Announcements:

- First midterm is a week from Monday (Oct 18), covering Chapters 1 to 6. Sample questions (and answers) have been on course website all along. Mixture of free-response and multiple choice. You are allowed one sheet of notes (both sides of page, typed or hand-written).
- Monday discussion *is* for credit the next four weeks.

Today: Section 6.4
Assessing the Statistical Significance of a 2x2 Table

Homework (due Friday, October 15):
Chapter 6: #44*, 45*, 50bcde, 52
*Count together as one problem.

Review from last time

What to do with two categorical variables:

- Create a "contingency table" with explanatory variable as rows, response variable as columns.
- Each combination of row and column is a *cell*.
- Use table to compute risk, relative risk, increased risk, odds, odds ratio.

Sometimes these measures don't make sense – just want to know if the two variables are related.

Today: How to determine if two categorical variables have a *statistically significant relationship*.

<u>Example</u> (Case Study 4.3, p. 129): Randomized experiment
Explanatory variable = wear nicotine patch or placebo
Response variable = Quit smoking after 8 weeks? Yes/ No

Results:

	Quit	Didn't	**Total**	% Quit
Nicotine	56	64	120	46%
Placebo (baseline)	24	96	120	20%
Total	80	160	240	33%

Relative "risk" of quitting with nicotine patch is .46/.20 = 2.3

Question: Could the observed relationship be due to chance, or is there really a difference in proportions who would quit in the *population* from which this *sample* was taken?

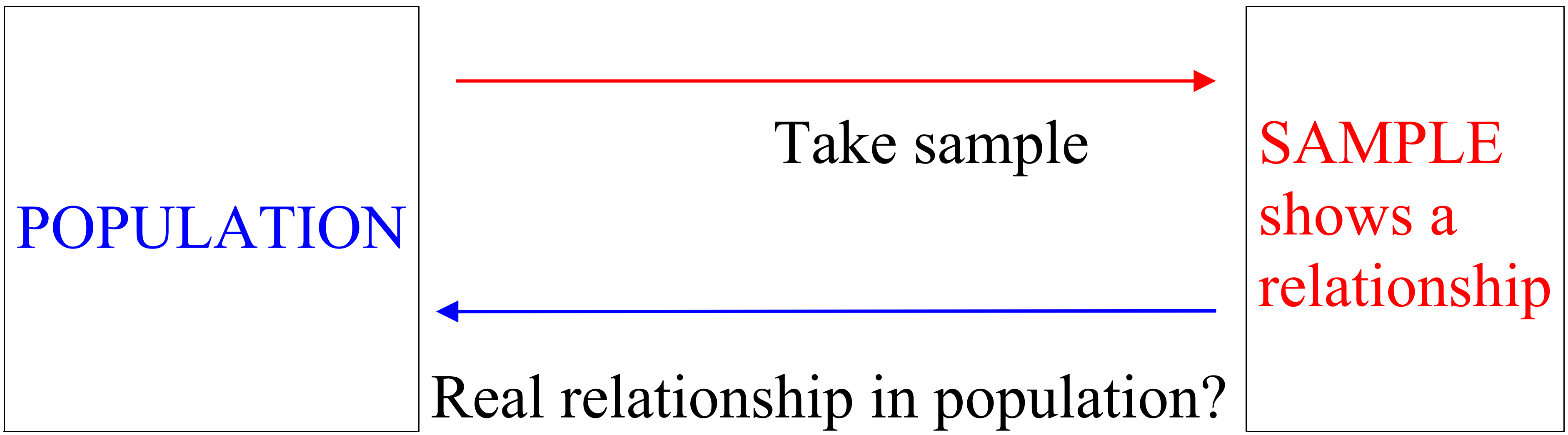
Definitions (from Chapters 1 and 3):

A **population** is the entire group of units (college students, Old Faithful eruptions, babies, cities in US, smokers, …) about which information is desired.

A **sample** is the subset of the population for which measurements are available.

Nicotine Patch Example:
Population: Smokers with a desire to quit
Sample: 240 smokers at Mayo clinics in Minnesota, Florida and Arizona, who volunteered to participate.

Goal of **statistical inference**: Use the data from the sample to make conclusions (*inferences*) about the population.



So far, *Descriptive statistics*

Now: *Inferential statistics*

- Confidence intervals (Chapter 3, then Chs 10, 11)
- Hypothesis tests (Chapter 6, then Chs 12, 13)

Confidence interval – An interval of values that we are "confident" covers the truth about a population value. (Ch 3)

Hypothesis test (also called a significance test) – based on *sample* determine if there is a relationship, difference, etc., in the *population*.

<u>Definitions</u>:

A ***statistic*** is a numerical summary of the data in a *sample*.
Ex: mean, median, correlation, etc, computed from *sample.*

A ***parameter*** is a number association with a *population*.
Ex: mean of a *population*, such as male heights for *all* college students.

A ***test statistic*** is a statistic that summarizes sample data in a way that can be used in a hypothesis test.

A ***chi-square statistic***

- The *test statistic* we use to assess the strength of the relationship in a two-way table, and to decide if the relationship is “statistically significant”.
- More complicated summary than seen so far, but still, just a numerical summary of sample data!
- Measures how far the *observed* numbers in the cells are from what we would *expect* if there is no relationship between the explanatory and response variables.

Nicotine Patch Example: What to Expect if No Relationship

Observed counts	**Quit**	**Didn't**	**Total**	**% Quit**
Nicotine	56	64	120	46%
Placebo (baseline)	24	96	120	20%
Total	80	160	240	33%

- Note that 80/240 = 1/3 (or 33%) quit smoking overall
- *If there is no difference* in the effect of patch type, we *expect* to see 1/3 of each type quit. So, we would expect:

Expected counts	**Quit**	**Didn't**	**Total**	**% Quit**
Nicotine	40	80	120	33%
Placebo (baseline)	40	80	120	33%

Five Steps to Determining Statistical Significance (**page 208**)

Here is how to do a hypothesis test:

Step 1: Determine the *null* and *alternative* hypotheses.

Step 2: Verify necessary data conditions, and if met, summarize the data into an appropriate *test statistic*.

Step 3: Assuming the null hypothesis is true, find the *p-value.*

Step 4: Decide whether or not the result is *statistically significant* based on the p-value.

Step 5: Report the *conclusion in the context* of the situation.

For two categorical variables (two-way table):

Step 1: Determine the null and alternative hypotheses.

In general:

- Null hypothesis is "nothing going on," status quo, no difference, etc.
- Alternative hypothesis is what researchers hope to show, that something interesting *is* going on.

For contingency tables:

- Null hypothesis: The two variables are *not* related in the population.
- Alternative hypothesis: The two variables *are* related in the population.

Step 1 for the Nicotine Patch Example:

Population: the hypothetical behavior of *all* smokers with a desire to quit, *if* given nicotine patch compared with *if* given placebo patch.

Null hypothesis: In the population of smokers who want to quit, there is no relationship between patch type and whether or not someone quits smoking.

Alternative hypothesis: In this population, there *is* a relationship between patch type and whether or not someone quits smoking.

Step 2: Verify necessary data conditions, and if met, **summarize the data into an appropriate test statistic.**

For two categorical variables

- Data condition: All expected counts ≥ 1, at least three ≥ 5
- The test statistic is called the *chi-square statistic*.

Logic of the chi-square statistic:

- Compute *expected counts* under the assumption of *no relationship in population* , i.e when *null hypothesis* is true
- Compare these to *observed counts* in the cells of the table, using a summary measure (to be shown)
- If they are very different (far apart), conclude there *is* a relationship between explanatory and response variables.

O = Observed count in each cell = actual sample data

E = Expected count (if null is true) in each cell =

$$\frac{(Row\ total)(Column\ total)}{Total\ n}$$

	Quit	**Did not quit**	**Total**
Nicotine	**56** **(120)(80)/240** **= 40**	**64** **(120)(160)/240** **= 80**	**120**
Placebo	**24** **(120)(80)/240** **= 40**	**96** **(120)(160)/240** **= 80**	**120**
Total	**80**	**160**	**240**

NOTE: Only need compute E for one cell, others determined by totals

Why do these "expected counts" make sense if the null hypothesis is true?

- Overall, 80/240 = 1/3 quit (see "Total" row).
- If *no relationship*, we would expect 80/240 to have quit in *each* treatment (each row of the table).
- So, we expect 120 × 80/240 = 40 to have quit in each treatment (row) and 120 × 160/240 = 80 to have *not quit* in each treatment. These match "expected count" formula.

	Quit	**Did not quit**	**Total**
Nicotine	**56 (Observed)** **40 (Expected by chance)**	**64** **80**	**120**
Placebo	**24** **40**	**96** **80**	**120**
Total	**80**	**160**	**240**

Continuing Step 2, Creating the test statistic:

- For each cell, summarize difference between "observed" counts (O) and "expected" counts (E), using

$$\frac{(O-E)^2}{E}$$

- Sum these over all cells.

Chi-square statistic: ***Notation, Greek letter "chi"***

$$\chi^2 = \sum_{all\ cells} \frac{(O-E)^2}{E}$$

Example: How far are *observed* numbers who quit from what we *expect* if there is no difference for patch types?

$\frac{(56-40)^2}{40} = \frac{256}{40} = 6.4$	$\frac{(64-80)^2}{80} = \frac{256}{80} = 3.2$
$\frac{(24-40)^2}{40} = \frac{256}{40} = 6.4$	$\frac{(96-80)^2}{80} = \frac{256}{80} = 3.2$

So, $\chi^2 = 6.4 + 3.2 + 6.4 + 3.2 = 19.2$

Does that indicate a large difference or a small one??? A strong relationship or no relationship at all in the population?

Step 3: Assuming the null hypothesis is true, find the *p-value*.

Decide *how unlikely* such a big difference *in the sample* would be *if* there is no *real relationship in the population*. This is a black box to you for now! Called the **p-value**.

Using R Commander (See handout on website):
Statistics -> Contingency Table -> Enter and analyze two-way table

Example: `X-squared = 19.2, df = 1, p-value = 1.177e-05` [.00001177]

Note: You can use Excel, but you need to find the expected counts yourself first. See page 212 in book.

Step 4: Decide whether or not the result is statistically significant, based on the p-value.

Possible conclusions:

Do not reject the null hypothesis and conclude there isn't enough evidence to convince us that there is a relationship in the population. Conclude this IF $p\text{-value} > .05$.

(Use .05 or other "level of significance")

Reject the null hypothesis and conclude there *is* a relationship in the population Conclude this IF $p\text{-value} \leq .05$.

Equivalent ways to say we *do not reject the null hypothesis*:

- There is *not enough evidence* to support the alternative hypothesis
- There is *not enough evidence* to reject the null hypothesis
- The relationship is *not statistically significant*

NOTE: It is *not okay* to "*accept the null hypothesis*."

Equivalent ways to say we *reject* the null hypothesis:

- We *accept* the alternative hypothesis
- There is a *statistically significant* relationship between the two variables.

Step 4 for the nicotine patch example:
The p-value of .00001177 is *much* less than .05, so relationship *is* statistically significant. We reject the null hypothesis. We accept the alternative hypothesis.

Step 5: Report the conclusion in the context of the situation.

Step 5 for the nicotine patch example:
There is a statistically significant relationship between type of patch worn and the ability to quit smoking.

And, because this was a randomized experiment, we can conclude that wearing nicotine patches would *cause* more people to quit smoking than wearing a placebo patch.

Caution #1: *p*-value depends on sample size. Easier to detect real difference with *larger* sample. Therefore, *failure to detect a statistically significant relationship does not mean there is no relationship.*

Example: Aspirin and heart attacks (Case Study 1.6):

- $\chi^2 = 25.4$, p-value ≈ 0, clearly there *is* a relationship between aspirin (yes/no) and heart attack (yes/no).
- Suppose the sample size is cut by a factor of 10 but same pattern, i.e. all observed counts are divided by 10 as well.

  Chi-square statistic = 2.54, p-value = .111, *not* statistically significant.

Caution #2:
Statistical significance is not the same thing as *practical* significance (importance). With a *very large sample* even a *minor* relationship will be statistically significant.

<u>Example</u>: Suppose drug compared to placebo

	Cured	Not Cured	Total	% Cured
Drug	5100	4900	10000	51%
Placebo	4900	5100	10000	49%
Total	10000	10000	20000	50%

Relative "risk" of cure = 51/49 = 1.04
Chi-square statistic = 8.0, *p*-value = .0047
Clearly reject the null hypothesis, conclude drug works!
But the difference is of little *practical* importance.

New example: Question asked in Discussion 1 (4pm)

Explanatory: Sex (Male/Female)
Response: Do you vote in U.S. government elections? Yes/No

	Vote	Don't vote	Total
Male	16	3	19
Female	11	9	20
Total	27	12	39

Population: All college students similar to those who take Statistics 8 at UCI.

Step 1: Determine the null and alternative hypotheses
Two versions are shown here for each hypothesis.

Null hypothesis:

- For the *population* of students, voting behavior *does not differ* for males and females.
- For the *population* of students, voting behavior and sex *are not related*.

Alternative hypothesis:

- For the population of students, voting behavior differs for males and females.
- For the population of students, voting behavior and sex *are* related.

Steps 2 and 3: Compute the test statistic and p-value

	Vote	Don't vote	Total
Male	16	3	19
Female	11	9	20
Total	27	12	39

Results from R Commander:

```
X-squared = 3.9029, df = 1, p-value = 0.0482
Expected Counts
          1          2
1  13.15385   5.846154
2  13.84615   6.153846
```

Compare these to observed counts in table.

For example, (19)(27)/39 = 13.15

Note 16, 9 larger than expected; 11, 3 smaller.

```
Chi-square Components
     1    2
1  0.62 1.39
2  0.59 1.32
```

(Interesting to look for cells with large values here)

Steps 4 and 5: Conclusion in statistical terms and context

<u>Step 4: Statistical conclusion</u>:
p-value = .0482 is less than .05, so *reject* the null hypothesis. The relationship *is* statistically significant.

<u>Step 5: Conclusion in context</u>:

There is a statistically significant relationship between sex and voting behavior in US government elections. *Note that a lower proportion of women vote.*

We can conclude that the relationship holds in the *population* represented by this sample.

A Few More Details

1. If you have to compute the test statistic by hand, there is a "short-cut" formula; see page 647:

	Column 1	Column 2	Total
Row 1	A	B	$A+B = R_1$
Row 2	C	D	$C+D = R_2$
Total	$A+C = C_1$	$B+D = C_2$	N

$$\chi^2 = \frac{N(AD - BC)^2}{R_1 R_2 C_1 C_2}$$

2. For a 2 x 2 table only (2 rows and 2 columns):
   p-value $\leq .05$ if and only if chi-square value ≥ 3.84.
   So *statistically significant relationship* if $\chi^2 \geq 3.84$.
   In our example, $\chi^2 = 3.9029 > 3.84$