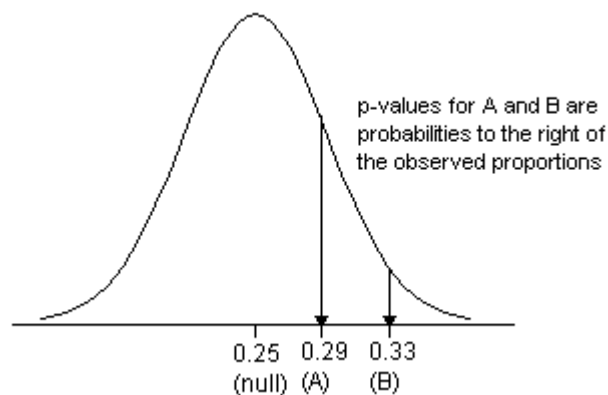


CHAPTER 11
EXERCISE SOLUTIONS

- 11.1** a. Null hypothesis.
b. Alternative hypothesis.
c. Alternative hypothesis.
d. Null hypothesis.
- 11.3** a. No. A null hypothesis is a statement about a population value, not a sample value as in the statement for this exercise.
b. Yes. It's a statement of no difference and could be interpreted to apply to all newborns.
c. No. This statement would more appropriately be an alternative hypothesis (a statement of a difference).
- 11.5** a. There are 5 choices so $p = 1/5 = .2$. Random selection means that each of the five choices is equally likely to be chosen.
b. $H_0: p = .2$ (random selection)
 $H_a: p < .2$ (less often than random selection would give)
- 11.6** $H_0: p = .65$ (same as standard)
 $H_a: p > .65$ (better than standard)
The null hypothesis could also be written as $H_0: p \leq .65$.
- 11.7** The decision to use a one-sided or two-sided alternative hypothesis should be made before looking at the sample data. The alternative hypothesis usually expresses the research hypothesis, which in turn gives the reason for collecting the data. The reason for collecting the data should be defined before the data are collected. *Note:* In Section 11.4 it will become clear that looking at the data first and making the alternative hypothesis consistent with it may distort the p -value, leading to the conclusion of statistical significance more often than is warranted.
- 11.9** a. Population is all school-aged children living in the state at the time of the study.
Proportion of interest = proportion of them who live with one or more grandparents.
b. Population consists of all people in the region being studied in the new research.
Proportion of interest = proportion of these people who have the unique genetic trait.
c. Population consists of all pieces of this type of candy made by the company.
Proportion of interest = the proportion of them that are red.
- 11.10** a. 0.03.
b. $\alpha = 0.05$.
c. $\hat{p} = 61/100 = .61$.
d. $n = 100$.
- 11.12** a. The woman is not pregnant.
b. The woman is pregnant.
c. She is not pregnant.
d. She is pregnant.
- 11.15** a. $H_0: p = .10$ (same as proportion in general population)
 $H_a: p > .10$ (greater than proportion in general population)
 p = population proportion of artists who are left-handed.
b. Sample proportion is $\hat{p} = 18/150 = .12$ that are left-handed.
c. The p -value is the probability that the sample proportion would be .12 or larger (for a sample of $n = 150$) if the population proportion actually is .10.

- 11.17**
- Cannot reject the null hypothesis. The p -value (0.35) is greater than 0.05. The observed result is not statistically significant.
 - Reject the null hypothesis (or accept the alternative hypothesis). The p -value (0.001) is less than 0.05. The observed result is statistically significant.
 - Reject the null hypothesis (or accept the alternative hypothesis). The p -value (0.04) is less than 0.05. The observed result is statistically significant.
- 11.18** The p -value will be smaller for Researcher B because that researcher's sample proportion is farther from $p = .25$ (the null value). So, it is stronger evidence against the null hypothesis. In general, the stronger the evidence against a null hypothesis the smaller the p -value. If the null hypothesis were true, the distribution of possible sample proportions would be approximately a normal curve centered at .25 (by the Rule for Sample Proportions, Chapter 9). The area to the right of .33 is smaller than the area to the right of .29 (so the p -value is smaller for Researcher B). The following figure illustrates the situation.

Figure for Exercise 11.18



- 11.19**
- Null:* Probability = .5 that prediction is correct
Alternative: Probability > .5 that prediction is correct.
 - The necessary probability (the p -value) is the probability of 6 or more correct predictions in 10 tries if the long-run probability for a correct prediction is .5.
 - No, this does not prove the skeptic's point. The sample size is too small for the results to be conclusive one way or the other. With $n = 10$ tries the margin of error is large, so we can't estimate the physician's true success rate very precisely. *Note:* Using the appropriate confidence interval methods for a small sample, Minitab gives the "exact" 95% confidence interval as .262 to .878 for the physician's true proportion of correct predictions. Based on this interval, both the null and alternative hypotheses are plausible.
 - The null hypothesis cannot be rejected. The p -value is not smaller than the specified level of significance ($\alpha = .05$).
 - Make predictions for a larger sample of pregnancies. Notice that he correctly predicted the sex for $6/10 = 60\%$ of the babies. If the physician were able to correctly predict the sex 60% of the time over many tries (200 for instance), the result would be statistically significant.
- 11.23**
- The proportion of all students favoring a new alcohol policy on campus.
 - The proportion of all company employees whom are left handed.
 - The proportion of all stockbrokers believing the market will go up next year.
 - The proportion of all mall visitors that buy something.

$$11.24 \quad \text{a. } \frac{.6 - .5}{\sqrt{\frac{.5(1-.5)}{30}}} = 1.10.$$

$$\text{b. } \frac{.1 - .25}{\sqrt{\frac{.25(1-.25)}{60}}} = -2.68.$$

$$\text{c. } \frac{.3 - .2}{\sqrt{\frac{.2(1-.2)}{500}}} = 5.59.$$

$$\text{d. } \frac{.5 - .8}{\sqrt{\frac{.8(1-.8)}{200}}} = -10.61.$$

- 11.25
- a. $2 \times P(Z < -1.10) = 2(.1357) = .2714$. This is total of the area to the left of -1.10 and the right of $+1.10$.
- b. $P(Z < -2.68) = .0037$.
- c. $P(Z > 5.59) \approx 0$. Using the “In the Extreme” portion of Table A.1, we see that $P(Z < 5.61) = .99999999$, so $P(Z > 5.59) \approx 1 - .99999999 = .00000001$, or about 1 in 100 million. In other words, about 0.
- d. $P(Z < -10.61) \approx 0$. This value of z is even more extreme than “In the Extreme.”

- 11.27
- a. $H_0: p \leq .50$ (one-half or less do)
 $H_a: p > .50$ (more than one-half do)

p = proportion of all adult American Catholics who favor allowing women to be priests.

Usually the “yes” answer to the research question is the alternative hypothesis, and that is what has been done here.

b. The sample was randomly selected, which is one of the necessary conditions. The sample size is large enough, which is the other necessary condition. Both np_0 and $n(1-p_0)$ are greater than 10, as they should be to use a z -statistic. Here, $n = 507$ and $p_0 = .50$.

$$\text{c. } z = \frac{\text{Sample estimate} - \text{Null value}}{\text{Null standard error}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.59 - .50}{\sqrt{\frac{.5(1-.5)}{507}}} = \frac{.09}{.0222} = 4.05$$

d. p -value $\approx .00003$. This is the probability (area) to the right of $z = 4.05$. To find the p -value exactly, use software or a calculator that can give normal curve probabilities. Table A.1 can be used to approximate the p -value. Due to symmetry, the area to the right of $z = 4.05$ equals the area to the left of $z = -4.05$. Near the bottom of left-side page of Table A.1, probabilities (areas) to the left of -3.72 and -4.25 are given as .0001 and .00001 respectively. The correct p -value is between these two values. Typically, this information may be stated as p -value $< .0001$.

e. Reject the null hypothesis and decide in favor of the alternative hypothesis. The p -value is smaller than 0.05. This is evidence that more than one-half ($p > .50$) of all adult American Catholics favor allowing women to be priests.

$$\text{f. Standard error} = s.e.(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.58(1-.58)}{507}} = .022.$$

An approximate 95% confidence interval is $\hat{p} \pm 2 \times \text{standard error}$, which is $.59 \pm (2)(.022)$, or .55 to .63. All values in this confidence interval are greater than .50. This confirms the conclusion that a majority of adult American Catholics favor allowing women priests.

- 11.28
- a. We can't be certain that the conditions are met because we don't know whether the sample was selected randomly. We do know that the sample size is large enough. Both np_0 and $n(1-p_0)$ are

greater than 10, as they should be to use a z -statistic. Here, $n = 180$ and $p_0 = .1$ (the proportion in the general population).

b. Step 1: $H_0: p = 0.1$ (proportion left-handed same for artists as in general population)
 $H_a: p > 0.1$

Step 2: See part (a) for discussion of the necessary conditions.

The test statistic is $z = \frac{\text{Sample estimate} - \text{Null value}}{\text{Null standard error}}$

Sample estimate = $\hat{p} = 15/180 = .12$

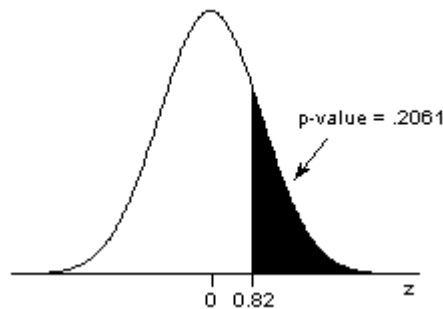
Null value = $p_0 = .1$

Null standard error = $\sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{.1(1-.1)}{150}} = .02449$

$z = \frac{\text{Sample estimate} - \text{Null value}}{\text{Null standard error}} = \frac{.12 - .10}{.0245} = \frac{.02}{.02449} = 0.82$

Step 3: p -value $\approx .21$. This is the probability that $z > 0.82$, which is illustrated in the following figure. By symmetry, $P(z > 0.82) = P(z < -0.82)$. Table A.1 gives $P(z < -0.82) = .2061$.

Figure for Exercise 11.28b



Note: An exact p -value can be found as the probability that $X \geq 18$ in a binomial distribution with $n = 150$ and $p = .10$. The exact p -value given by Minitab is .242.

Step 4: Cannot reject the null hypothesis. The result is not statistically significant because the p -value (.2061) is greater than .05.

Step 5: We cannot conclude that artists are more likely to be left-handed than people in the general population.

- 11.29** The p -value would be $2 \times .04 = .08$. As long as the p -value is less than .5, the p -value for a two-sided test is twice the p -value for a one-sided test of the same proportion. We're assuming that the null value p_0 is the same for both the one-sided and two-sided tests.
- 11.35** Very large. A very large sample may provide enough evidence to detect small and possibly unimportant differences from the null value. This leads to a conclusion of statistical significance, even though the result may not have practical "real world" significance.
- 11.36** Very small. With a small sample, the standard error is relatively large. As a result the sample result may not be statistically significant, even when in truth the null hypothesis is false.