Set 3: Informed Heuristic Search

ICS 271 Fall 2013

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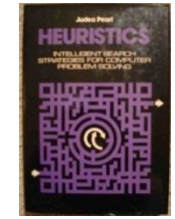
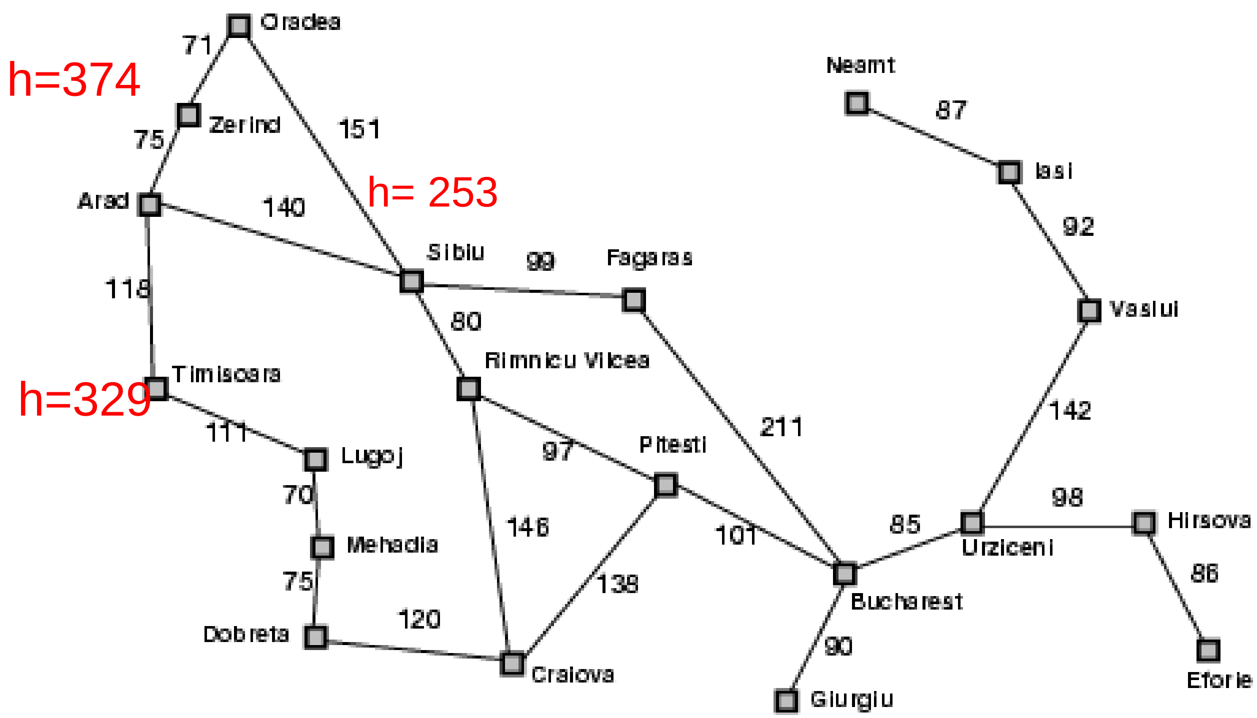
Overview

- Heuristics and Optimal search strategies
  - heuristics
  - hill-climbing algorithms
  - Best-First search
  - A*: optimal search using heuristics
  - Properties of A*
    - admissibility,
    - consistency,
    - accuracy and dominance
    - Optimal efficiency of A*
  - Branch and Bound
  - Iterative deepening A*
  - Automatic generation of heuristics

Heuristic Search



- State-Space Search: every problem is like search of a map
- A problem solving robot finds a path in a state-space graph from start state to goal state, using heuristics

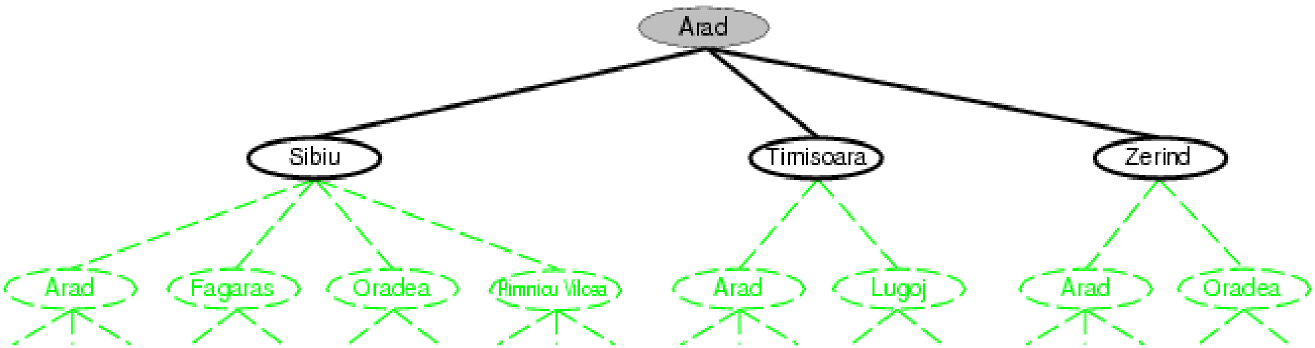


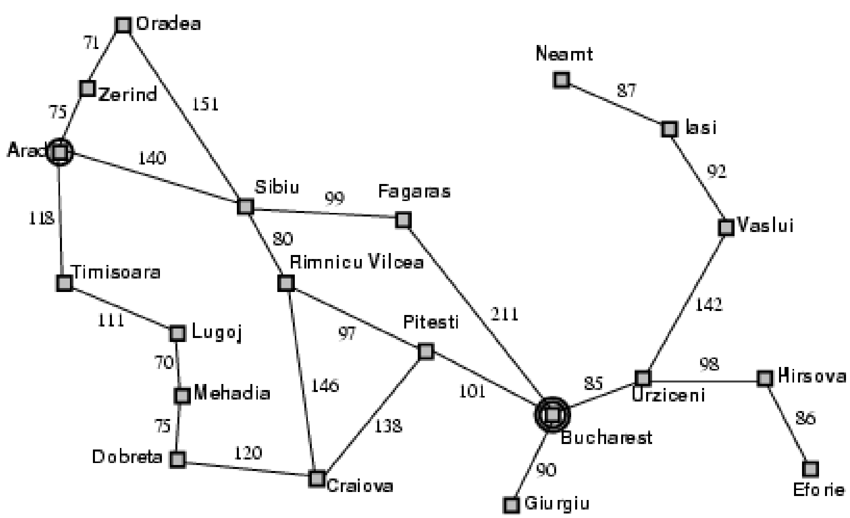
Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

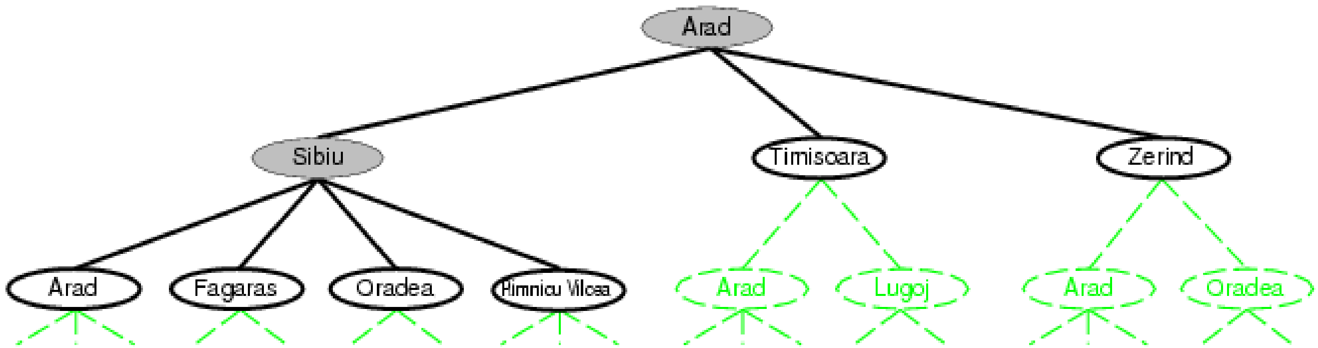
Heuristic = straight-line distance

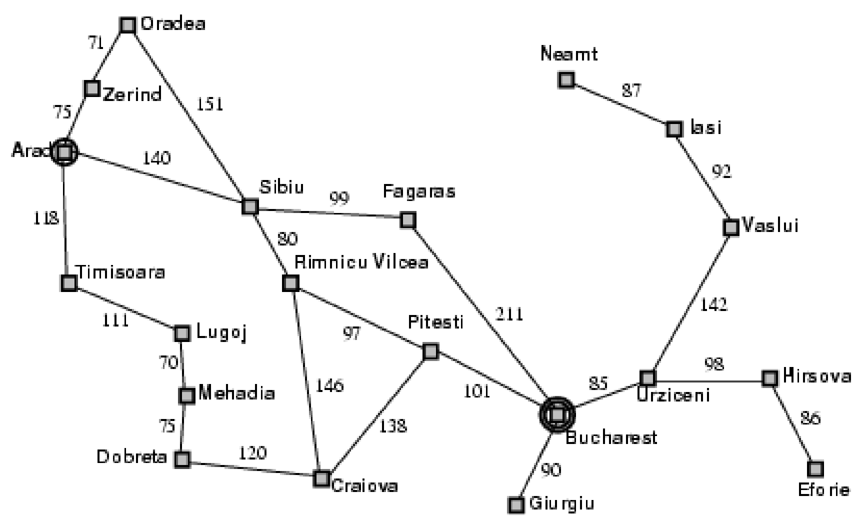
State Space for Path Finding in a Map



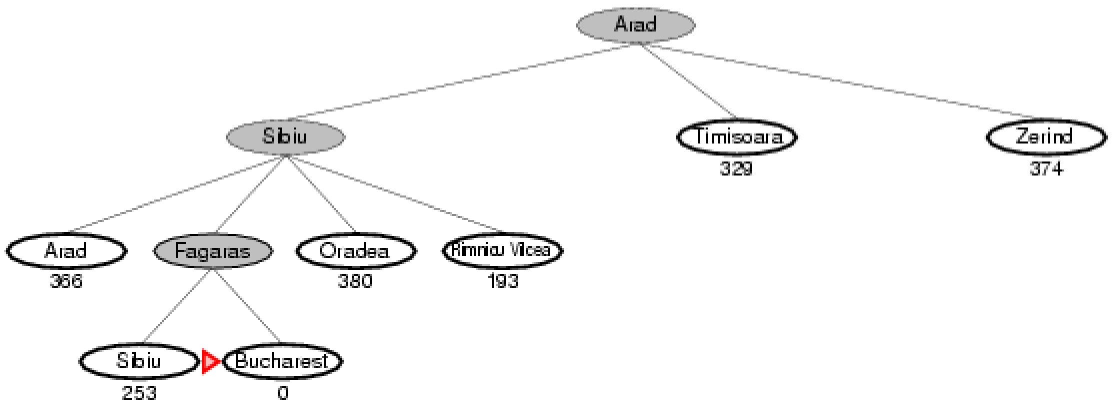


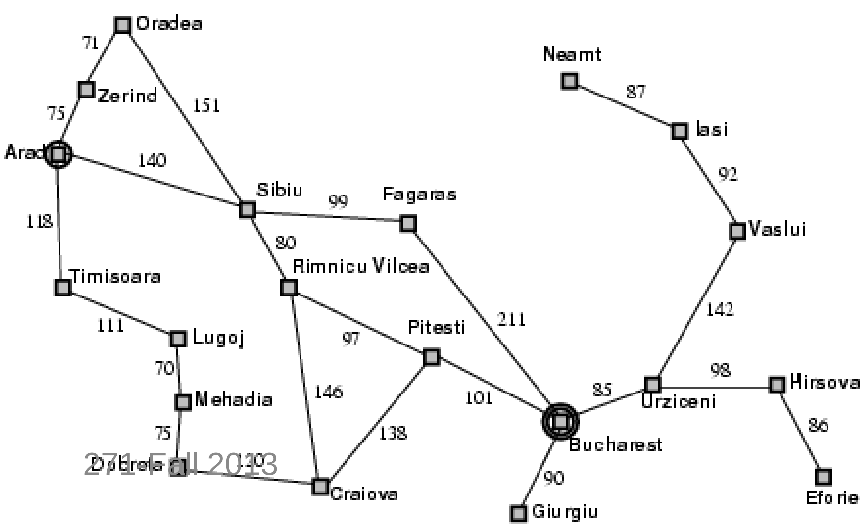
State Space for Path Finding in a Map





Greedy Search Example





State Space of the 8 Puzzle Problem

Initial state

goal

1	2	3
4	5	6
7	8	

8-puzzle: 181,440 states
15-puzzle: 1.3 trilion
24-puzzle: 10^25

Search space exponential

Use Heuristics as people do

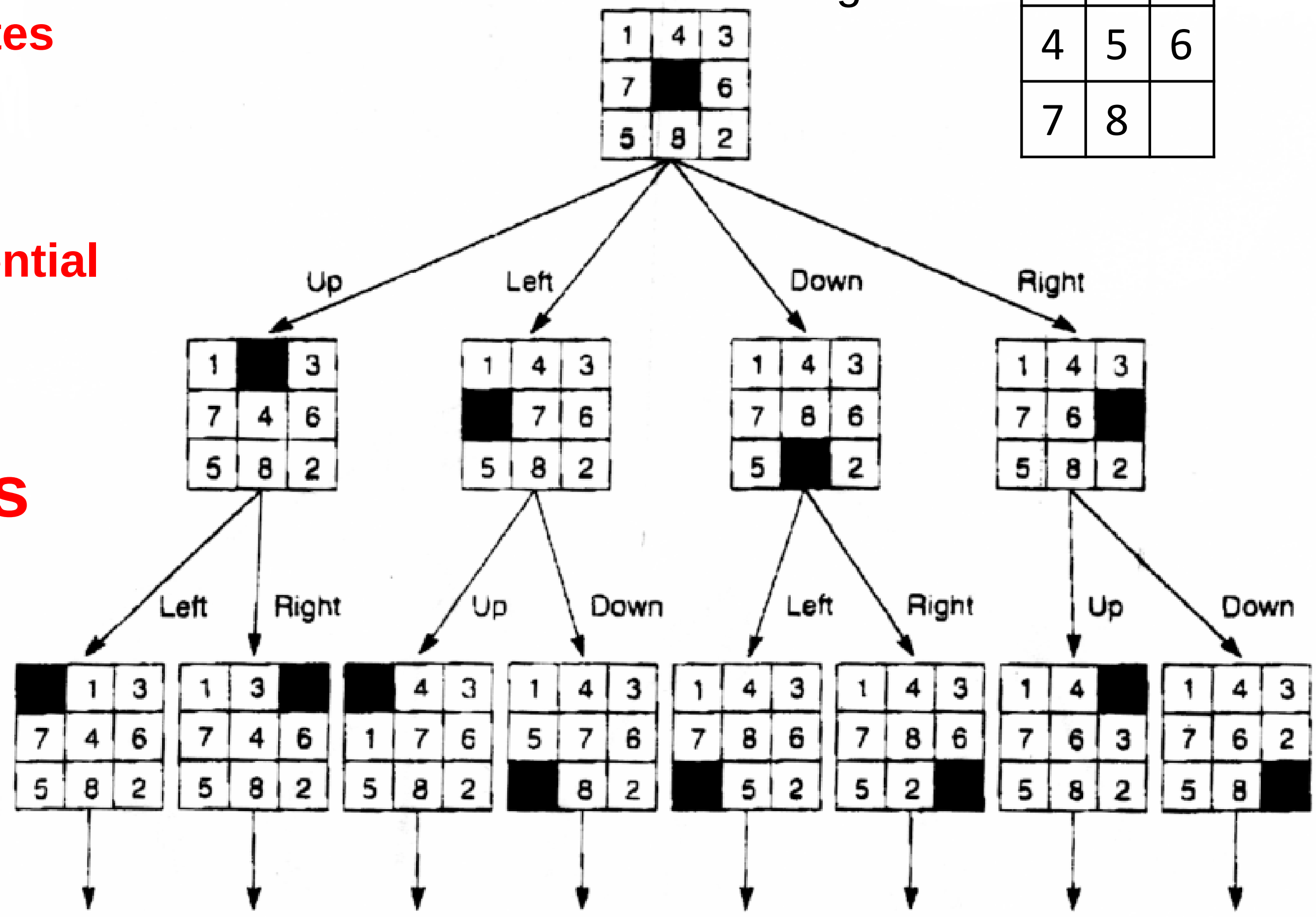


Figure 3.6 State space of the 8-puzzle generated by "move blank" operations.

State Space of the 8 Puzzle Problem

1	2	3
4	5	6
7	8	

h1 = number of misplaced tiles

h2 = Manhattan distance

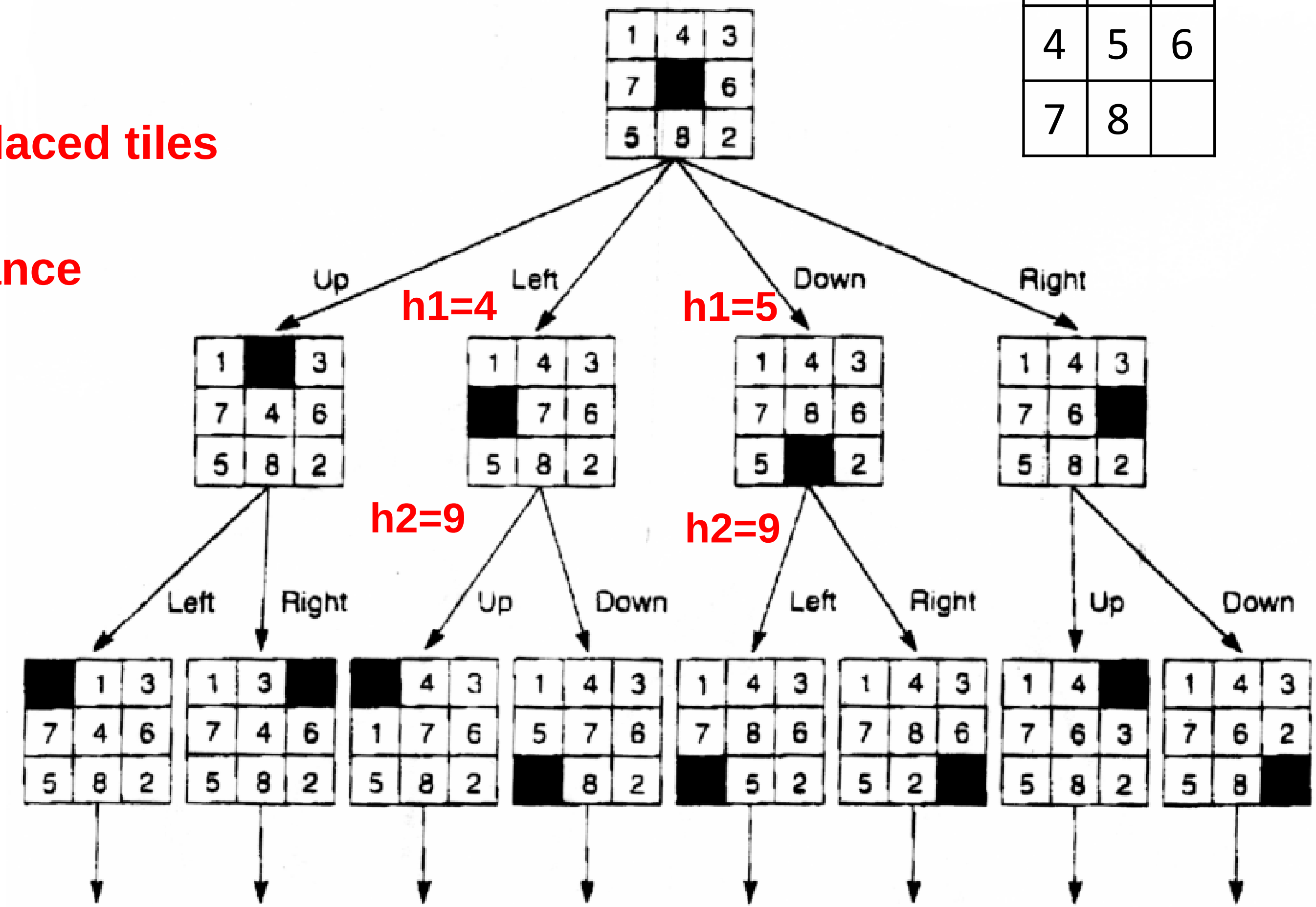


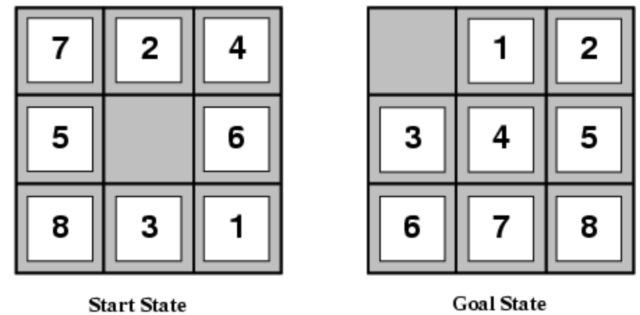
Figure 3.6 State space of the 8-puzzle generated by "move blank" operations.

What are Heuristics

- **Rule of thumb, intuition**
- **A quick way to estimate how close we are to the goal. How close is a state to the goal..**
- Pearl: "the ever-amazing observation of how much people can accomplish with that simplistic, unreliable information source known as *intuition*."

8-puzzle

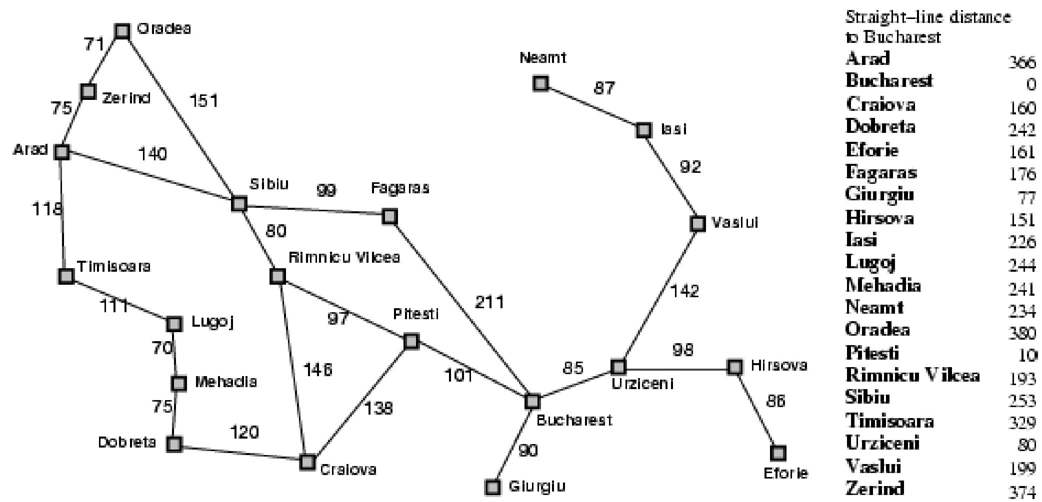
- **h1(n): number of misplaced tiles**
- **h2(n): Manhattan distance**



$h_1(S) = ?$ 8

$h_2(S) = ?$ 3+1+2+2+2+3+3+2 = 18

- Path-finding on a map
  - Euclidean distance

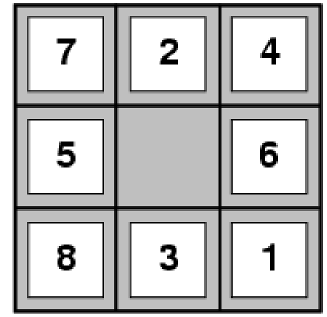


Problem: Finding a Minimum Cost Path

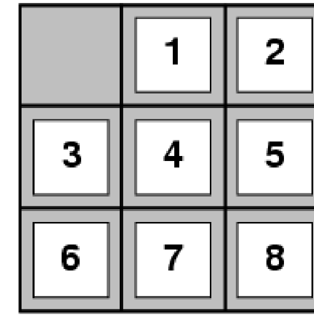
- Previously we wanted an arbitrary path to a goal or best cost. Now, we want the minimum cost path to a goal G
  - Cost of a path = sum of individual transitions along path
- Examples of path-cost:
  - Navigation
    - path-cost = distance to node in miles
      - minimum => minimum time, least fuel
  - VLSI Design
    - path-cost = length of wires between chips
      - minimum => least clock/signal delay
  - 8-Puzzle
    - path-cost = number of pieces moved
      - minimum => least time to solve the puzzle
- Algorithm: Uniform-cost search… still somewhat blind

Heuristic Functions

- 8-puzzle
  - Number of misplaced tiles
  - Manhatten distance
  - Gaschnig’s

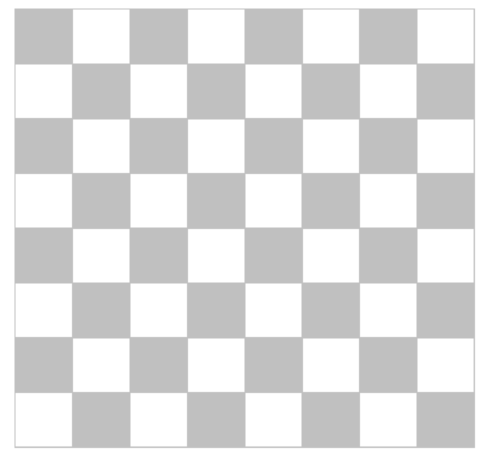


Start State

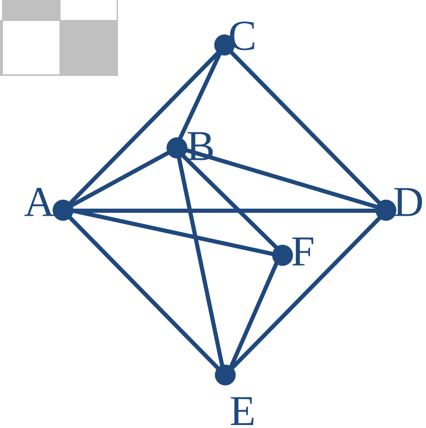


Goal State

- 8-queen
  - Number of future feasible slots
  - Min number of feasible slots in a row
  - Min number of conflicts (in complete assignments states)



- Travelling salesperson
  - Minimum spanning tree
  - Minimum assignment problem



Best-First (Greedy) Search:
f(n) = number of misplaced tiles

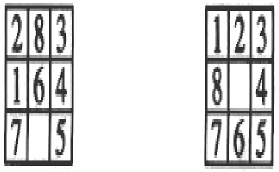
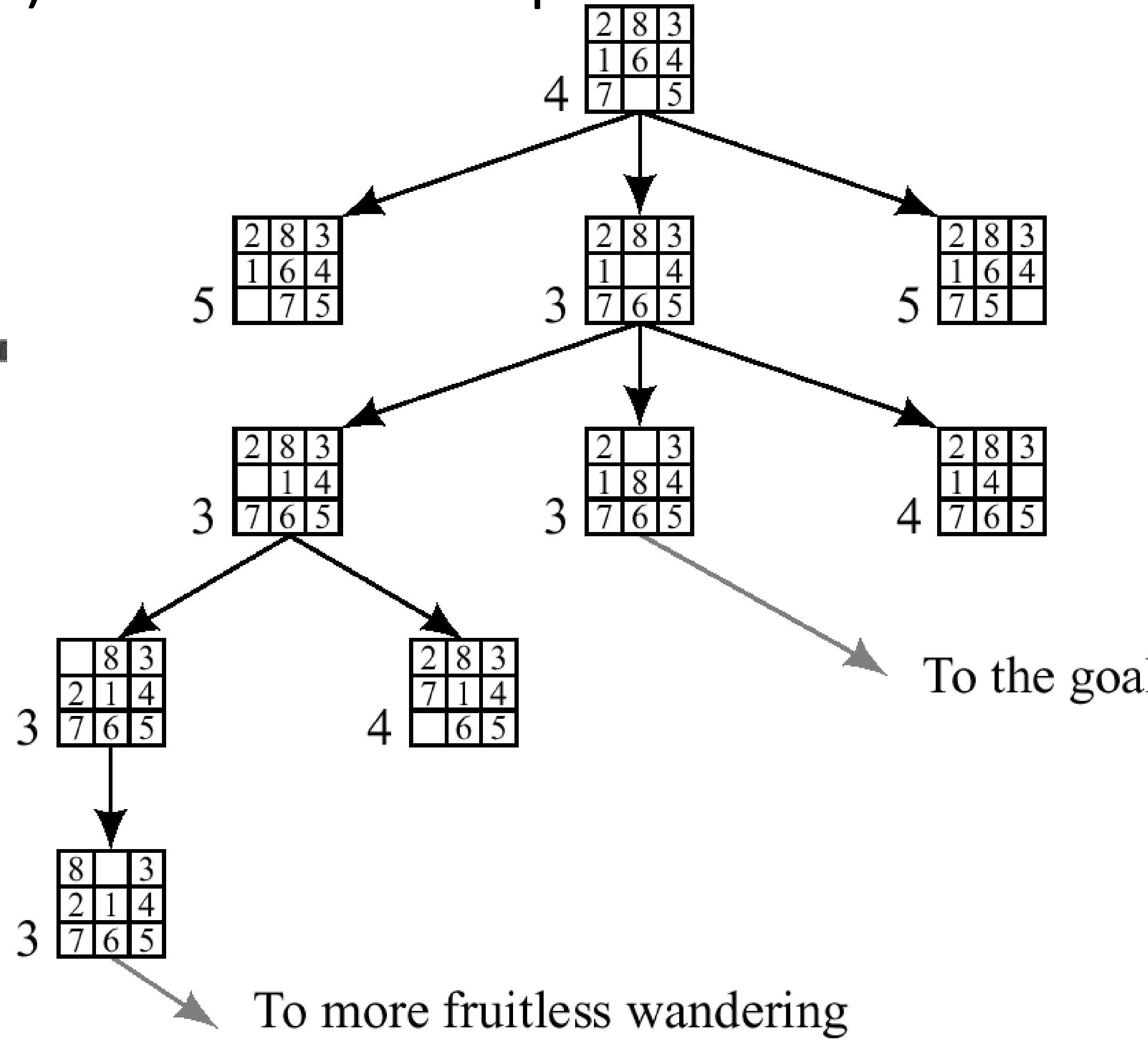
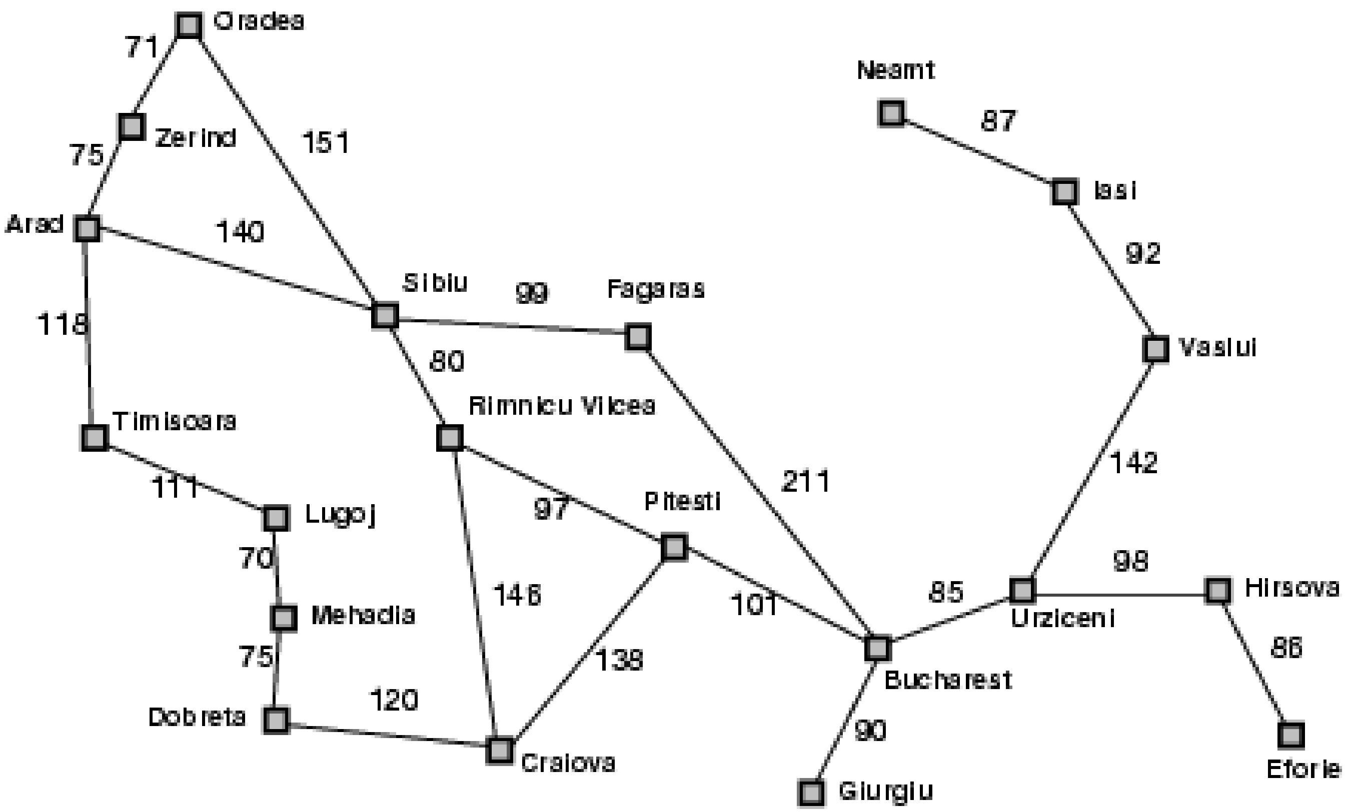


Figure 8.1

Start and Goal Configurations for the Eight-Puzzle



Romania with Step Costs in km



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Greedy Best-First Search

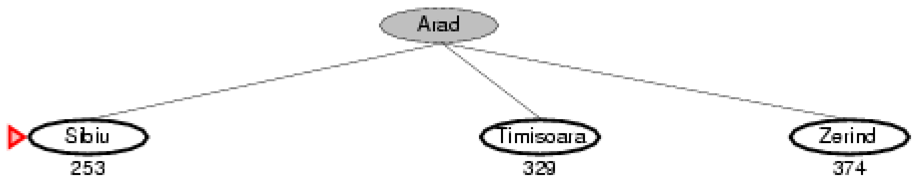
- Evaluation function $f(n) = h(n)$ (heuristic)
- = estimate of cost from *n* to *goal*

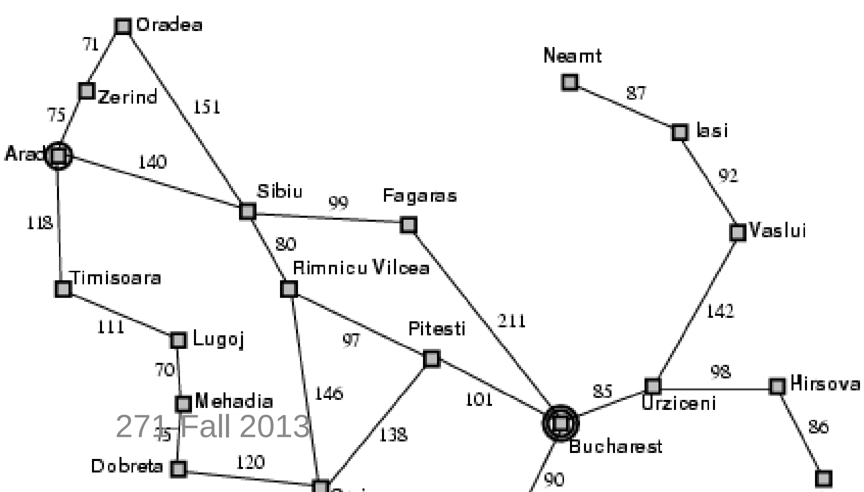
- e.g., $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest

- Greedy best-first search expands the node that appears to be closest to goal

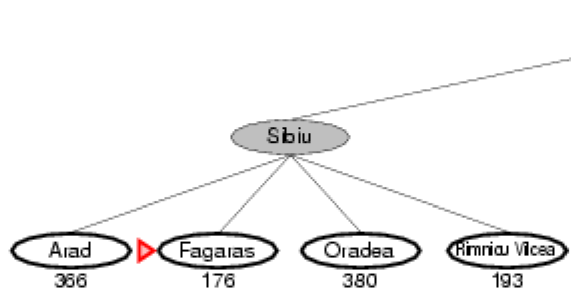
Greedy Best-First Search Example

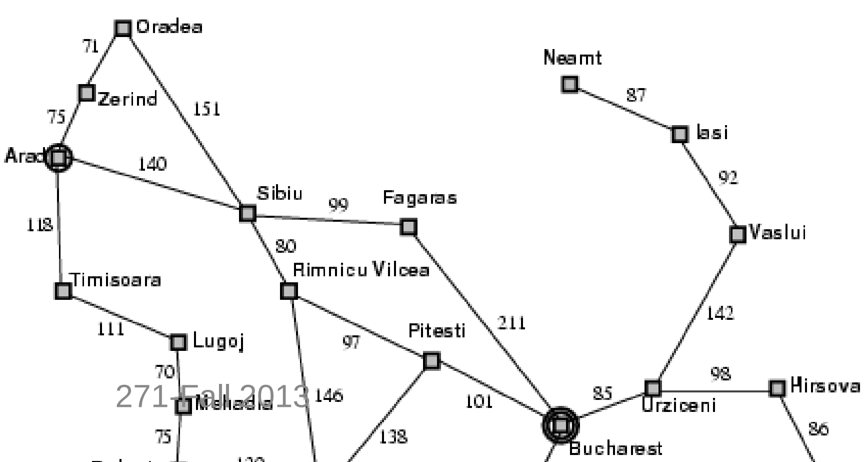
Greedy Best-First Search Example





Greedy Best-First Search Example





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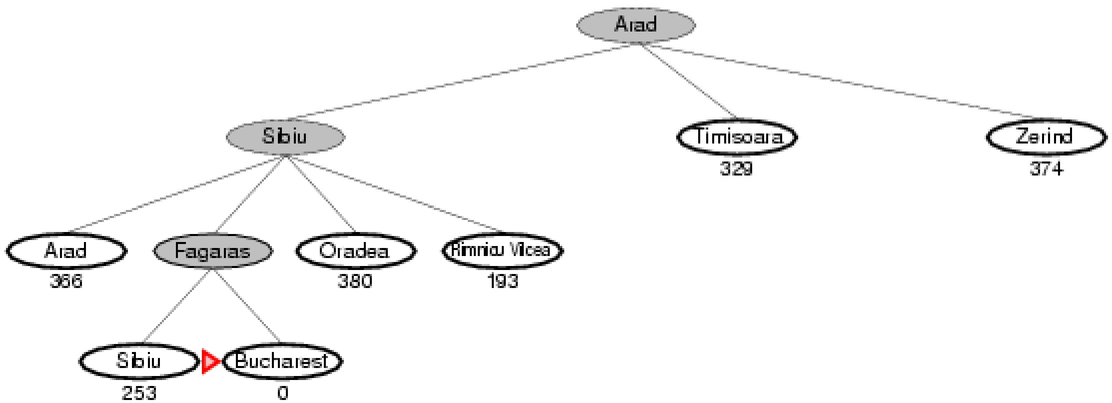
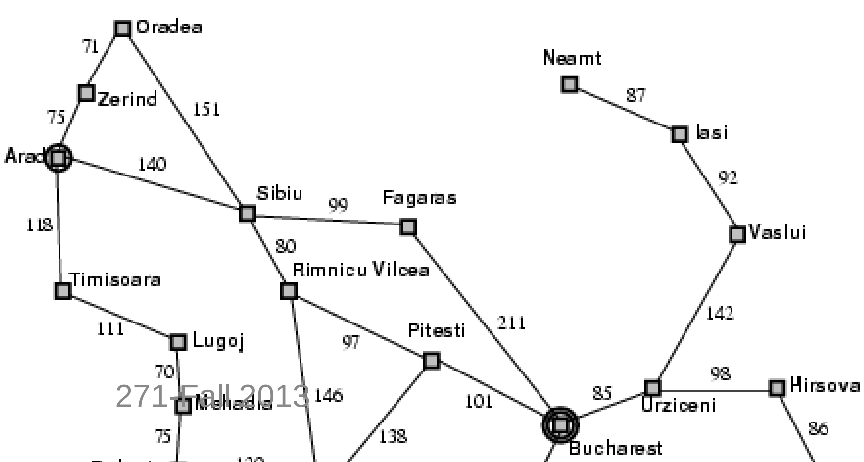




Problems with Greedy Search

- Not complete
- Get stuck on local minimas and plateaus,
- Irrevocable,
- Infinite loops
- Can we incorporate heuristics in systematic search?

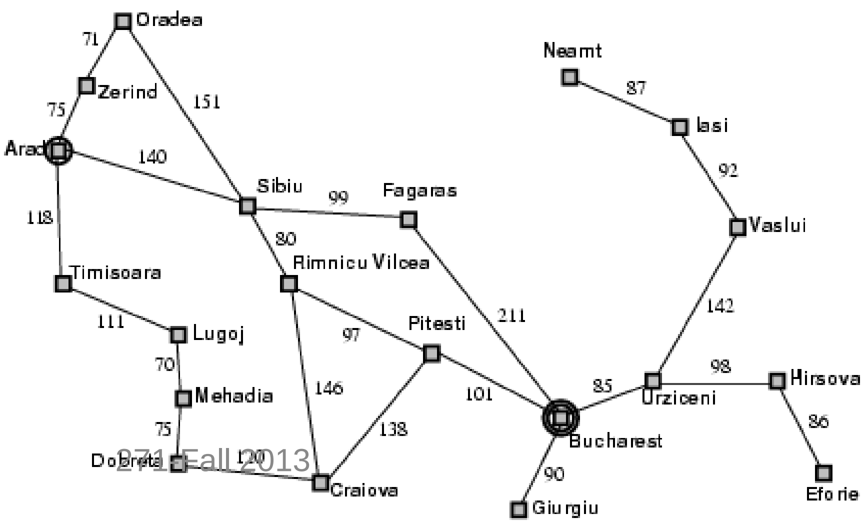
Informed Search - Heuristic Search

- How to use heuristic knowledge in systematic search?
- Where ? (in node expansion? hill-climbing ?)
- Best-first:
  - select the best from **all** the nodes encountered so far in OPEN.
  - "good" use heuristics
- Heuristic estimates value of a node
  - promise of a node
  - difficulty of solving the subproblem
  - quality of solution represented by node
  - the amount of information gained.
- f(n)- heuristic evaluation function.
  - depends on n, goal, search so far, domain

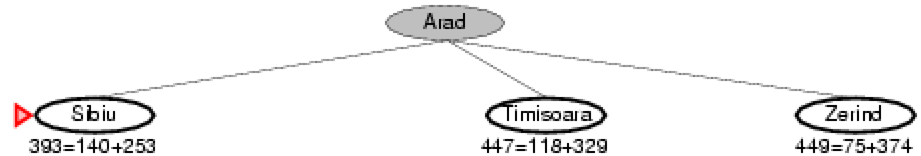
A^* Search

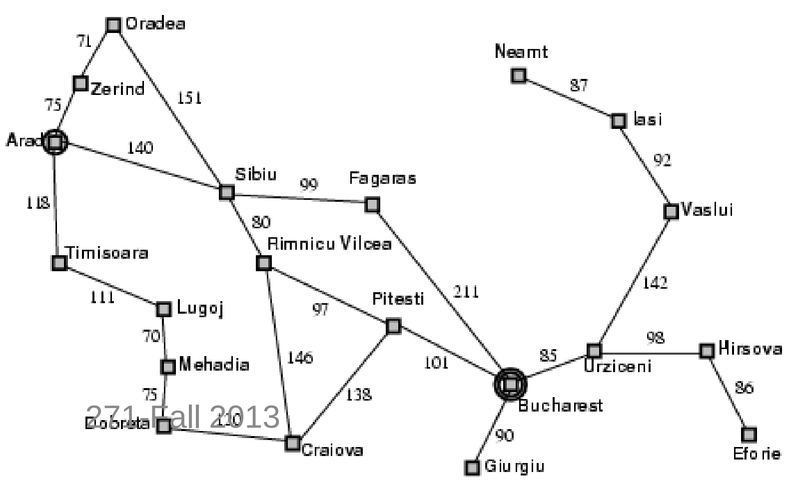
- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal

A* Search Example

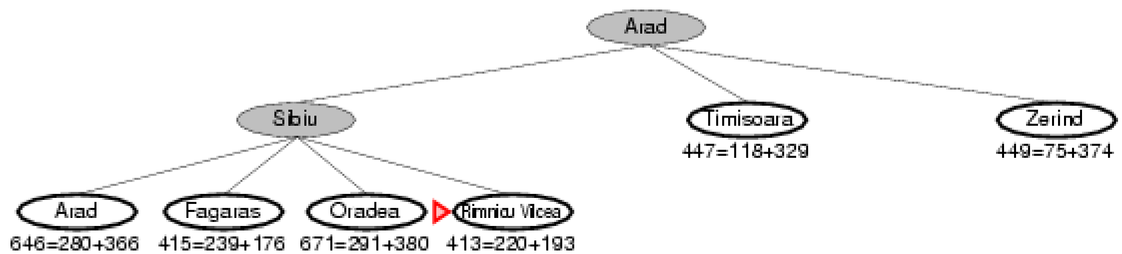


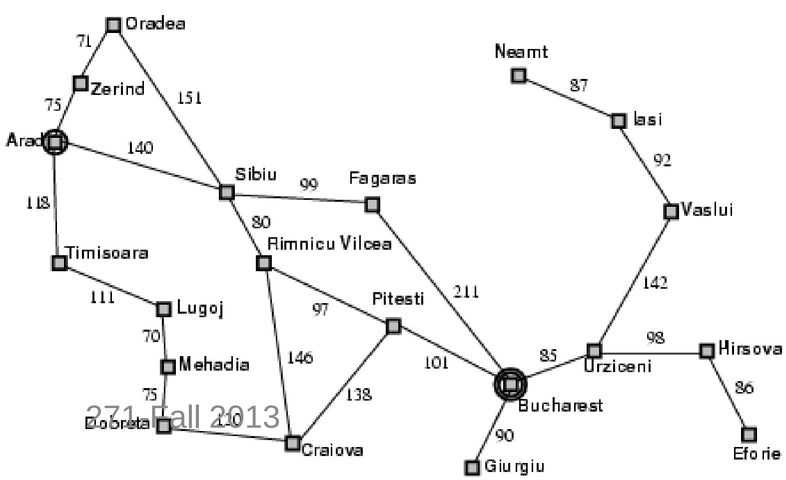
A* Search Example



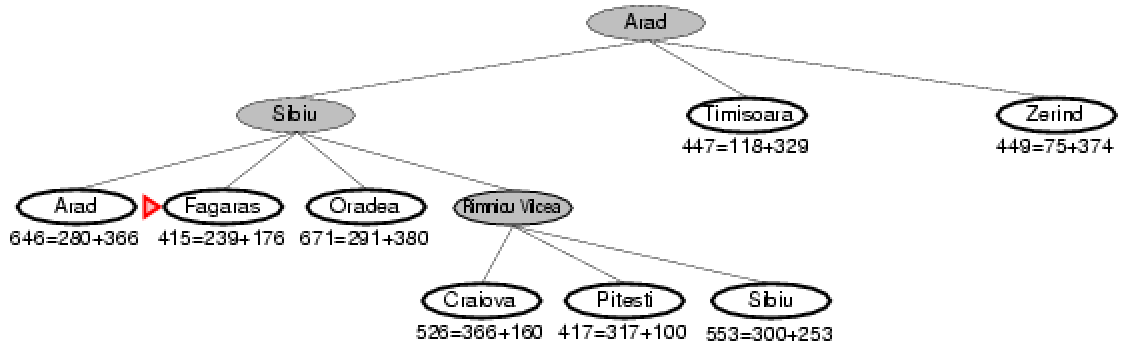


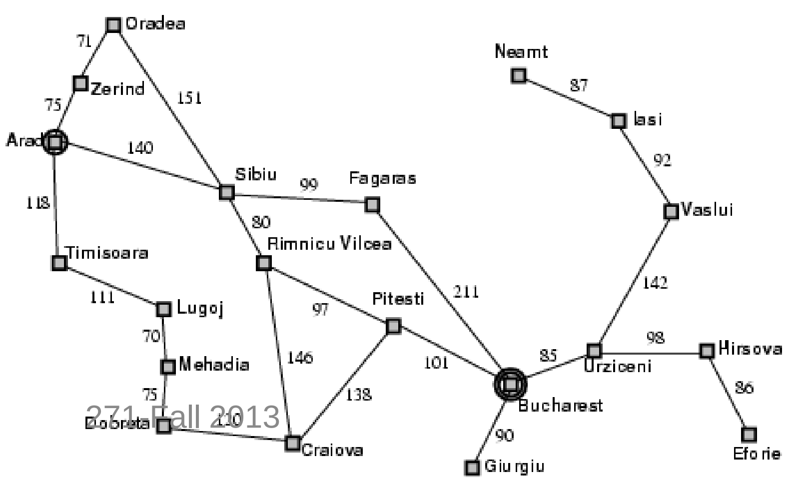
A* Search Example





A* Search Example





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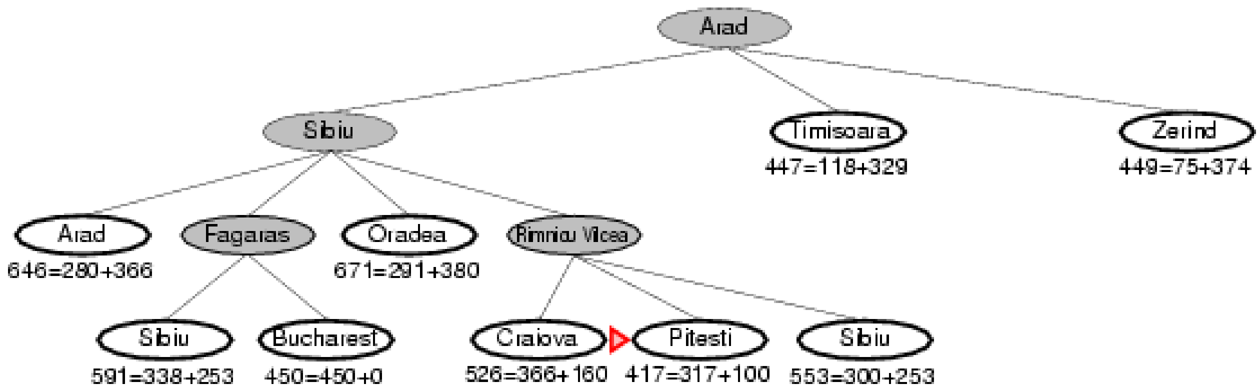
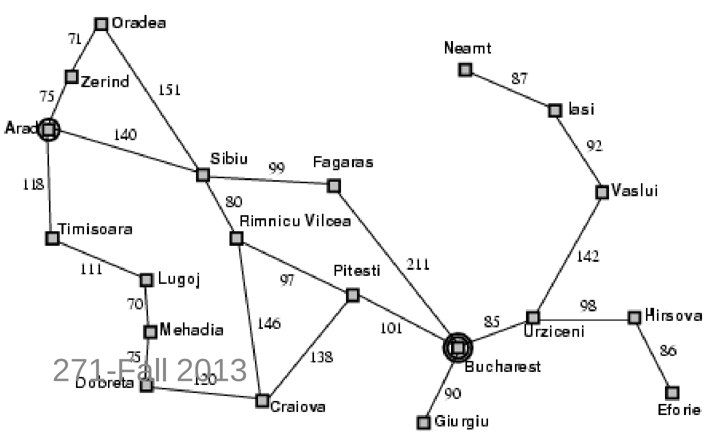
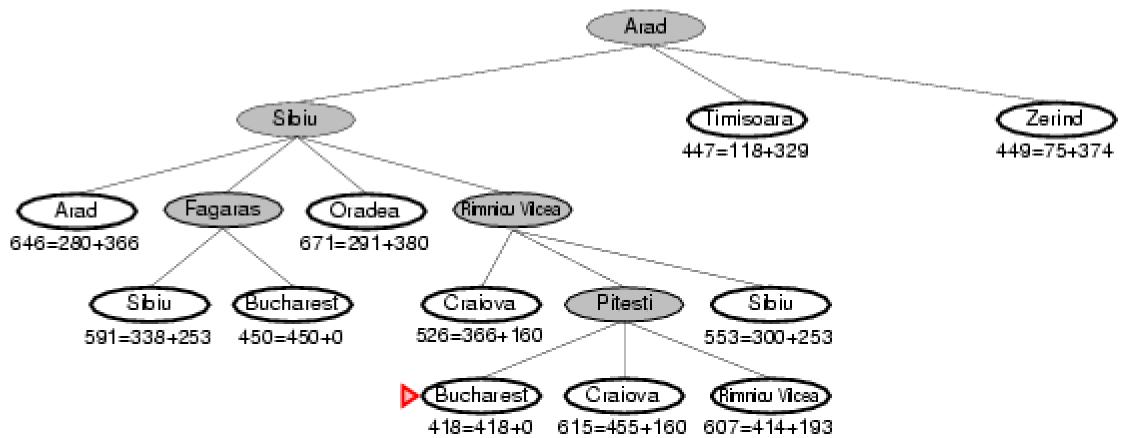




A^* Search Example

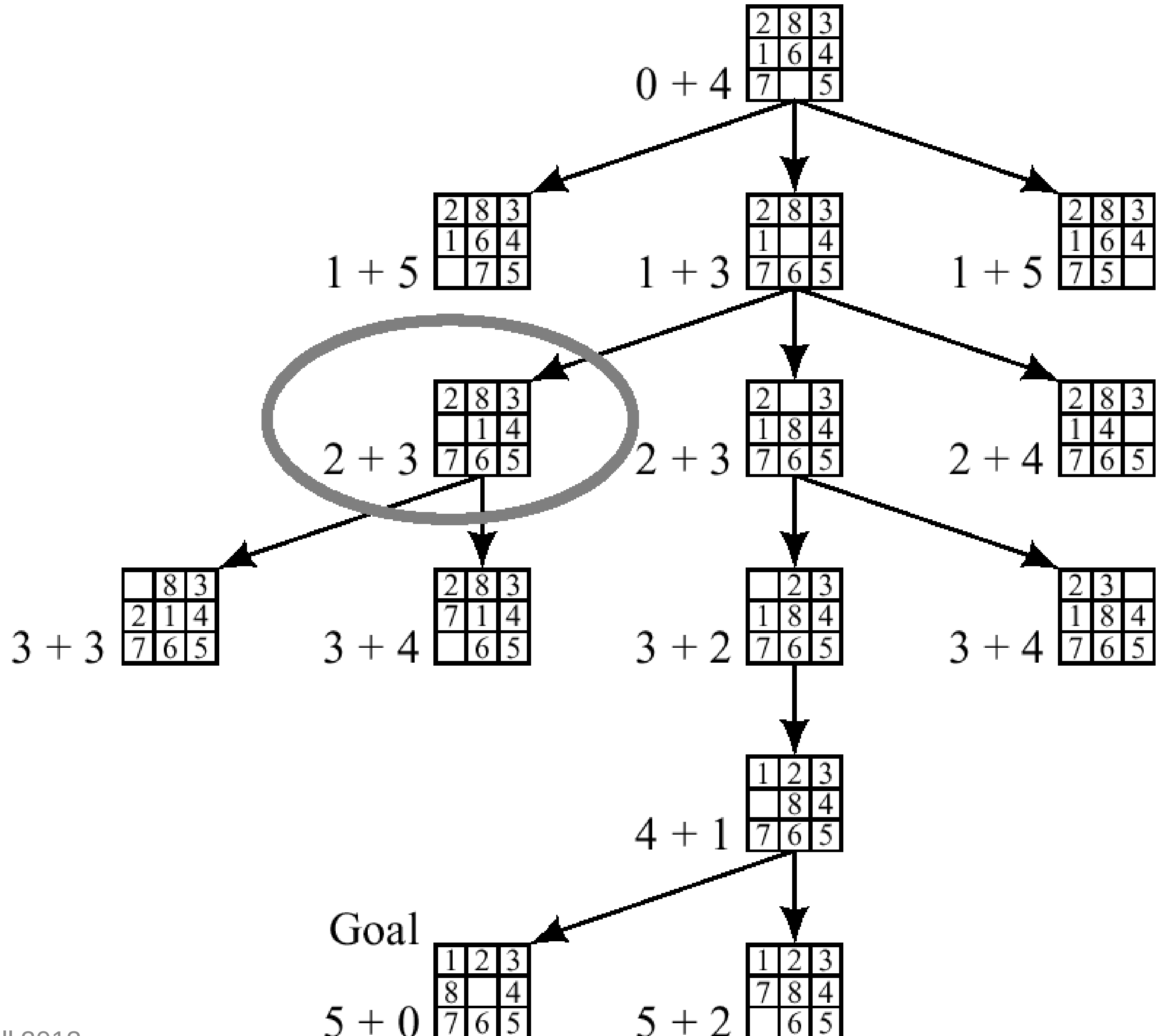


A*- a Special Best-First Search

- Goal: find a minimum sum-cost path
- Notation:
  – c(n,n') - cost of arc (n,n')
  – g(n) = cost of current path from start to node n in the search tree.
  – h(n) = estimate of the cheapest cost of a path from n to a goal.
  – Special evaluation function: f = g+h
- f(n) estimates the cheapest cost solution path that goes through n.
  – h*(n) is the true cheapest cost from n to a goal.
  – g*(n) is the true shortest path from the start s, to n.

- If the heuristic function, h always underestimate the true cost (h(n) is smaller than h*(n)), then A* is guaranteed to find an optimal solution.

A* on 8-Puzzle with h(n) = # misplaced tiles



Algorithm A* (with any h on search Graph)

- Input: an implicit search graph problem with cost on the arcs
- Output: the minimal cost path from start node to a goal node.
  - 1. Put the start node s on OPEN.
  - 2. If OPEN is empty, exit with failure
  - 3. Remove from OPEN and place on CLOSED a node n having minimum f.
  - 4. If n is a goal node exit successfully with a solution path obtained by tracing back the pointers from n to s.
  - 5. Otherwise, expand n generating its children and directing pointers from each child node to n.
    - For every child node n’ do
      - evaluate h(n’) and compute $f(n') = g(n') + h(n') = g(n) + c(n,n') + h(n')$
      - If n’ is already on OPEN or CLOSED compare its new f with the old f. If the new value is higher, discard the node.
      - Else, put n’ with its f value in the right order in OPEN
  - 6. Go to step 2.

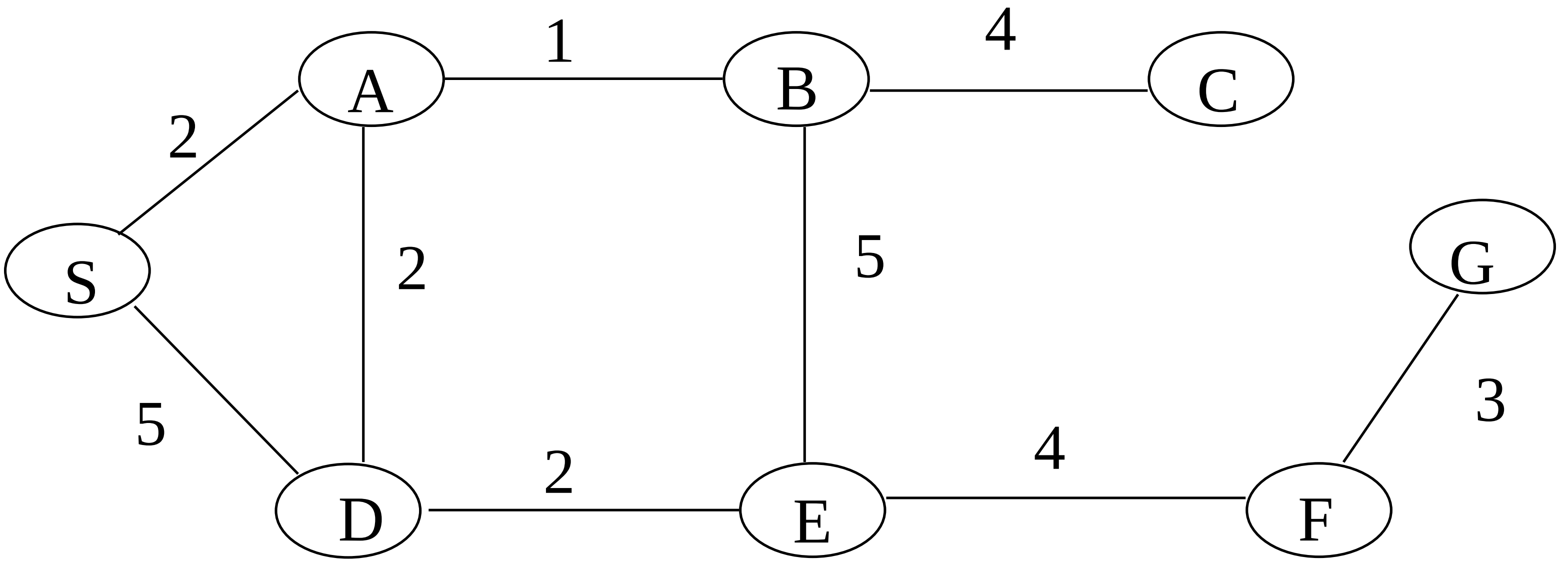
Best-First Algorithm *BF (*)*

1. Put the start node *s* on a list called *OPEN* of unexpanded nodes.
2. If *OPEN* is empty exit with failure; no solutions exists.
3. Remove the first *OPEN* node *n* at which f is minimum (break ties arbitrarily), and place it on a list called *CLOSED* to be used for expanded nodes.
4. Expand node *n*, generating all it's successors with pointers back to *n*.
5. If any of *n*'s successors is a goal node, exit successfully with the solution obtained by tracing the path along the pointers from the goal back to *s*.

6. For every successor *n'* on *n*:
   a. Calculate *f* (*n'*).
   b. if *n'* was neither on *OPEN* nor on *CLOSED*, add it to *OPEN*. Attach a pointer from *n'* back to *n*. Assign the newly computed *f(n')* to node *n'*.
   c. if *n'* already resided on *OPEN* or *CLOSED*, compare the newly computed *f(n')* with the value previously assigned to *n'*. If the old value is lower, discard the newly generated node. If the new value is lower, substitute it for the old (*n'* now points back to *n* instead of to its previous predecessor). If the matching node *n'* resided on *CLOSED*, move it back to *OPEN*.

7. Go to step 2.

* With tests for duplicate nodes.


A
B
C
S
G
D
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F
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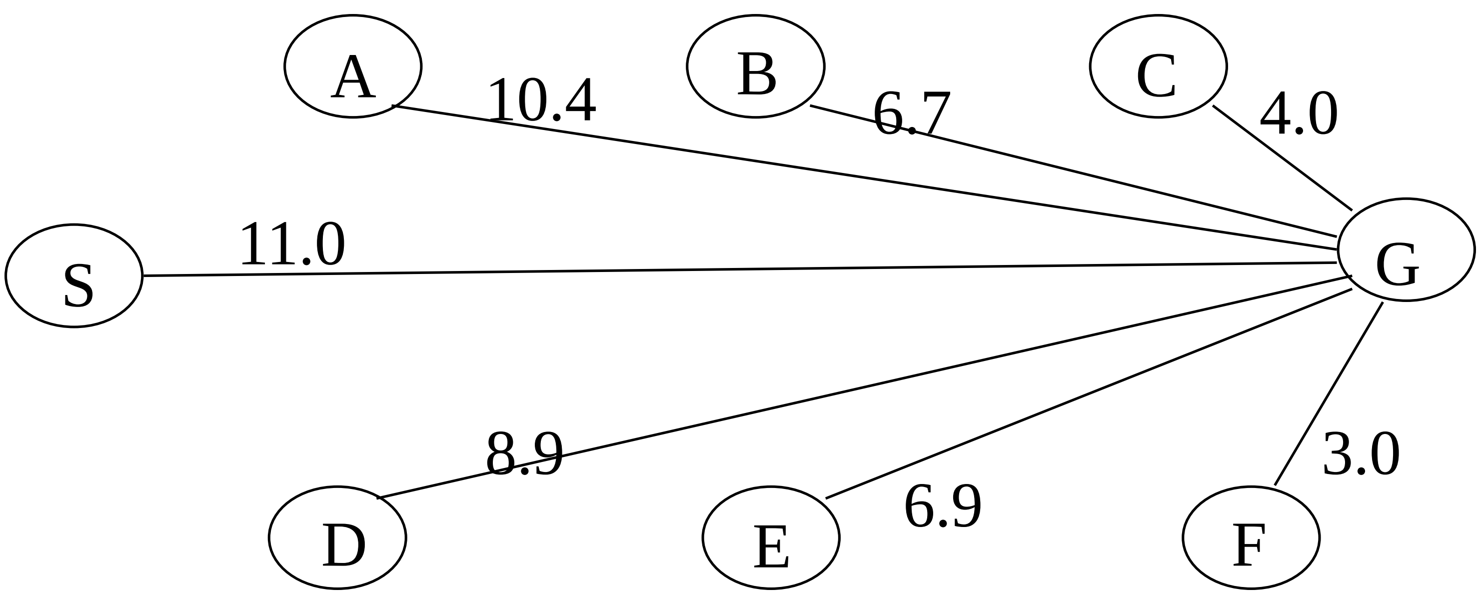
4
2
2
5
3
5
2
4


A
B
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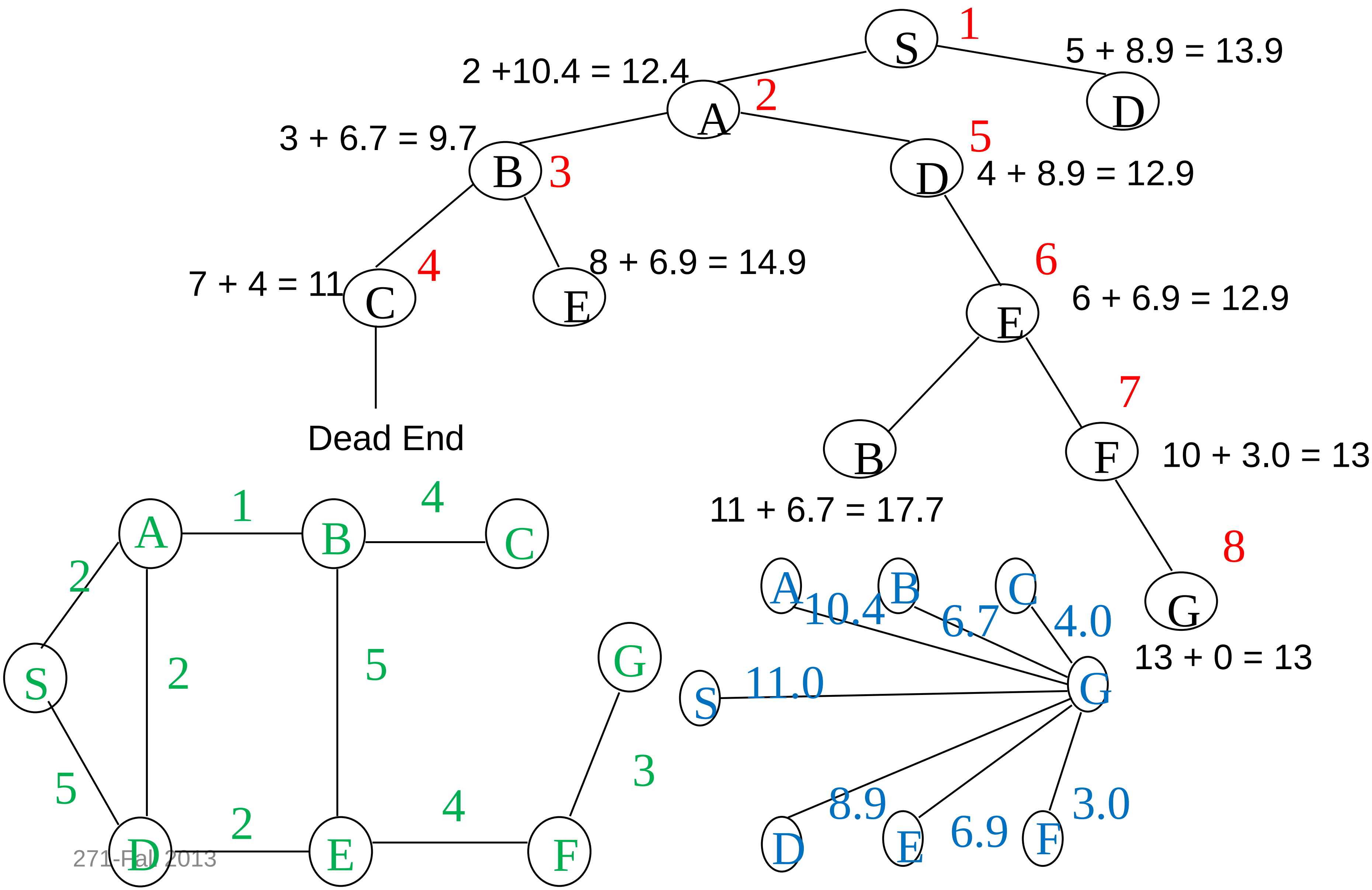
3.0

Example of A* Algorithm in Action



Behavior of A - Termination

- The heuristic function h(n) is called admissible if h(n) is never larger than h*(n), namely h(n) is always less or equal to true cheapest cost from n to the goal.

- A* is admissible if it uses an admissible heuristic, and h(goal) = 0.

- Theorem (completeness) (Hart, Nilsson and Raphael, 1968)
  - A* always terminates with a solution path (h is not necessarily admissible) if
    - costs on arcs are positive, above epsilon
    - branching degree is finite.
- Proof: The evaluation function f of nodes expanded must increase eventually (since paths are longer and more costly) until all the nodes on an optimal path are expanded.

Behavior of A* - Completeness

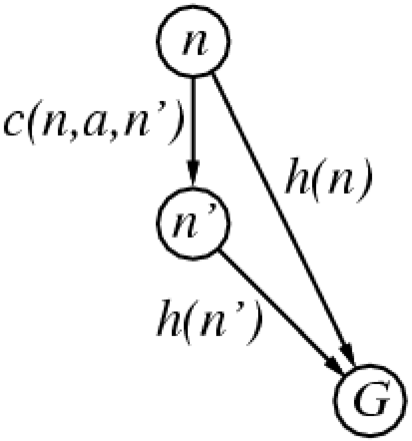
- Theorem (completeness for optimal solution) (HNL, 1968):
  - If the heuristic function is admissible than A* finds an optimal solution.

- Proof:
  - 1. A* will expand only nodes whose f-values are less (or equal) to the optimal cost path C* (f(n) is less-or-equal C*).
  - 2. The evaluation function of a goal node along an optimal path equals C*.
- Lemma:
  - Anytime before A* terminates there exists and OPEN node n' on an optimal path with f(n') <= C*.

Consistent (monotone) Heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

  $h(n) \le c(n,a,n') + h(n')$



- If *h* is consistent, we have

$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\ge g(n) + h(n) \\
&= f(n)
\end{aligned}
$$

- i.e., *f(n)* is non-decreasing along any path.

- Theorem: If *h(n)* is consistent, f along any path is non-decreasing.
- Corollary: the f values seen by A* are non-decreasing.

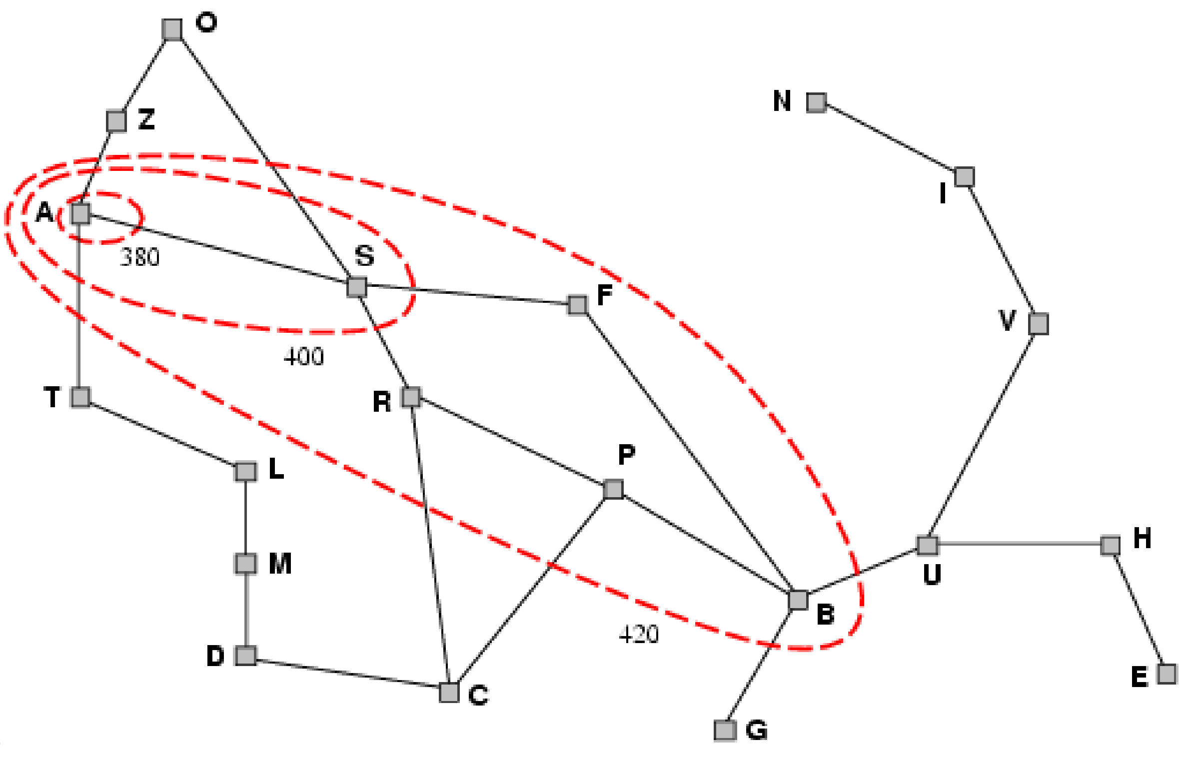
Consistent Heuristics

- If h is consistent) and h(goal)=0 then h is admissible
  - Proof: (by induction of distance from the goal)

- An A* guided by consistent heuristic finds an optimal paths to all expanded nodes, namely g(n) = g*(n) for any closed n.
  - Proof: Assume g(n) > g*(n) and n expanded along a non-optimal path.
  - Let n’ be the shallowest OPEN node on optimal path p to n →
  - g(n’) = g*(n’) and therefor f(n’)=g*(n’)+h(n’)
  - Due to consistency we get f(n’) <=g*(n’)+k(n’,n)+h(n)
  - Since g*(n) = g*(n’)+k(n’,n) along the optimal path, we get that
  - f(n’) <= g*(n) + h(n)
  - And since g(n) > g*(n) then f(n’) < g(n)+h(n) = f(n), contradiction

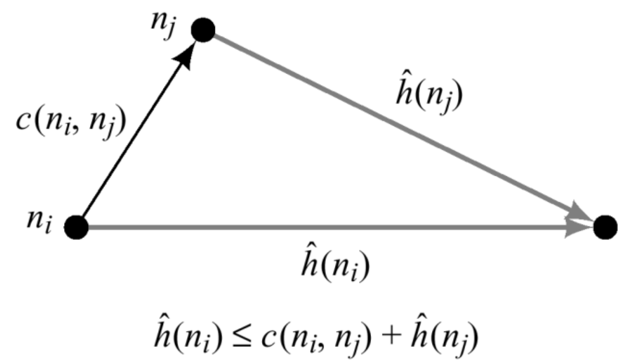
A^* with Consistent Heuristics

- A^* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Summary of Consistent Heuristics

- h is consistent if the heuristic function satisfies triangle inequality for every n and its child node n': h(ni) <= h(nj) + c(ni,nj)



- When h is consistent, the f values of nodes expanded by A* are never decreasing.
- When A* selected n for expansion it already found the shortest path to it.
- When h is consistent every node is expanded once (if check for duplicates).
- Normally the heuristics we encounter are consistent
  - the number of misplaced tiles
  - Manhattan distance
  - straight-line distance

Admissible and Consistent Heuristics?

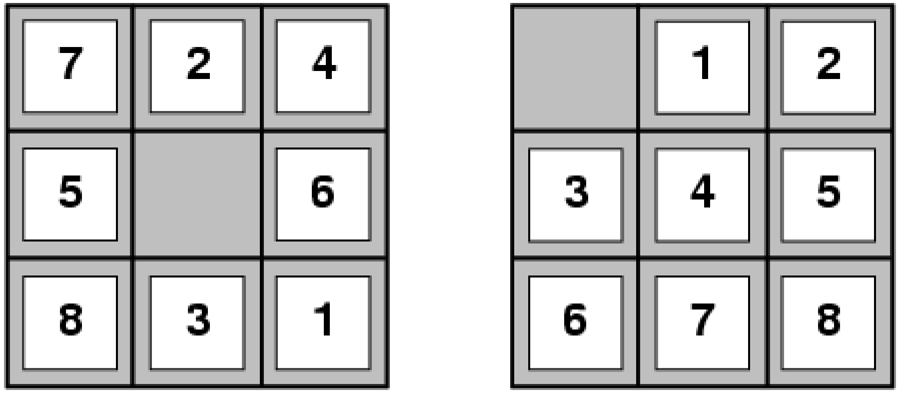
E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)

The true cost is 26.

Average cost for 8-puzzle is 22. Branching degree 3.



Start State Goal State

- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

Summary so far

- Heuristic (informed) search
  - Best-First (guided by heuristic eval fn f)
  - A* as a special case of BF (f=g+h)
  - A* is guaranteed to terminate as long as costs>0 and branching factor is finite (any h)
  - A* is guaranteed to find optimal if h is admissible
    - If not admissible, still useful, but optimality not guaranteed
  - Consistent (monotonic) h
    - Consistent implies admissible
    - When a node is selected for expansion, shortest path to it is found
    - More efficient, do not have to re-open expanded (closed) nodes
    - Expands nodes in increasing order of f

A* properties

- A* expands every path along which $f(n) < C^*$
- A* will never expand any node s.t. $f(n) > C^*$
- If h is consistent A* will expand any node such that $f(n) < C^*$
- Therefore, A* expands all the nodes for which $f(n) < C^*$ and a subset of the nodes for which $f(n) = C^*$.
- Therefore, if $h_1(n) < h_2(n)$ clearly the subset of nodes expanded by h_2 is smaller.

Complexity of A*

- A* is optimally efficient (Dechter and Pearl 1985):
  - It can be shown that all algorithms that do not expand a node which A* did expand (inside the contours) may miss an optimal solution
- A* worst-case time complexity:
  - is exponential unless the heuristic function is very accurate
- If h is exact (h = h*)
  - search focus only on optimal paths
- Main problem: space complexity is exponential
- Effective branching factor:
  - logarithm of base (d+1) of average number of nodes expanded.

Effectiveness of A* search

- How quality of heuristic impact search?
- What is the time and space complexity?
- Is any algorithm better? Worse?
- Case study: the 8-puzzle

Effectiveness of A* Search Algorithm

Average number of nodes expanded

d	IDS	A*(h1)	A*(h2)
2	10	6	6
4	112	13	12
8	6384	39	25
12	364404	227	73
14	3473941	539	113
20	------------	7276	676
24	------------	39135	1641

Average over 100 randomly generated 8-puzzle problems
h1 = number of tiles in the wrong position
h2 = sum of Manhattan distances

Dominance

- Definition: If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 dominates h_1

- *Is* h_2 better for search?

- Typical search costs (average number of nodes expanded):

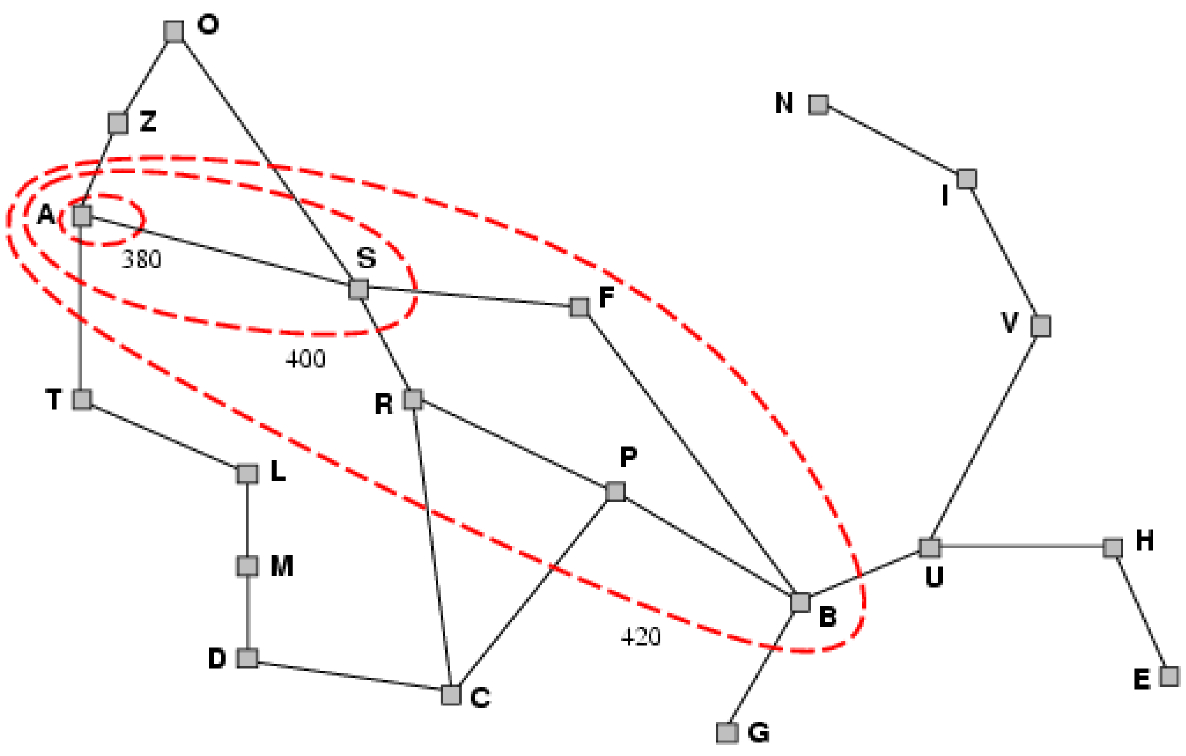
- $d=12$ IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- $d=24$ IDS = out of memory
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Heuristic's Dominance and Pruning Power

- Definition:
  - A heuristic function h_2 (strictly) dominates h_1 if both are admissible and for every node n, $h_2(n)$ is (strictly) greater than $h_1(n)$.
- Theorem (Hart, Nilsson and Raphale, 1968):
  - An A* search with a dominating heuristic function h_2 has the property that any node it expands is also expanded by A* with h_1.
- Question: Does Manhattan distance dominate the number of misplaced tiles?
- Extreme cases
  - $h = 0$
  - $h = h^*$

A* with Consistent Heuristics

- Contours of stronger (dominating) heuristics are inside contours of weaker heuristics



Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

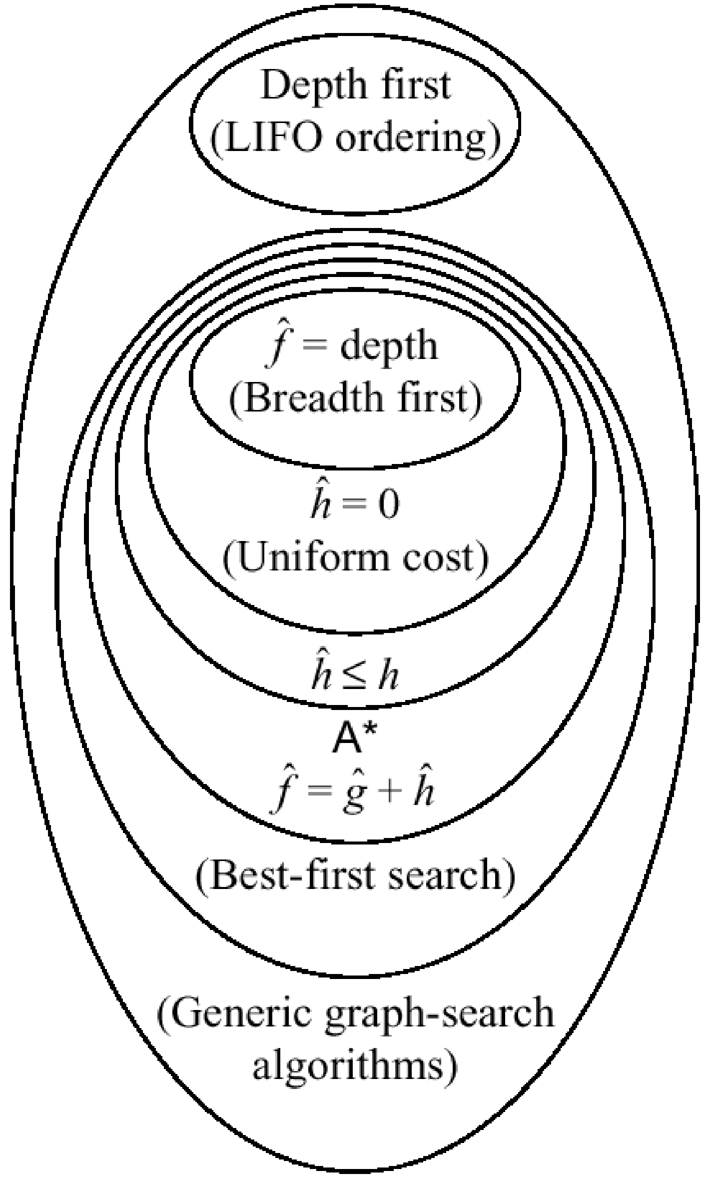
Time?? Exponential in [relative error in h $\times$ length of soln.]

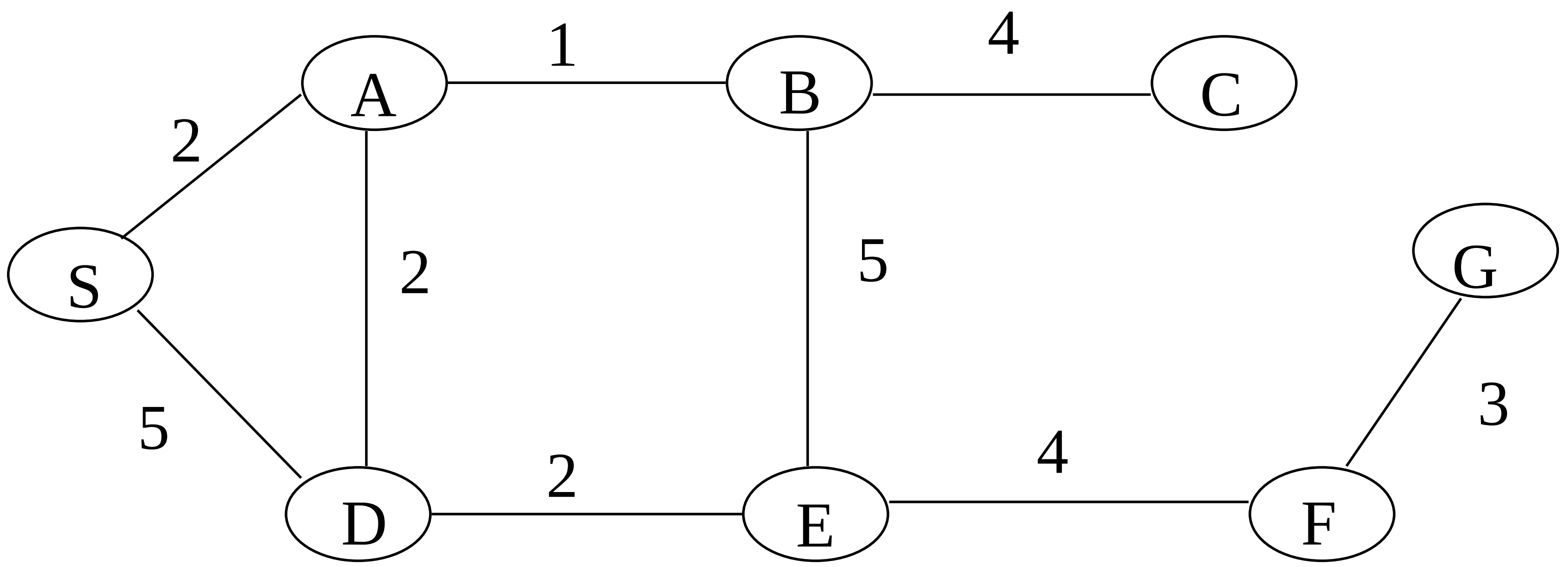
Space?? Keeps all nodes in memory

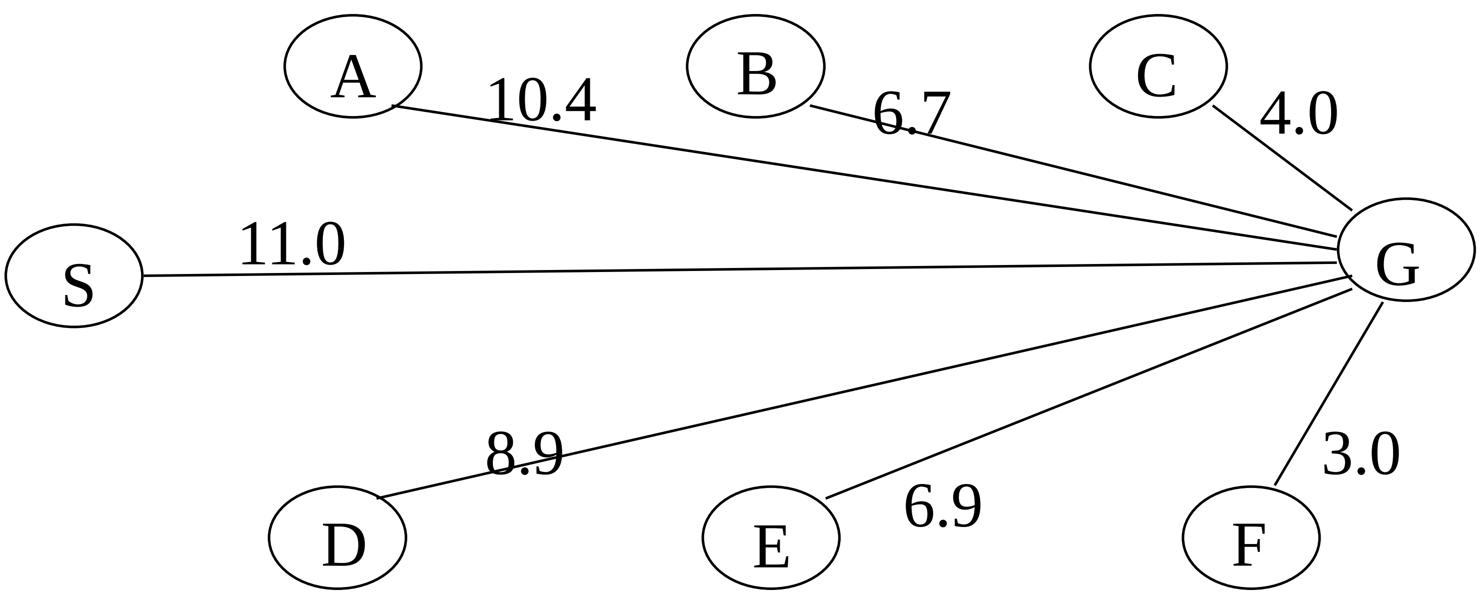
Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$

Relationships among Search Algorithms




A
B
C
S
G
D
E
F
1
4
2
2
5
5
3
2
4


A
B
C
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D
E
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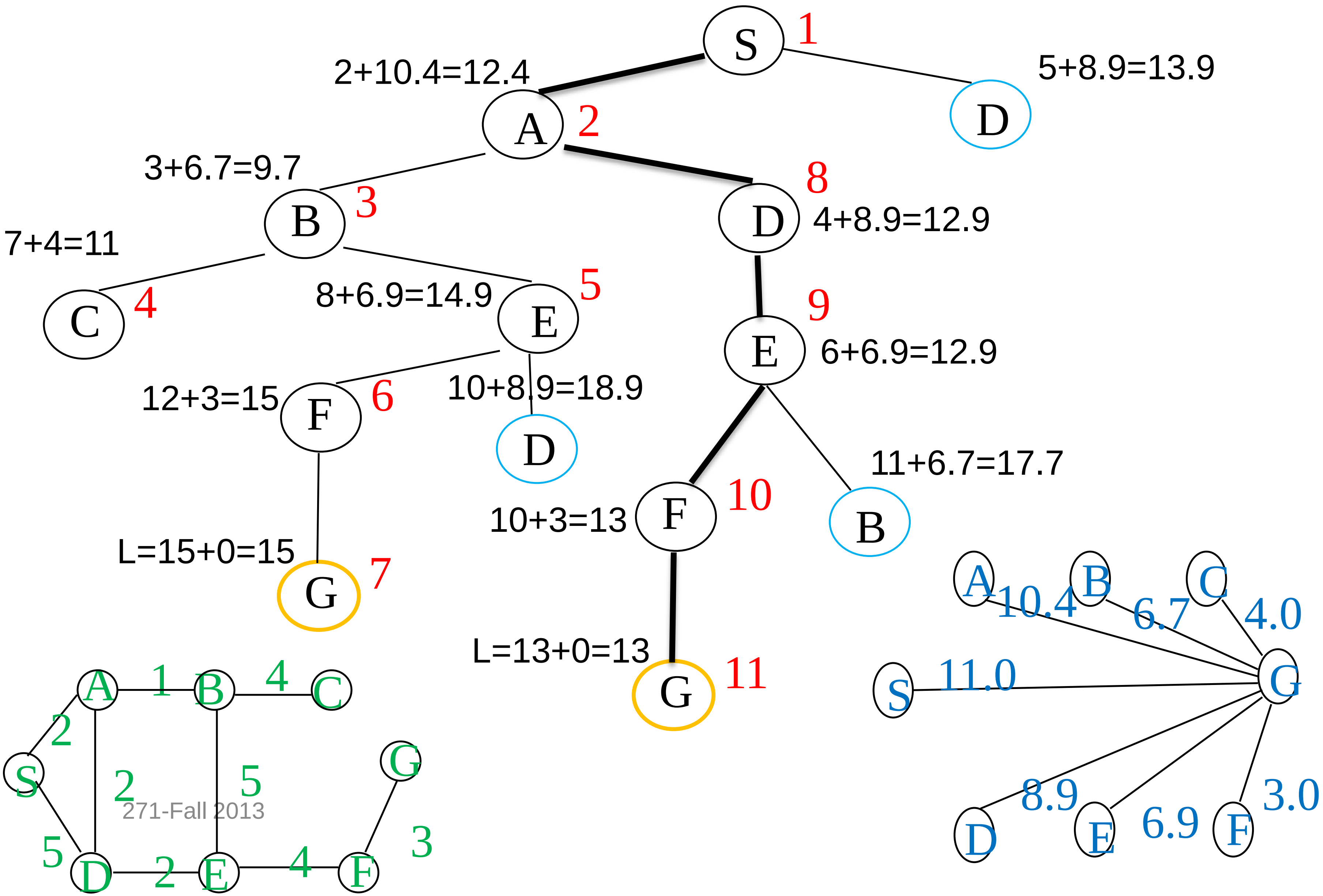
10.4
6.7
4.0
11.0
8.9
6.9
3.0

Example of Branch and Bound in action



Pseudocode for Branch and Bound Search (An informed depth-first search)

```
Initialize: Let Q = {S}, L=∞

While Q is not empty
        pull Q1, the first element in Q
        if f(Q1)>=L, skip it
        if Q1 is a goal compute the cost of the solution and update
                L <-- minimum (new cost, old cost)
        else
                child_nodes = expand(Q1),
                <eliminate child_nodes which represent simple loops>,
                For each child node n do:
                        evaluate f(n).  If f(n) is greater than L discard n.
                end-for
                Put remaining child_nodes on top of queue in the order of their f.
        end
Continue
```

Properties of Branch-and-Bound

- Not guaranteed to terminate unless
  - has depth-bound
  - consistent f and reasonable L
- Optimal:
  - finds an optimal solution (f is admissible)
- Time complexity: exponential
- Space complexity: can be linear
- Advantage:
  - anytime property
- Note : unlike A*, BnB may (will) expand nodes f>C*.

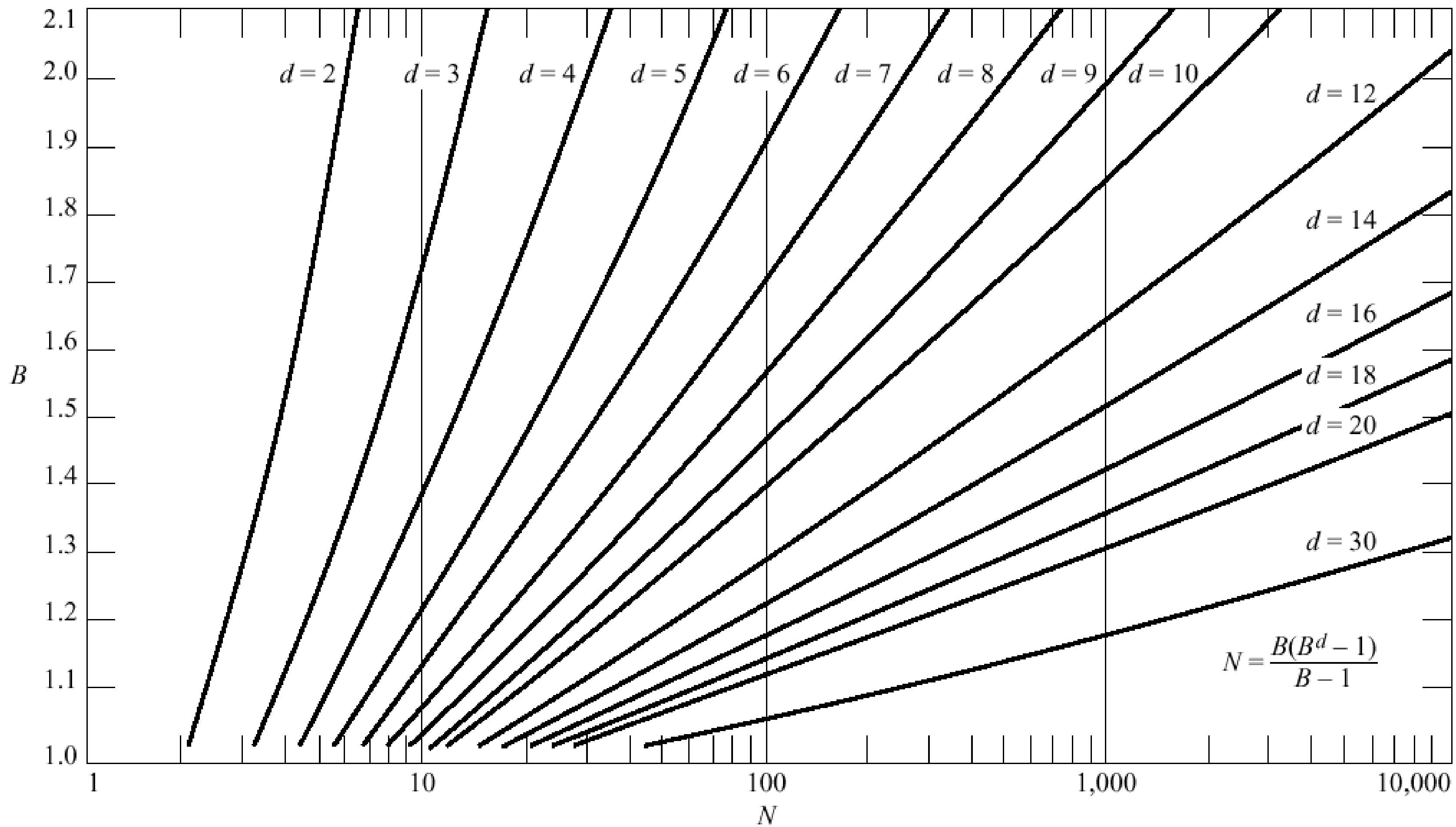
Iterative Deepening A* (IDA*) (combining Branch-and-Bound and A*)

- Initialize: f <-- the evaluation function of the start node
- until goal node is found
  - Loop:
    - Do Branch-and-bound with upper-bound L equal to current evaluation function f.
    - Increment evaluation function to next contour level
  - end

- Properties:
  - Guarantee to find an optimal solution
  - time: exponential, like A*
  - space: linear, like B&B.

  - Problems: The number of iterations may be large.

The Effective Branching Factor



Inventing Heuristics automatically

- Examples of Heuristic Functions for A*
  - The 8-puzzle problem
    - The number of tiles in the wrong position
      - is this admissible?
    - Manhattan distance
      - is this admissible?
  - How can we invent admissible heuristics in general?
    - look at “relaxed” problem where constraints are removed
      - e.g.., we can move in straight lines between cities
      - e.g.., we can move tiles independently of each other

Inventing Heuristics Automatically (cont.)

- How did we
  - find h1 and h2 for the 8-puzzle?
  - verify admissibility?
  - prove that straight-line distance is admissible? MST admissible?
- Hypothetical answer:
  - Heuristic are generated from relaxed problems
  - Hypothesis: relaxed problems are easier to solve
- In relaxed models the search space has more operators or more directed arcs
- Example: 8 puzzle:
  - Rule : a tile can be moved from A to B, iff
    - A and B are adjacent
    - B is blank
  - We can generate relaxed problems by removing one or more of the conditions
    - … A and B are adjacent & B is blank
    - ... if B is blank

Relaxed Problems

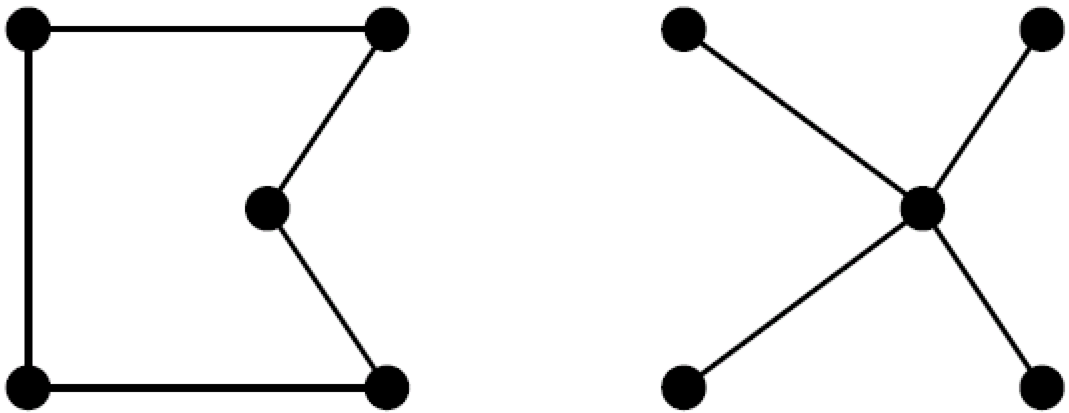
- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ *(number of misplaced tiles)* gives the shortest solution
- If the rules are relaxed so that a tile can move to any h/v adjacent square, then $h_2(n)$ *(Manhatten distance)* gives the shortest solution

Generating heuristics (cont.)

- Example: TSP
- Find a tour. A tour is:
  - 1. A graph with subset of edges
  - 2. Connected
  - 3. Total length of edges minimized
  - 4. Each node has degree 2
- Eliminating 4 yields MST.

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in $O(n^2)$
and is a lower bound on the shortest (open) tour

Automating Heuristic generation

- Use STRIPs language representation:
- Operators:
  - pre-conditions, add-list, delete list
- 8-puzzle example:
  - on(x,y), clear(y) adj(y,z) ,tiles x1,…,x8
- States: conjunction of predicates:
  - on(x1,c1),on(x2,c2)….on(x8,c8),clear(c9)
- move(x,c1,c2) (move tile x from location c1 to location c2)
  - pre-cond: on(x1,c1), clear(c2), adj(c1,c2)
  - add-list: on(x1,c2), clear(c1)
  - delete-list: on(x1,c1), clear(c2)
- Relaxation:
  - Remove from precondition: clear(c2), adj(c2,c3) → #misplaced tiles
  - Remove clear(c2) → Manhattan distance
  - Remove adj(c2,c3) → h3, a new procedure that transfers to the empty location a tile appearing there in the goal
- The space of relaxations can be enriched by predicate refinements
  - adj(y,z) = iff neigbour(y,z) and same-line(y,z)

Heuristic generation

- Theorem: Heuristics that are generated from relaxed models are consistent.

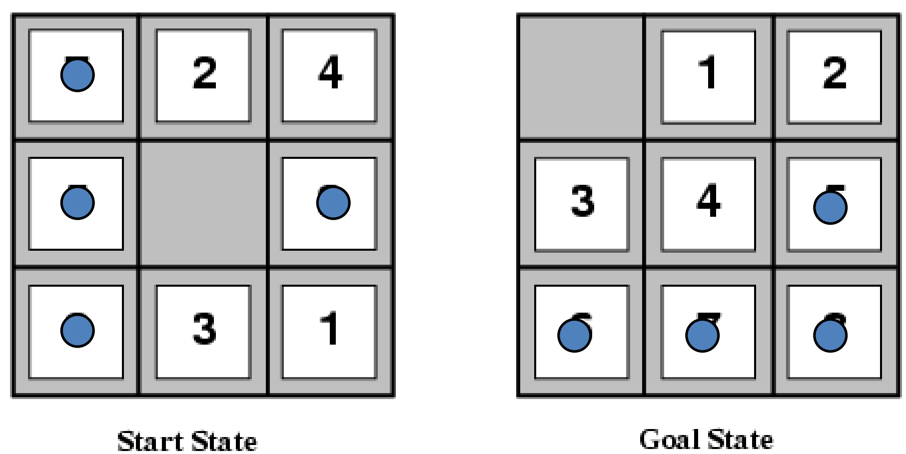
- Proof: h is true shortest path in a relaxed model
  - h(n) <=c’(n,n’)+h(n’) (c’ are shortest distances in relaxed graph)
  - c’(n,n’) <=c(n,n’)
  - → h(n) <= c(n,n’)+h(n’)

Heuristic generation

- Total (time) complexity = heuristic computation + nodes expanded
- More powerful heuristic – harder to compute, but more pruning power (fewer nodes expanded)
- Problem:
  - not every relaxed problem is easy
    - How to recognize a relaxed easy problem
    - A proposal: a problem is easy if it can be solved optimally by a greedy algorithm
- Q: what if neither h_1 nor h_2 is clearly better? $\max(h_1, h_2)$
- Often, a simpler problem which is more constrained is easier; will provide a good upper-bound.

Improving Heuristics

- Reinforcement learning.
- Pattern Databases: you can solve optimally a sub-problem



Pattern Databases

- For sliding tiles and Rubic's cube
- For a subset of the tiles compute shortest path to the goal using breadth-first search
- For 15 puzzles, if we have 7 fringe tiles and one blank, the number of patterns to store are 16!/(16-8)! = 518,918,400.
- For each table entry we store the shortest number of moves to the goal from the current location.
- Use different subsets of tiles and take the max heuristic during IDA* search. The number of nodes to solve 15 puzzles was reduced by a factor of 346 (Culberson and Schaeffer)
- How can this be generalized? (a possible project)

Problem-reduction representations AND/OR search spaces

- The erratic vacuum world (actions are non-deterministic)
- Graphical models
- Decomposable production systems (Natural language parsing)

  Initial database: (C,B,Z)

  Rules: R1: C $\rightarrow$(D,L)

  R2: C$\rightarrow$ (B,M)

  R3: B$\rightarrow$ (M,M)

  R4: Z $\rightarrow$ (B,B,M)

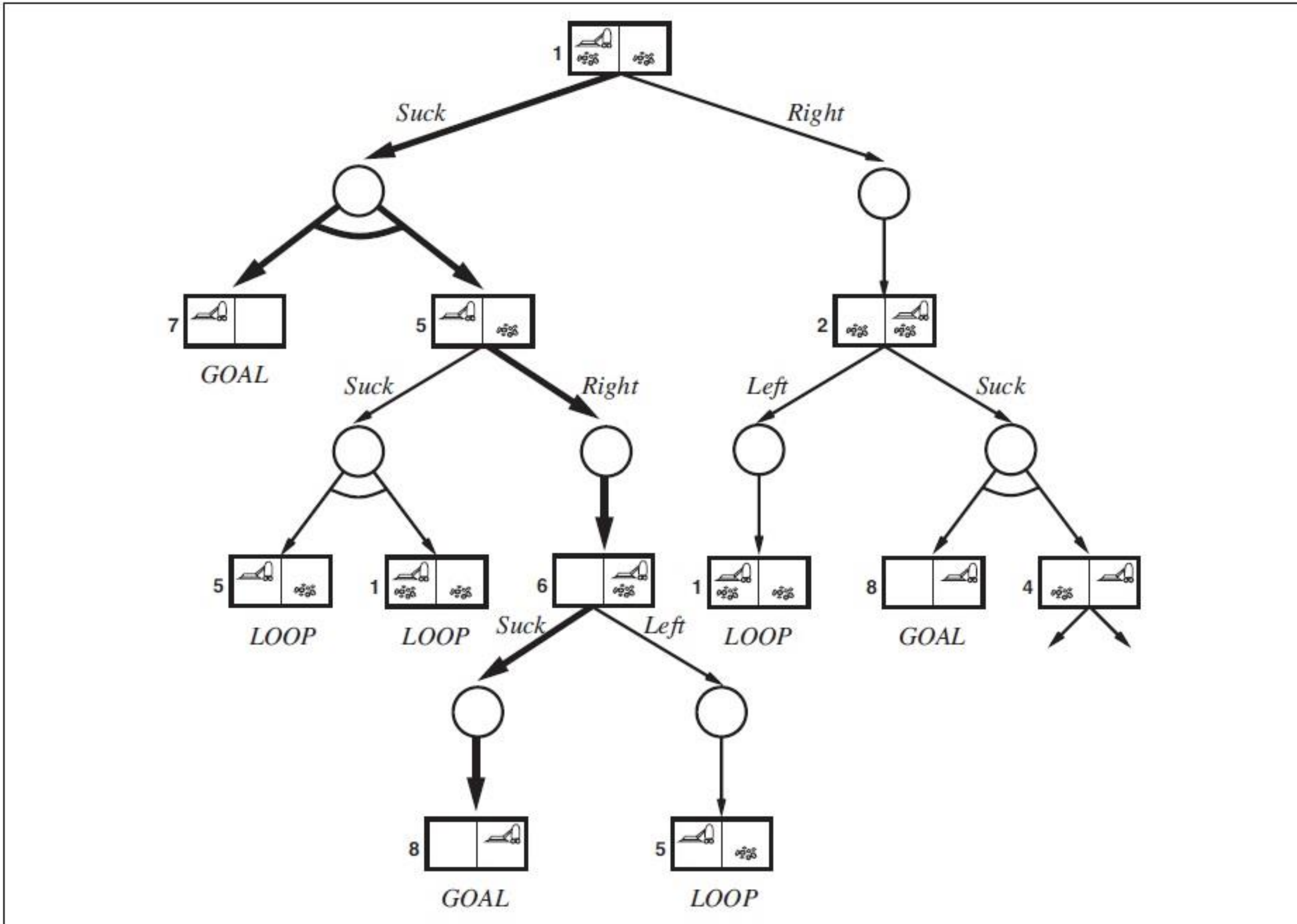
  Find a path generating a string with M's only.
- The tower of Hanoi

  To move n disks from peg 1 to peg 3 using peg 2

  Move n-1 pegs to peg 2 via peg 3,

  move the nth disk to peg 3,

  move n-1 disks from peg 2 to peg 3 via peg 1.


1
Suck
Right
7
GOAL
5
Suck
Right
2
Left
Suck
5
LOOP
1
LOOP
6
Suck
Left
1
LOOP
8
GOAL
4
8
GOAL
5
LOOP

AND/OR Graphs

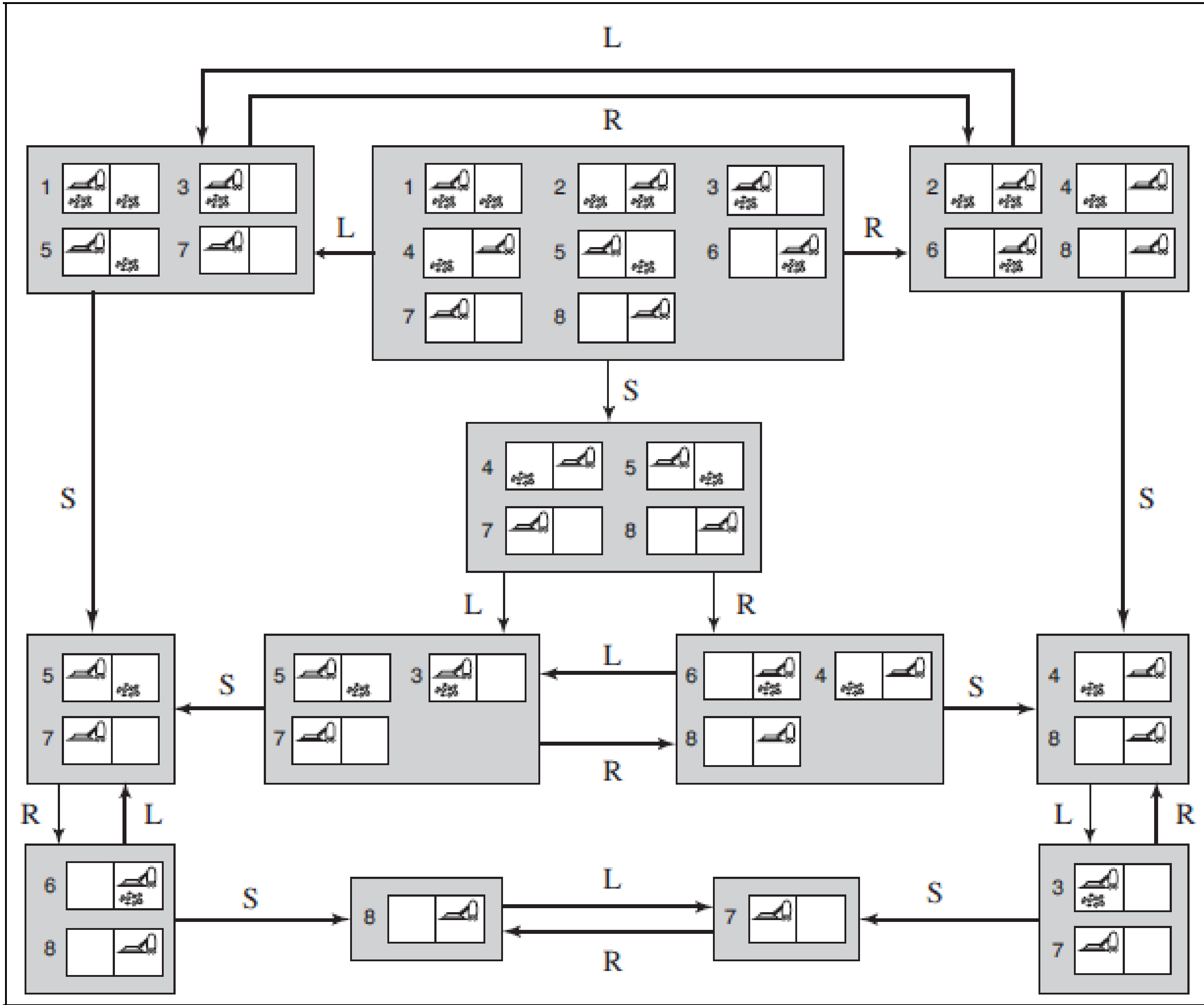
- Nodes represent subproblems
  - AND links represent subproblem decompositions
  - OR links represent alternative solutions
  - Start node is initial problem
  - Terminal nodes are solved subproblems
- Solution graph
  - It is an AND/OR subgraph such that:
    - It contains the start node
    - All its terminal nodes (nodes with no successors) are solved primitive problems
    - If it contains an AND node A, it must contain the entire group of AND links that leads to children of A.

Algorithms searching AND/OR graphs

- All algorithms generalize using hyper-arc successors rather than simple arcs.
- AO*: is A* that searches AND/OR graphs for a solution subgraph.
- The cost of a solution graph is the sum cost of it arcs. It can be defined recursively as: $k(n,N) = c_n+k(n1,N)+\ldots k(n_k,N)$
- $h^*(n)$ is the cost of an optimal solution graph from n to a set of goal nodes
- $h(n)$ is an admissible heuristic for $h^*(n)$
- Monotonicity:
- $h(n) \le c+h(n1)+\ldots h(nk)$ where n1,…nk are successors of n
- AO* is guaranteed to find an optimal solution when it terminates if the heuristic function is admissible

Beyond Classical Search (chapter 4)

- Local search for optimization
  - Greedy hill-climbing search, simulated annealing, local beam search, genetic algorithms.
  - Local search in continuous spaces
  - SLS : "Like climbing Everest in thick fog with amnesia"
- Searching with non-deterministic actions
  - The erratic vacuum cleaner example
  - Using AND/OR search spaces; solution is a contingent plan.
- Searching with partial observations
  - Using belief states
- Online search agents and unknown environments
  - Actions, costs, goal-tests are revealed in state only
  - Exploration problems. Safely explorable


L
R
1 3 5 7
1 2 3 4 5 6 7 8
2 4 6 8
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4 5 7 8
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5 7
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5 3 7
L
R
6 4 8
S
4 8
R
L
L
R
6 8
S
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L
R
7
S
3 7

Summary

- In practice we often want the goal with the minimum cost path
- Exhaustive search is impractical except on small problems
- Heuristic estimates of the path cost from a node to the goal can be efficient in reducing the search space.
- The A* algorithm combines all of these ideas with admissible heuristics (which underestimate) , guaranteeing optimality.
- Properties of heuristics:
  - admissibility, consistency, dominance, accuracy
- Reading
  - R&N Chapters 3-4