

Each problem is worth 10 points. Open book and notes.

Questions 1 to 5 refer to this scenario: An ESP experiment is done in which a participant guesses which of 4 cards the researcher has randomly picked, where each card is equally likely. This is repeated for 220 trials and the participant guesses correctly 72 times. The null hypothesis is that the subject is guessing, while the alternative is that the subject has ESP and can guess at higher than the chance rate.

1. State the null and alternative hypotheses in symbols.

If the participant were just guessing, the probability that he or she would get the answer right on each trial is $\frac{1}{4}$. So, p = probability that the participant gets the right answer and the null hypothesis is that this is $\frac{1}{4}$.

$$H_0: p = .25$$

$$H_a: p > .25$$

2. Calculate the value of the test statistic.

$$\hat{p} = \frac{72}{220} = .327, \text{ null standard error} = \sqrt{\frac{(.25)(.75)}{220}} = .029, \text{ so } z = \frac{.327 - .25}{.029} = 2.65$$

3. Find the p-value for the test. Draw a sketch of the appropriate distribution showing this value.

The p-value is the area to the right of z , because this is a one-tailed test with $>$ as the alternative hypothesis. Therefore, the p-value is the area above 2.65. From Table A.1 this is .004.

The sketch should illustrate a standard normal curve with a very small area above 2.65 shaded.

4. Make a conclusion and state it in the context of the problem.

Because the p-value is less than .05, reject the null hypothesis. Conclude that the participant can guess at a higher than chance rate.

5. Explain what a Type 1 error would be in this situation.

A type 1 error is made when the null hypothesis is true, but it is rejected anyway. In this context, a type 1 error is made when a participant who does not have ESP does well enough in the test to convince us that he or she does.

Questions 6 to 10 refer to this scenario: A study was done to see if listening to Mozart would help increase IQ in the short-term, compared to listening to a relaxation tape. (Honest! The data presented here are simplified, but reflect what was found.) Participants came to the laboratory and listened to a relaxation tape for half an hour one week and Mozart for half an hour a different week. (The order was randomized for each participant to rule out learning effects.) The difference in score on an IQ test for the two conditions was computed for each participant, with a positive difference meaning that the IQ was higher after listening to Mozart. The mean of the differences was 4.3 points and the standard deviation of the differences was 5.8 points. There were 10 participants.

6. State the null and alternative hypotheses in symbols. Be sure to use the correct symbols.

This is a paired difference situation. Define μ_d = hypothetical mean difference in IQ for the population after listening to Mozart or listening to a relaxation tape.

$$H_0: \mu_d = 0$$

$$H_a: \mu_d > 0$$

7. Calculate the value of the test statistic.

$$t = \frac{\bar{d} - 0}{s/\sqrt{n}} = \frac{4.3 - 0}{5.8/\sqrt{10}} = 2.34$$

8. Draw a sketch of the appropriate distribution showing the p-value and find the p-value range for the test. (You can't find an exact p-value with the tables you have available.)

The p-value is the area above the value of the test statistic in a t-distribution with $df = n - 1 = 9$. From Table A.3, we can tell that $0.015 < p\text{-value} < 0.022$, because the test statistic is between 2.33 and 2.58.

The sketch should show a t-distribution (which looks like a standard normal distribution) with the area above 2.34 shaded.

9. Make a conclusion and state it in the context of the problem.

Because the p-value is $< .05$, reject the null hypothesis. Conclude that the mean IQ in the population if one were to listen to Mozart would be higher than if one were to listen to a relaxation tape.

10. Explain what a Type 2 error would be in this situation.

A type 2 error would occur if the Mozart listening really does raise IQ but the experiment failed to detect this.