

Student ID: _____

Q1: [Data conversion]

9 points

(a) Convert the following decimal number to binary using divide-by-2 method (3 points):

151

$$151/2 = 75 + 1$$

$$75/2 = 37 + 1$$

$$37/2 = 18 + 1$$

$$18/2 = 9 + 0$$

$$9/2 = 4 + 1$$

$$4/2 = 2 + 0$$

$$2/2 = 1 + 0$$

$$1/2 = 0 + 1$$

Answer: 1 0 0 1 0 1 1 1

(b) Convert the following binary number to decimal (3 points):

1 1 0 1 0 1 1 0

$$2^0 * 0 = 0$$

$$2^1 * 1 = 2$$

$$2^2 * 1 = 4$$

$$2^3 * 0 = 0$$

$$2^4 * 1 = 16$$

$$2^5 * 0 = 0$$

$$2^6 * 1 = 64$$

$$2^7 * 1 = 128$$

Sum = 214

(c) Convert the following hexadecimal number to decimal (3 points):

4AF

$$16^0 * F = 1 * 15$$

$$16^1 * A = 16 * 10$$

$$16^2 * 4 = 256 * 4$$

Sum = 1024 + 160 + 15 = 1199

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Q2: [Boolean algebra]

10 points

(a) Prove the following Boolean equation using Boolean algebra (5 points):

$$xyz + xyz' + x'yz + xyz' + x'yz' = y$$

$$xyz + xyz' + x'yz + xyz' + x'yz' = xyz + (\mathbf{xyz'} + \mathbf{xyz'}) + x'yz + x'yz' \leftarrow \text{idempotent}$$

$$xyz + xyz' + x'yz + xyz' + x'yz' = xyz + \mathbf{xyz'} + x'yz + x'yz'$$

$$xyz + xyz' + x'yz + x'yz' = xy(\mathbf{z + z'}) + x'y(\mathbf{z + z'}) \leftarrow \text{distribution}$$

$$xy + x'y = y(\mathbf{x + x'}) \leftarrow \text{distribution}$$

$$y(\mathbf{x + x'}) = y$$

(b) Use algebraic manipulation to convert the following equation to sum-of-product form (5 points) :

$$y(z + w)' + x(z' + y')(w)' + (zw)(x'y)'$$

$$= y(z + w)' + x(z' + y')(w)' + (zw)(x'y)'$$

$$= y\mathbf{z'w'} + x(z' + y')(w)' + (zw)(x'y)' \leftarrow \text{DeMorgan's}$$

$$= yz' \mathbf{w'} + (\mathbf{xz'} + \mathbf{xy'})(\mathbf{w'})} + (zw)(x'y)' \leftarrow \text{distribution}$$

$$= yz' \mathbf{w'} + xz'w' + xy'w' + (zw)(\mathbf{x + y'})} \leftarrow \text{DeMorgan's}$$

$$= yz' \mathbf{w'} + xz'w' + xy'w' + \mathbf{zwx + zwy'}} \leftarrow \text{distribution}$$

$$= yz' \mathbf{w'} + xz'w' + xy'w' + zwx + zwy'}$$

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Q3: [Combinational logic design]

10 points

Consider

Your favorite team is about to make it to the playoffs, however, its playoff ticket depends on the results from the last game of the teams in your group. There are three other teams in your group (A, B, C). Your team will qualify under the following conditions:

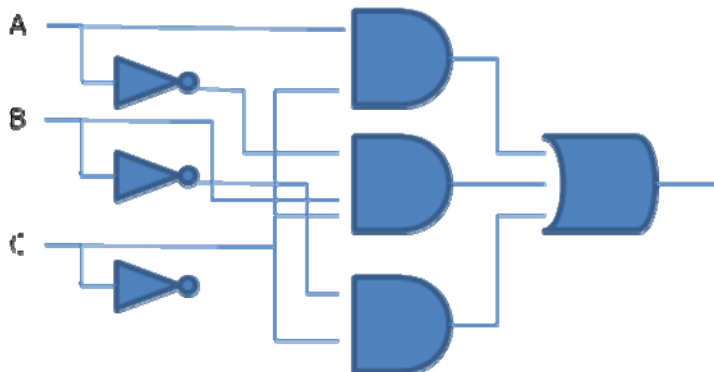
- 1) If both A and C lose their last game or
- 2) A wins, and both B and C lose their last game or
- 3) C loses and B wins their respective last games

(a) Write a Boolean equation to represent the qualification condition. Use the name of the teams as the variables of your equation. You DO NOT have to simplify the function. (5 points)

Note: Variable A = 1 if team A loses the last game, 0 otherwise

$$AC + A'BC + B'C$$

(b) Design a circuit based on the Boolean equation from part (a) using And, Or and Not gates. (5 points)



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Q4: [Combinational design]

6 points

We want to design a combinational circuit that computes the function $f(X) = 3X+1$ for a 2-bit X , where X is a number greater or equal to 0:

(a) What is the minimum number of bits required to represent all outputs from $f(X)$? (2 points)

$X = 2$ bits, maximum value for $x = (11)_2 = 3$

$F(3) = 3(3) + 1 = 10$, maximum number of needed bits = 4.

(b) Draw the truth table for this function. (4 points)

0 1
1 4
2 7
3 10

x1	x0	y3	y2	y1	y0
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	1	1	1
1	1	1	0	1	0