer.

Also note that you have to also scale the increments from d(prev) to d(new).

If E is chosen

$$2d(new) = 2d(prev) + 2dy$$

If NE is chosen

$$2d(new) = 2d(prev) + 2dy - 2dx$$

Example

$(x_0, y_0) = (5, 8)$

$(x_1, y_1) = (9, 11)$

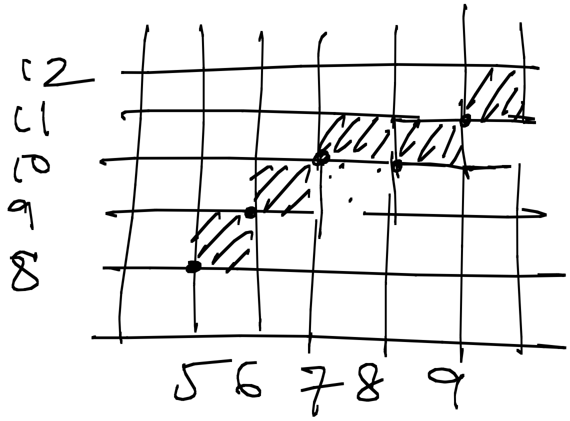
$$y = \frac{3}{4}x + \frac{17}{4}$$

or $3x - 4y + 17 = 0$

$\therefore A = dy = 3$

$B = -dx = -4$

$\therefore dx = 4$



$(x_0, y_0) = (5, 8)$

$(x_1, y_1) = (9, 11)$

$dy = 11 - 8 = 3$

$dx = 9 - 5 = 4$

1. $d = 2dy - dx = 2 \times 3 - 4 = 2 > 0$

$\Rightarrow NE$

2. $d = 2 + 2dy - 2dx$

$= 2 + 6 - 8 = 0 \Rightarrow NE$

3. $d = 0 + 2dy - 2dx = 6 - 8 = -2 < 0$

$\Rightarrow E$

4. $d = 6 + 2dy = 6 + 6 = 0 \Rightarrow NE$