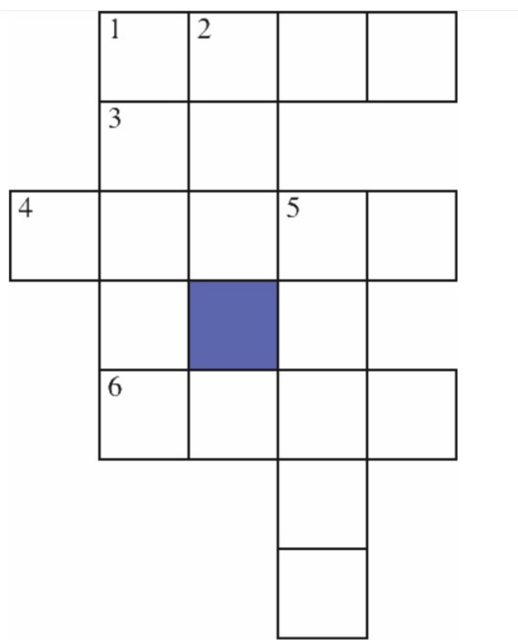
ICS 271
Fall 2018
Instructor : Kalev Kask
Homework Assignment 4
Due Sunday 11/4

1. (15 points) Consider the crossword puzzle



We represent the problem as a CSP where there is a variable for each of the positions where a word is supposed to go (1-across, 2-down, 3-across, 4-across, 5-down, 6-across). Domains of variables are words from this list : at, eta, be, hat, he, her, it, him, on, one, desk, dance, usage, easy, dove, first, else, loses, fuels, help, haste, given, kind, sense, soon, sound, this, think. The constraints are that the letter is the same where the words intersect.

Given a complete assignment of values to variables, define a cost function of the assignment and a neighborhood relation. Then simulate execution of Stochastic Local Search on this problem. Start with v1=this,

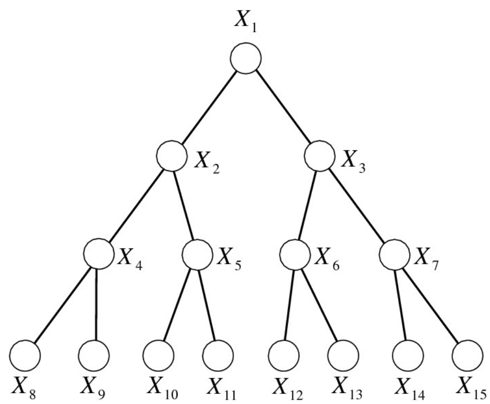


Figure 1: A constraint tree problem

v2=eta, v3=at, v4=dance, v5=haste, v6=dove. Show the first 3 iterations of SLS. For each iteration, show the current cost, and the cost of each node in the neighborhood, as well as the new assignment chosen.

2. (15 points) Consider the following binary constraint network: There are 4 variables: X_1, X_2, X_3, X_4, with the domains:

   $D_1 = \{1, 2, 3, 4\}$, $D_2 = \{3, 4, 5, 8, 9\}$, $D_3 = \{2, 3, 5, 6, 7\}$, $D_4 = \{3, 5, 7, 8\}$.

   The 3 constraints are: $X_1 \geq X_2$, $X_2 > X_3$ or $X_3 - X_2 = 3$, $X_3 \neq X_4$.

   (a) Write the constraints in a relational form and draw the constraint graph.

   (b) Is the network arc-consistent? if not, compute the arc-consistent network.

   (c) Is the network consistent (i.e. does it have a solution)? If yes, give a solution.

3. (20 points) Consider a constraint satisfaction problem with 15 variables $X_1, \ldots, X_{15}$, and domains of variables $D_1, \ldots, D_{15} = \{1, 2, 3, 4, 5, 6, 8, 9\}$, *the constraint graph is a perfect binary tree with degree 2*, and the binary constraints C_{ij} are $X_i \geq X_j + 2$ where i is the parent of j in the tree (the root is X_1, see Figure 1).

   (a) Is the network arc-consistent? if not, enforce arc-consistency.

(b) Is the network consistent? If yes, compute a solution.

(c) Can you suggest a variable ordering for which backtrack search will be backtrack-free, assuming the problem is arc-consistent?

(d) Bound the complexity of solving any problem whose constraint graph is the same as for this problem.

4. (15 points) (Problem 6.5 in Russel and Norvig) Solve the cryptarithmetic problem in Figure 6.2 by hand, using backtracking with MRV variable order heuristic (use Forward Checking) and with least constraining value heuristic.

   For each step of the algorithm show

   - Depth of the current node in search tree
   - Current variable (with its MRV score); if no variable (deadend) write $\emptyset$
   - Current domain of the variable, sorted in the order ot decreasing preference
   - All other un-assigned variables, each with its MRV score

5. (15 points) Consider the following algorithms

   - Arc consistency with domain splitting. This algorithm (recursively, until each variable has domain of size at most 1) splits the problem into disjoint smaller subproblems by splitting the domain of some variable; then each subproblem is solved separately, and solutions to each subproblem are combined to form a solution to the original problem. Before each domain splitting, we enfore arc consistency.
   - Variable elimination. This algorithm (recursively, until one variable remains) picks a variable to eliminate, combines all constraints involving that variable with *join*, projects the resulting combined constraint onto remaining variables, and eliminates the variable from the problem.
   - Stochastic local search.
   - Backtracking search.

Which of these algorithms can

(a) Assuming the problem is inconsistent, determine that there is no solution?

(b) Find a solution if one exists?

(c) Guarantee to find all solutions?