# Results from the 2016 AP Statistics Exam

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## The six free-response questions for 2016

- Question #1: Robin's tips
  - Describe histogram; effect of changing a value on mean & median
- Question #2: Effect of ad on children's snack choices
  - Chi-square test; describe the effect of ads on children's choices
- Question #3: Association between smoking & Alzheimer's
  - Explanatory, response, type of study, confounding variable
- Question #4: Super ignitor for launching model rockets
  - Probability for independent events, time to first failure, reasoning
- Question #5: Poll for environment vs economy statement
  - ► Confidence interval for proportion; reason for conditions
- Question #6 (investigative task): Semesters and salaries
  - ► Association between two variables, with and without a 3<sup>rd</sup> variable





#### Plan for each question

- State question
- Present solution
- Describe common student errors
- Suggest teaching tips
- Report average score (all at the end)





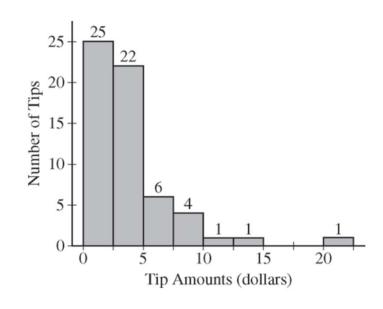
Question #1:
Robin's tips
Describe histogram; effect of
changing a value on mean & median





#### Question 1, Part (a)

Robin works as a server in a small restaurant, where she can earn a tip (extra money) from each customer she serves. The histogram shows the distribution of her 60 tip amounts for one day of work.



(a) Write a few sentences to describe the distribution of tip amounts for the day shown.

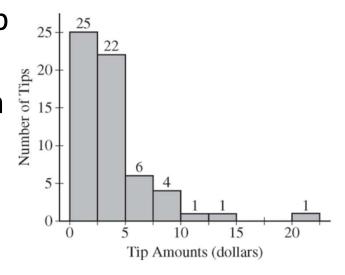


# **Solution Part (a)**

The distribution of Robin's tip amounts is skewed to the right. There is a gap between the largest tip amount (in the \$20 to \$22.50 interval) and the second largest tip amount (in the \$12.50 to \$15 interval), and the largest tip amount appears to be an outlier. The median tip amount is

between \$2.50 and \$5.00. Robin's tip amounts vary from a minimum of between \$0 and \$2.50 to a maximum of between \$20.00 and \$22.50.

About 78% of the tip amounts are between \$0 and \$5.





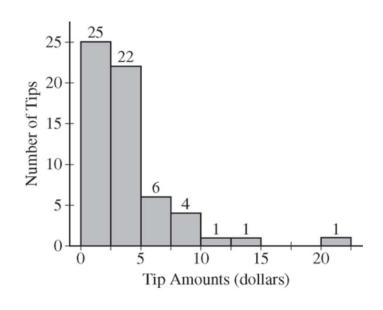
## **Common Student Errors – Part (a)**

- Had difficulty describing variability from a histogram
- Made no mention of the gap/outlier
- ▶ Didn't include context (tip amounts) or mistook units (dollars) for context
- Provided an acronym for describing the distribution (like SOCS or CUSS), but didn't discuss all the elements
- Described the frequency of the values in each interval (the heights of the histogram bars)



# **Question Part (b)**

Robin works as a server in a small restaurant, where she can earn a tip (extra money) from each customer she serves. The histogram shows the distribution of her 60 tip amounts for one day of work.



(b) One of the tip amounts was \$8. If the \$8 tip had been \$18, what effect would the increase have had on the following statistics? Justify your answers.

The mean:

The median:



# **Solution Part (b)**

► The mean: If the \$8 tip had been \$18, the mean would increase by \$10 divided by 60, or \$1/6, or about 17 cents.

The median: If the \$8 tip had been \$18, the median would not change because the current median is between \$2.50 and \$5.00 and both \$8 and \$18 are bigger than that.



#### **Common Student Errors- Part (b)**

- ► Thinking a new value of \$18 was added to the data set.
- ▶ Not using the values (\$8 and \$18) in their justifications
- Not clearly justifying why mean would increase
- Arguing that the mean is not resistant to extreme values, but not indicating that \$18 is an extreme value.
- Not clearly justifying why median would stay the same
- Thought the median only changes if n changes or if \$8 or \$18 was the median
- Included inappropriate modifiers (mean is likely to increase or median probably won't change)
- Made up a data set that was unrelated to the histogram provided to investigate effect on mean and median, but never connected back to Robin's tip distribution





# **Question 1 Teaching Tips**

- ➤ Give more exposure to analyzing distributions of quantitative data in which some/most of the data values are unknown.
- Provide more opportunities to discuss distributions in context without simply listing features
- Stress that context is about the variables of interest.
- Give students ample practice making statistical arguments and receiving critical feedback.
- ▶ Be sure students can distinguish the effects of:
  - Changing one value in a data set
  - Adding or removing a value from a data set
  - Transforming all the values in a data set via addition, subtraction, multiplication, division





# **More Question 1 Teaching Tips**

- Build in spiral review throughout the course. Revisit topics and emphasize connections, comparisons, contrasts.
- Distinguish when it is appropriate to give a specific numerical value rather than an approximate value when describing some aspect of a distribution.
- Show students the physical interpretation of the mean as the balance point of the distribution.



# Question #2: Effect of ad on children's snack choices

Chi-square test; describe the effect of ads on children's choices





#### **Question 2**

Product advertisers studied the effects of television ads on children's choices for two new snacks. The advertisers used two 30-second television ads in an experiment. One ad was for a new sugary snack called Choco-Zuties, and the other was for a new healthy snack called Apple-Zuties.

For the experiment, 75 children were randomly assigned to one of three groups, A, B, or C. Each child individually watched a 30-minute television program that was interrupted for 5 minutes of advertising.



#### **Question 2, continued**

The advertising was the same for each group with the following exceptions.

- The advertising for group A included the Choco-Zuties ad but not the Apple-Zuties ad.
- The advertising for group B included the Apple-Zuties ad but not the Choco-Zuties ad.
- ► The advertising for group C included neither the Choco-Zuties ad nor the Apple-Zuties ad.



#### **Question 2, continued**

After the program, the children were offered a choice between the two snacks. The table below summarizes their choices.

Group	Type of Ad	Number Who Chose Choco-Zuties	Number Who Chose Apple-Zuties
Α	Choco-Zuties only	21	4
В	Apple-Zuties only	13	12
С	Neither	22	3



#### **Problem 2, continued**

## Part (a)

Do the data provide convincing statistical evidence that there is an association between type of ad and children's choice of snack among all children similar to those who participated in the experiment?

## Part (b)

Write a few sentences describing the effect of each ad on children's choice of snack.





#### **Solution** Part (a)

Step 1: State a correct pair of hypotheses.

H<sub>0</sub>: the proportion of children who would choose each snack is the same regardless of which type of ad is viewed.

 H<sub>a</sub>: the proportion of children who would choose each snack differs based on which type of ad is viewed.

#### **Solution** Part (a), continued:

Step 2: Identify a correct test procedure (by name or formula) and check appropriate conditions.

Appropriate test: chi-square test of homogeneity

The conditions for this test are satisfied because

- (1) the question states that the children were randomly assigned to groups, and
- (2) all expected counts are at least 5.

18.67	6.33
18.67	6.33
18.67	6.33

#### **Solution** Part (a), continued:

$$\chi^2$$
 = 10.291,  $df$  = 2,  $p$ -value = 0.006

Because the p-value is very small (for instance, much smaller than  $\alpha = 0.05$ ), we would reject the null hypothesis at the 0.05 level (and at the 0.01 level) and conclude that the sample data provide strong evidence that the proportions who would choose each snack differ based on which ad is viewed.

#### **Solution** Part (b)

When neither ad was viewed, 22/25 or 88% of the children chose Choco-Zuties whereas only 12% chose Apple-Zuties.

When the Choco-Zuties ad was viewed, 84% of the children chose Choco-Zuties, which was very similar to the percentage that chose them without viewing any ad. So watching the Choco-Zuties ad did not affect the snack choice very much.

When the Apple-Zuties ad was viewed, only 13/25 or 52% of the children chose Choco-Zuties and 48% chose Apple-Zuties. So watching the Apple-Zuties ad seemed to increase the proportion of children choosing Apple-Zuties.

#### **Common Errors in Part (a)**

<u>Section 1</u> (Hypotheses, name of test, conditions)

- Hypotheses reversed
- Random sample vs. random assignment
- Expected values = 12.5, or missing
- Normality check vs. sample size check

Section 2 (Mechanics)

- Most students did a good job here
- Not using calculator efficiently
- Forgetting to report degrees of freedom

Section 3 (Linkage and conclusion)

- Students did a good job here
- Most introduced an  $\alpha$  level
- Many students reported .0058 > .05

#### **Common Errors in Part (b)**

#### Part (b) (Effectiveness of ads)

- Many incorrectly stated that A was effective
- Many just <u>summarized</u> what was in the table without assessing effectiveness
- Students tried to assess effectiveness of ads by comparing individual  $\chi^2$  contributions

# **Teaching Tips for Question 2**

#### Many issue regarding conditions:

- Address the specific conditions needed to justify each inference method and caution students to avoid a "laundry list" (shotgun) approach to listing and checking off conditions, such as:
  - ► Normal v
  - ► Independent √
  - ► SRS √
- ► Emphasize that a statement regarding conditions, or a comparison that is based on numerical results (e.g., expected counts > 5), must be justified with appropriate calculations shown or reference to the numbers given in the problem that support the statement.



# **More Teaching Tips for Question 2**

- Emphasize the distinction between random assignment and random sampling.
- ➤ Avoid the use of abbreviations such as "SRS" as an acceptable way of describing randomness conditions generally, and require that students describe in words (complete sentences) the conditions they are checking and whether or not they are satisfied.
- Inform students that degrees of freedom should always be reported when applicable
- Specify  $\alpha$  level when one is not given in the problem. It not only conveys that the p-value must be small, but also gives a threshold for "smallness."





## **More Question 2 Teaching Tips**

- Emphasize with students the need to clearly state hypotheses in terms of populations, and not use terms that imply that the hypotheses are about the data that was collected in this study or experiment.
- ▶ Address with students how to use technology to carry out the mechanics of tests, and in particular what should be reported from the calculator to justify their response. For instance, for a chi-square test this should include the value of the test statistic, the degrees of freedom, and the *p*-value. Students should avoid "calculator speak" in conveying information key information (e.g. the degrees of freedom).



Question #3:
Association between smoking &
Alzheimer's
Explanatory, response, type of study,
confounding variable





#### **Question 3**

Alzheimer's disease results in a loss of cognitive ability beyond what is expected with typical aging. A local newspaper published an article with the following headline.

Study Finds Strong Association Between Smoking and Alzheimer's

The article reported that a study tracked the medical histories of 21,123 men and women for 23 years. The article stated that, for those who smoked at least two packs of cigarettes a day, the risk of developing Alzheimer's disease was 2.57 times the risk for those who did not smoke.





#### Question 3 Part (a)

Identify the explanatory and response variables in the study.

Explanatory variable:

Response variable:

#### **Solution:**

The explanatory variable is the person's degree of cigarette smoking. The response variable is whether the person develops Alzheimer's disease during the course of the study.





## **Common Student errors in part (a):**

- ➤ Failure to describe the possible levels of the variable, that is, not understanding that a variable needs to take on more than one value (smoking and not smoking or Alzheimer's and not Alzheimer's)
- ➤ Failure to describe the variable as a characteristic of the observational unit (e.g. using the words "risk" or "chance" or "how many subjects" or "the people who" to describe developing Alzheimer's or not, when those words refer to a measurement taken from a sample)
- The response variable is not the conclusion of the study





#### Question 3 Part (b)

(b) Is the study described in the article an observational study or an experiment? Explain.

#### **Solution:**

This is an observational study, because the people in the study were not assigned to a certain degree of cigarette smoking. Rather, the degree of cigarette smoking for each person was passively observed and recorded, not manipulated by the researchers.





#### **Common Student errors part (b):**

- Circular argument "it is an observational study because it is not an experiment"
- Vague or general statements such as "no treatment" or "just manipulating the study"
- Believing a control group is necessary in an experiment
- Thinking that "random" assignment or "random selection" is necessary for an experiment
- Poor communication of what is meant by "imposing" treatments
- Thinking that following subjects over an extended period of time is what makes it an observational study





#### More part (b) student errors

- Not stating that it is "smoking status" that is ONLY observed
- Describing an observational study as a study that just observes an association or effect of smoking on Alzheimer's, which you would do regardless of the type of study
- Imprecise language referring to statistical concepts like "treatment," "controls," and "assignment"
- Repeating the stem of the problem as the justification: "tracked medical histories"



#### Question 3 Part (c)

Exercise status (regular weekly exercise versus no regular weekly exercise) was mentioned in the article as a possible confounding variable. Explain how exercise status could be a confounding variable in the study.





#### **Question 3 Part (c), Solution**

A confounding variable is one that is related to the explanatory variable and possibly influences the response variable. In this case, it seems plausible that **people who exercise more regularly** might be more health conscious; therefore less likely to smoke cigarettes than people who do not exercise regularly. Similarly, it's possible that people who exercise more regularly are less likely to develop Alzheimer's disease than people who do not exercise regularly. If both of these relationships turn out to be true, then smoking cigarettes would be associated with developing Alzheimer's disease due to the association of both variables with exercise, even if there were no cause-and-effect relationship between smoking cigarettes and developing Alzheimer's disease.





# Part (c)

Alzheimer's Smoking Exercise

# Common Student errors part (c):

- Defining a confounding variable as a variable that only affects the response variable without mentioning its relationship to the explanatory variable
- Not explaining how the confounding variable influences the response variable, e.g. lack of exercise affects health without indicating how it might influence the development of Alzheimer's



### What Students Did Well

- Using context when describing the variables.
- Recognizing that smoking status was related to the explanatory variable and Alzheimer's was related to the response variable.
- Many students correctly understood that a confounding variable is a variable that is associated with the explanatory variable and possibly influences the response variable.
- Recognizing that this was an observational study.
- Understanding that an experiment requires assignment of treatments.





# **Teaching Tips**

- Require students to use statistical language carefully. For example, "confounding" and "correlation" have specific statistical meanings.
- Be clear about the different vocabulary associated with experimental design vs. sampling (e.g. blocking vs. stratifying, randomization vs. random selection)
- Develop an understanding that a control group is not necessary in an experiment.
- Communicate clearly and always write responses in the context of the problem.



# Question #4: Super ignitor for launching model rockets

Probability for independent events, time to first failure, reasoning





#### **Question 4**

A company manufactures model rockets that require igniters to launch. Once an igniter is used to launch a rocket, the igniter cannot be reused. Sometimes an igniter fails to operate correctly, and the rocket does not launch. The company estimates that the overall failure rate, defined as the percent of all igniters that fail to operate correctly, is 15 percent.

A company engineer develops a new igniter, called the super igniter, with the intent of lowering the failure rate. To test the performance of the super igniters, the engineer uses the following process:

Step 1: One super igniter is selected at random and used in a rocket.

Step 2: If the rocket launches, another super igniter is selected at random and used in a rocket.





### **Question 4, continued**

Step 2 is repeated until the process stops. The process stops when a super igniter fails to operate correctly or 32 super igniters have successfully launched rockets, whichever comes first. Assume that super igniter failures are independent.

(a) If the failure rate of the super igniters is 15 percent, what is the probability that the first 30 super igniters selected using the testing process successfully launch rockets?





# Solution - Part (a)

If the failure rate for the super igniters is 15%, then the probability that each igniter fails is 0.15, and the probability that it does not fail is 0.85. Therefore, the probability that the first 30 igniters tested do <u>not</u> fail is

$$(0.85)^{30} = 0.0076$$

The solution can also be written as

$$(1 - 0.15)^{30} = 0.0076$$



### Common student errors, Question 4(a)

- Many students gave incomplete definitions of parameter values and values of the random variable when using calculator notation.
- Some students calculated the incorrect probability. Two specific cases are that they calculated P(X = 30) using the geometric distribution, or they used the binomial distribution with n = 32 trials.
- Many students confused the use of the binomial distribution and the geometric distribution when specifying the required distribution in terms of success and failure.
- Some students gave correct numerical answers but without any justification or work shown.
- Although asked to calculate a probability, many students gave the expected number of successes or failures instead.



## Question 4, Part (b)

Given that the first 30 super igniters successfully launch rockets, what is the probability that the first failure occurs on the thirty-first or thirty-second super igniter tested if the failure rate of the super igniters is 15%?

### Solution – Part (b)

Given that there are no failures in the first 30 tries, the probability that the first failure occurs on the  $31^{st}$  try is 0.15, and the probability that it does not occur on the  $31^{st}$  but occurs on the  $32^{nd}$  try is (0.85)(0.15) = 0.1275. Therefore, the probability of one or the other is

$$0.15 + 0.1275 = 0.2775$$

Note that this is equivalent to asking for the probability that the first failure occurs on the first or second try, which is

$$0.15 + (0.85)(0.15) = 0.2775$$



## Common student errors, Question 4(b)

- Many students did not realize that the first 30 trials had no influence on the 31<sup>st</sup> and 32<sup>nd</sup> trial.
- Many students did not realize that if a conditional probability approach was used, the denominator must be computed as well. (They only computed the numerator).
- ➤ Some students did not recognize that a single probability of the union of two events was required and reported two separate probabilities (for the 31<sup>st</sup> and 32<sup>nd</sup> trials).
- Some students failed to realize that the 31<sup>st</sup> and 32<sup>nd</sup> trials cannot both be the first failure.
- Some students gave answers without justification ("bald answers").





### Question 4, Part (c)

Given that the first 30 super igniters successfully launch rockets, is it reasonable to believe that the failure rate of the super igniters is less than 15 percent? Explain.

### Solution – Part (c)

The result of the probability calculation in part (a) provides reason to believe that the failure rate of the super igniters is less than 15%. The calculated probability of 0.0076 shows that there is less than a 1% chance that 30 or more igniters in a row would not fail if the failure rate was 15%. This probability is smaller than conventional significance levels such as  $\alpha$  = 0.05 or  $\alpha$  = 0.01, and thus is small enough to make it reasonable to believe that the failure rate of the super igniters is less than 15%.





### Common student errors, Question 4(c)

- ▶ No direct yes or no answer.
- ▶ No context in the response.
- No clear reference to the probability in part(a) or did not relate part (c) to part (a).
- Overall poor communication.
- ▶ Used the <u>expected number</u> of 4.5 failures for 30 trials and a 15% failure rate as justification, without giving a statistical argument for why 0 failures would therefore provide evidence that the failure rate is less than 15%.
- ➤ Some students asserted that evidence indicating that the failure rate is likely to be less than 15% <u>proves</u> that the failure rate is less than 15%.



## **Question 4, teaching tips**

- When solving probability problems involving a specific distribution, focus on probability notation rather than calculator notation. When discussing the distribution, require students to specify values for the parameters, and to give clear definitions of those parameters.
- Make sure students have a clear understanding of the characteristics of probability distributions and associated parameters.
- ➤ Give students lots of practice with problems that require the application of basic probability rules and make sure students have a clear understanding of independence and conditional probability, and a clear understanding of the probability of the union and intersection of events.



## Question 4, more teaching tips

- Remind students to always answer in context.
- Give students lot of practice with communicating the answer to a question in English. Make sure student responses are clear about the meaning of any pronouns used, such as "it" or "that."
- ▶ Give students practice with making predictions and decisions based on probability alone. Do some problems of this sort after inference so students learn that not everything needs a hypothesis test.
- ➤ Remind students to give complete justification for answers. Give criteria, comparison and conclusion to support a decision. Show your work and cross out what you don't want scored.



Question #5:
Poll for environment vs economy statement
Confidence interval for proportion; reason for conditions





### **Question 5**

A polling agency showed the following two statements to a random sample of 1,048 adults in the United States.

Environment statement: Protection of the environment should be given priority over economic growth

Economy statement: Economic growth should be given priority over protection of the environment

The order in which the statements were shown was randomly selected for each person in the sample. After reading the statements, each person was asked to choose the statement that was most consistent with his or her opinion. The results are shown in the table. [next slide]





# **Question 5, continued**

The results are shown in the table.

	Environment Statement	_	No Preference
Percent of sample	58%	37%	5%

(a) Assume the conditions for inference have been met. Construct and interpret a 95 percent confidence interval for the proportion of all adults in the United States who would have chosen the economy statement.



### Question 5, part (a), Solution

The appropriate procedure is a one-sample z-interval for a population proportion. The problem stated the conditions for inference have been met, so they do not need to be checked. A 95% confidence interval for the population proportion is given by

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$
 which is

$$0.37 \pm 1.96 \sqrt{\frac{(0.37)(0.63)}{1048}} \approx (0.34,0.40)$$

We are 95% confident that the population proportion of all adults in the U.S. who would have chosen the economy statement is between 0.34 and 0.40.



### Common Errors + Teaching Tips, Part (a)

- ► Not providing name or providing incorrect name/formula

  Teaching Tip: Use full names of the procedures. "One Sample Z Interval for Proportions" rather than One Samp Z-Int, "Two Sample Z Interval for a difference in proportion" rather than 2 Prop Z Int, etc. If students are writing formulas in addition to the name, make sure they are correct.
- ➤ Attempting to use a t-value as the critical value for a proportion procedure OR Using a z critical value for a different % confidence level

**Teaching Tip:** Focus on using the correct critical value for the procedure (e.g. t\*, z\*) and how they are found



### **More Errors + Teaching Tips, Part (a)**

➤ Stating the estimation is for a proportion of the adults "sampled", "asked", "who chose" (indicating the sample) rather than the population

**Teaching Tip:** Focus on identifying the population of interest

▶ Not checking if the interval calculated is reasonable.

**Teaching Tip:** Have students ask themselves, does this interval make sense given the type of interval you are computing? Does my point estimate fall in the interval?

➤ Combining the interpretation of interval and level "In repeated samples, we are 95% confident that this interval will contain..."

**Teaching Tip:** Stress the difference between interpreting an interval versus interpreting the level of confidence.



# Question 5 – Part (b) + Solution

One of the conditions for inference that was met is that the number who chose the economy statement and the number who did not choose the economy statement are both greater than 10. Explain why it is necessary to satisfy that condition.

### Part (b) Solution:

The condition is necessary because the formula for the confidence interval relies on the fact that the binomial distribution can be approximated by a normal distribution which then results in the sampling distribution of  $\hat{p}$  being approximately normal. The approximation does not work well unless both  $n\hat{p}$  and  $n(1-\hat{p})$  are at least 10.





### Common Errors + Teaching Tips, Part (b)

Incorrectly focusing on the sample size requirement leading to a representative sample rather than the sampling distribution becoming approximately normal

**Teaching Tip:** Explain why each interval condition is necessary in addition to checking them. It may be helpful to have the student write a sentence for why it is necessary to have each condition. For example, "Why is it important that a random sample is selected for this study", or "Why is it important that the sample size be greater than 30." etc.

Struggling to specify which distribution is approximately normal (e.g., "sample distribution" vs. "sampling distribution")

**Teaching Tip:** Focus on using language that reinforces the difference between a sample distribution and a sampling distribution.





# Question 5 – Part (c)

A suggestion was made to use a two-sample z-interval for a difference between proportions to investigate whether the difference in proportions between adults in the United States who would have chosen the economy statement is statistically significant. Is the two-sample z-interval for a difference between proportion an appropriate procedure to investigate the difference? Justify your answer.





### Question 5, Part (c), Solution

The suggested procedure is not appropriate since one of the requirements for using a two-sample z-interval for a difference between proportions is that the two proportions are based on two independent samples. In the situation described in this problem the two proportions come from a single sample, and thus are not independent.





### Common Errors + Teaching Tips, Part (c)

- Incorrectly believing there is a difference between
  - ► a two sample z interval for a difference in proportions and
  - ► a two sample z proportion interval based on calculator use.

    Teaching Tip: Use the full names of the procedures. "One Sample Z Interval for Proportions" rather than One Samp Z-Int, "Two Sample Z Interval for a difference in proportion" rather than 2 Prop Z Int, etc.
- ► Not fully communicating what is needed and why that is different than what is currently offered

**Teaching Tip:** Stress good communication. Practice forming responses that include the answer to the question AND a full <u>explanation</u> of why that answer is correct. For example, don't just state the procedure is inappropriate because only one sample is available but also state what would make it an appropriate is two independent samples.





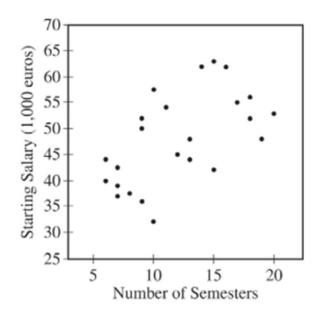
Question #6 (Investigative task)
Semesters and Salaries
Association between variables, with
and without a 3<sup>rd</sup> variable





### **Question 6 (Investigative task)**

A newspaper in Germany reported that the more semesters needed to complete an academic program at the university, the greater the starting salary in the first year of a job. The report was based on a study that used a random sample of 24 people who had recently completed an academic program. Information was collected on the number of semesters each person in the sample needed to complete the program and the starting salary, in thousands of euros, for the first year of a job. The data are shown in the scatterplot below.







### Question 6, Part (a)

Does the scatterplot support the newspaper report about number of semesters and starting salary? Justify your answer.

### Part (a) Solution:

The scatterplot supports the newspaper report about number of semesters needed to complete an academic program and starting salary, because it shows a positive association between these two variables.





# **Common Errors, Question 6 part (a)**

Most students recognized the positive association between number of semesters and starting salary shown in the scatterplot and correctly stated that the scatterplot supports the newspaper report. However, some students went on to discuss additional characteristics of the scatterplot (strength, form) that were unnecessary to answer this part. Because the association isn't very strong, this led some students to incorrectly say that the report wasn't supported or only weakly supported.

**Teaching Tip:** Make sure that students answer the question that is asked—and only that question. In this case, the question was about the <u>direction</u> of the association only, not the strength.





### Question 6, Part (b)

The table below shows computer output from a linear regression analysis on the data.

Predictor	Coef	SE Coef	T	P	
Constant	34.018	4.455	7.64	0.000	
Semesters	1.1594	0.3482	3.33	0.003	
S = 7.37702	R-Sq = 33.5%	R-Sq(adj) = 30.5%			

Identify the slope of the least-squares regression line, and interpret the slope in context.



### **Question 6, Part (b) Solution**

The slope is 1.1594. For each additional semester needed to complete an academic program, the predicted starting salary increases by €1,159.40.

Predictor	Coef	SE Coef	T	P
Constant	34.018	4.455	7.64	0.000
Semesters	1.1594	0.3482	3.33	0.003
S = 7.37702	R-Sq = 33.5%	R-Sq(	adj) = 30.5%	



# **Common Errors, Question 6 part (b)**

- ➤ Some students weren't able to identify the numerical value of the slope from the computer output.
  - **Teaching Tip:** Make sure students are familiar with computer output, especially in a regression context.
- ► Instead of interpreting the slope, some students simply described the positive association between the two variables.
  - **Teaching Tip:** Make sure students know that the slope of a least-squares regression line describes the change in the predicted value of the response variable for each 1-unit increase in the explanatory variable—and can interpret the slope in context.



## **More Errors, Question 6 part (b)**

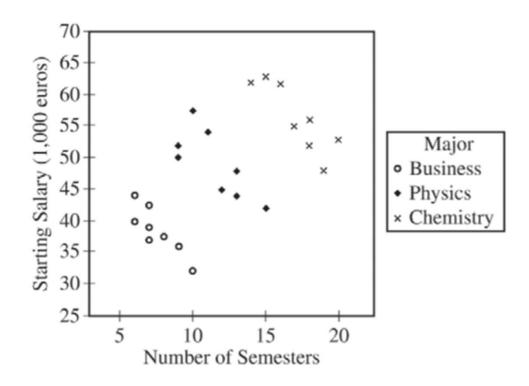
- ▶ Most students did not use non-deterministic language for the change in the value of the response variable. For example, many students said something like "For each additional semester, the starting salary will increase 1159.40 euros." A correct response implies that the actual values vary above or below the value provided by the model. For example, "For each additional semester, the *predicted* starting salary will increase 1159.40 euros" or "For each additional semester, the starting salary will increase 1159.40 euros, on average."
  - ► Teaching Tip: Make sure students understand that statistical models provide <u>estimated</u> or predicted values and that actual values are likely to differ from the value provided by the model.





# Question 6, Stem for Parts (c) and (d)

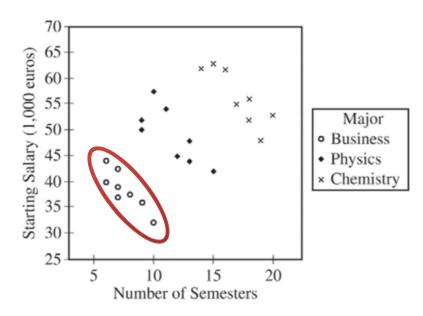
An independent researcher received the data from the newspaper and conducted a new analysis by separating the data into three groups based on the major of each person. A revised scatterplot identifying the major of each person is shown below.





### Question 6, Part (c) and solution

(c) Based on the people in the sample, describe the association between starting salary and number of semesters for the business majors.



#### **Solution:**

For the business majors alone, there is a strong, negative, linear association between number of semesters and starting salary. Business majors who need a greater number of semesters to complete an academic program tend to have smaller starting salaries.





## **Common Errors, Part (c)**

➤ Many students were able to identify one characteristic of the association (direction), but didn't address the other two (strength, form).

**Teaching Tip:** When describing scatterplots, make sure to address direction, form, strength, and any unusual features (e.g., outliers, clusters).

➤ Some students didn't describe the association in context by referring to the variable names (number of semesters, starting salary).

**Teaching Tip:** Make sure to use context for any description, comparison, interpretation, explanation, and justification.



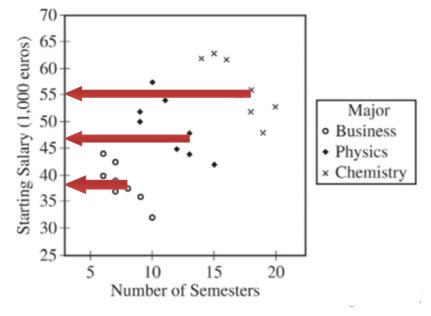


### Question 6, Part (d) and Solution

(d) Based on the people in the sample, compare the median starting salaries for the three majors.

**Solution:** Business majors have the smallest median starting salary at around €38,000, followed by physics majors at around €48,000, and then chemistry majors with the largest median starting salary at around €55,000.







## **Common Errors, Part (d)**

▶ Many students didn't make any comparison or only made a partial comparison. For example, some students only listed the median salaries for each major but never ranked them. Other students said something like "Chemistry has a higher median starting salary than business or physics" but never compared business to physics.

**Teaching Tip:** When asked to make a comparison, make sure to use words like "greater than" or "less than" and make sure to compare all groups.





### **Question 6, Part (e) and solution**

Based on the analysis conducted by the independent researcher, how could the newspaper report be modified to give a better description of the relationship between the number of semesters and the starting salary for the people in the sample?

**Solution:** The newspaper report should be modified to account for major. Overall, majors that take longer to complete tend to have higher starting salaries, with chemistry the highest, physics the next highest, and business the lowest. However, within a major, students who take a greater number of semesters tend to have lower starting salaries.





### **Question 6, Part (e) Common Errors**

➤ Many students were too general in their description of how to modify the newspaper report. Students said things like "the newspaper should account for major" or "the newspaper should include a scatterplot for each major" without providing any additional details.

**Teaching Tip:** When students are asked to modify, change, improve something, they should include specific details about what needs modification and how it should be modified.

**Teaching Tip:** Remind students that the investigative task typically has a flow, where students are asked to put different parts together in the end. When pulling things together, students should explicitly refer to previous parts and not expect a reader to make the connection for them.



### **Score Summary**

Q #	1	<b>2</b>	3	4	5	6
	Tips	Snack ads	Smoking	Rockets	Env/Econ	Salaries
Mean score (Possible 0 to 4)	1.73	1.22	1.55	0.99	1.27	1.61

Exam Score	1	2	3	4	5
%	24.0%	15.7%	24.7%	21.7%	13.9%

#### **Final Notes:**

- Consider becoming an AP Statistics Reader!
   http://apcentral.collegeboard.com
- Questions/Comments: jutts@uci.edu



