

PROBLEM 1: In a study reported in Example 3.13 of the textbook, researchers were able to get information about voting behavior of a sample of registered voters. They knew whether or not these people voted in the November 1986 election. Seven months after the election, they surveyed these people and asked them whether or not they had voted in that election. Of those who actually *had* voted, 96% said that they did and 4% said they did not. Of those who had *not* voted, 40% claimed that they did vote and 60% admitted that they did not. Suppose these people are representative of the population of registered voters, and that in fact 50% of the population actually voted in the election.

Use the following abbreviations for events for a randomly selected person from this population:

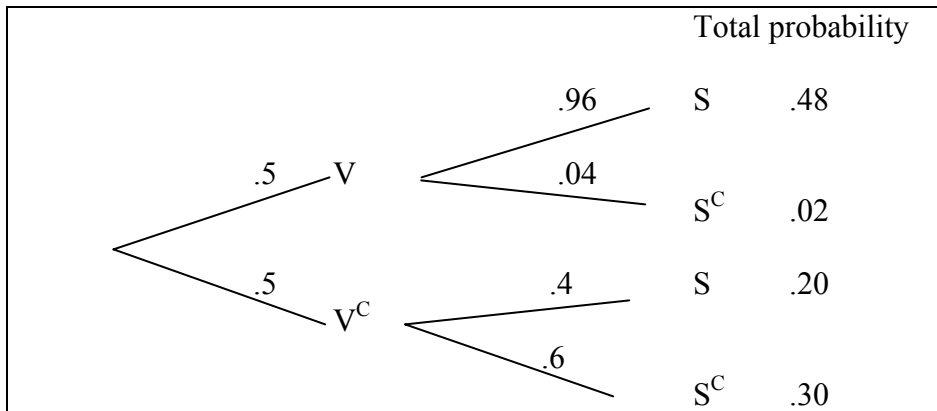
V = voted, V^C = didn't vote

S = said they voted, S^C = said they did not vote

Given that someone said they voted, what is the probability that they actually did vote? In other words, find $P(V | S)$

SOLUTION:

It's easier to solve if you first draw a tree diagram for this situation **or** construct a "hypothetical hundred thousand" table.



	Said they voted	Said they didn't vote	Total
Voted	48,000	2,000	50,000
Did not vote	20,000	30,000	50,000
Total	68,000	32,000	100,000

From the table, it's easy to see that $P(V | S)$ = the proportion of voters in the "Said they voted" column =

$$\frac{48,000}{68,000} = .706.$$

From the tree diagram, we first need to find $P(S)$. It's the sum of the probabilities for branches ending in "S" = $.48 + .20 = .68$.

$$\text{Then } P(V | S) = \frac{P(V \text{ and } S)}{P(S)} = \frac{.48}{.68} = .706$$

In other words, only 70.6% of the people who said they voted actually did!

PROBLEM 2 (from lecture 13 but not solved): Suppose you drive on a certain freeway daily. The speed limit is 65. You drive over 65 all the time, but over 75 about 30% of the time.

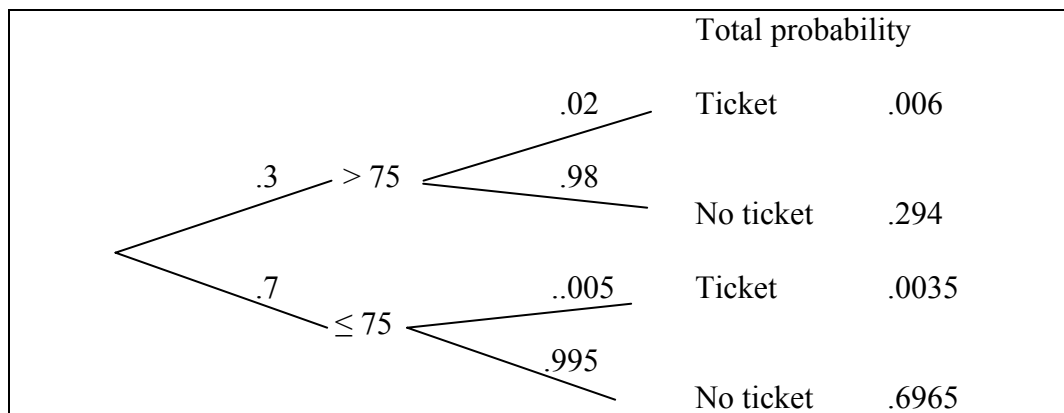
$$P(\text{ticket} \mid \text{over } 75) = 1/50 = .02$$

$$P(\text{ticket} \mid 65 \text{ to } 75) = 1/200 = .005.$$

1. What is the probability you get a ticket on a randomly selected day?
2. Given that you get a ticket, what is the probability that you were going over 75?

SOLUTION:

It's easier to solve if you first draw a tree diagram for this situation **or** construct a “hypothetical hundred thousand” table.



	Ticket	No ticket	Total
Over 75	$(.02)(30,000) = 600$	29,400	30,000
Under 75	$(.005)(70,000) = 350$	69,650	70,000
Total	950	99,050	100,000

1. From the table, it's easy to see that $P(\text{Ticket}) = \frac{950}{100000} = .0095$

From the tree diagram, $P(\text{Ticket}) = .006 + .0035 = .0095$.

2. From the table, it's easy to see that $P(\text{Over } 75 \mid \text{Ticket}) = \frac{600}{950} = .632$

From the tree diagram, $P(\text{Over } 75 \mid \text{Ticket}) = \frac{.006}{.006 + .0035} = \frac{.006}{.0095} = .632$

PROBLEM 3: The speeds of cars at a certain location on an interstate highway are approximately normal with $\mu = 67$ miles per hour and $\sigma = 6$ miles per hour. When the highway patrol is looking for speed violators, they will stop cars going over 75 miles per hour.

1. What proportion of cars are going over 75 miles per hour?
2. The speed limit is 65 miles per hour. What proportion of cars are going at or under the speed limit?
3. If three cars are randomly selected at different times, what is the probability that they are all going over 67 mph?

SOLUTION

1. What proportion of cars are going over 75 miles per hour?

First, convert the value of 75 to a z-score:

$$z = (75 - 67)/6 = 1.33$$

$$\text{so } P(X > 75 \text{mph}) = P(Z > 1.33) = P(Z < -1.33) = .0918$$

2. The speed limit is 65 miles per hour. What proportion of cars are going at or under the speed limit?

First, convert the value of 65 to a z-score:

$$z = (65 - 67)/6 = -.33$$

$$\text{so } P(X \leq 65 \text{mph}) = P(Z \leq -.33) = .3707$$

3. If two cars are randomly selected at different times, what is the probability that they are both going over the speed limit of 65 mph?

For each car, the probability that it is going over the speed limit is $1 - .3707 = .6293$.

The speeds are independent, so the probability that they are both going over the speed limit is the product of the individual probabilities, which is $(.6293)^2 = .3960$

PROBLEM 4: There are 2000 tickets sold for a raffle. Three winning tickets are chosen and those ticket holders each win \$400. Define the random variable X = amount won by purchasing one ticket.

1. Write the pdf for X .
2. Find $E(X)$ = the expected value of X . Show your work.
3. If you buy 2 tickets
 - a. What is the probability that you win for *both tickets*?
 - b. What is the probability that *neither* ticket is a winner?
 - c. Using the above 2 answers, write the pdf for Y = number of winning tickets (of the 2 you bought).

SOLUTION:

1. Write the pdf for X .

There are two possible values for X . If the ticket is a winning ticket, $X = \$400$, otherwise $X = 0$. There are 3 winning tickets, so $P(X = \$400) = 3/2000$. So the pdf can be written as:

k	$P(X=k)$
\$400	3/2000
0	1997/2000

2. Find $E(X)$ = the expected value of X . Show your work.

$$E(X) = \$400(3/2000) + 0(1997/2000) = \$1200/2000 = \$.60 \text{ or } 60 \text{ cents.}$$

3. If you buy 2 tickets:

- a. What is the probability that you win for *both tickets*?

First, recognize that winning with Ticket 1 and winning with Ticket 2 are not independent events so you can't use Rule 3b. Instead you have to use the more general Rule 3a. (You could also do this using a tree diagram, similar to the one for choosing 2 left-hand children in one of the homework problems.) Think of this problem as if there is a deck of 2000 cards, and you are going to draw 2 without replacement. There are 3 winning cards in the deck, and 1997 losing cards. Define events:

A = your first ticket is a winner B = your second ticket is a winner

$$P(A \text{ and } B) = P(A) P(B | A)$$

$$P(A) = 3/2000$$

$P(B | A) = 2/1999$ because once we know A has happened, there are 1999 tickets left, and 2 of them are winning tickets.

$$\text{So from Rule 3a, } P(A \text{ and } B) = P(A) P(B | A) = (3/2000) (2/1999) = 6/3998000 = .000015$$

- b. What is the probability that *neither* ticket is a winner?

$$\text{Using the same reasoning, it is } (1997/2000)(1996/1999) = 3986012/3998000 = .9970015$$

- c. Parts (a) and (b) give $P(Y = 2)$ and $P(Y = 0)$ so the only remaining option is $Y = 1$. That probability can be found by subtraction to be $1 - (.000015 + .9970015)$. So the pdf for Y is:

k	0	1	2
$P(Y=k)$.9970015	.0029835	.000015