Homework 8 Solutions
Chapter 11: #40, 53, 78 (Use R Commander; counts double)
Chapter 13: #15, 24ac, 25
Chapter 15: #6, 26bd, 35

Assigned Fri, Nov 19

11.40 **a.** Confidence interval is about 0.82 to 2.78 inches, computed as 1.8 ± (2.09)(0.47).
Parameter is μ_d = mean difference between desired and actual height in population of women.

Compute interval as Sample estimate ± Multiplier × Standard error, which here is $\bar{d} \pm t^* \frac{s_d}{\sqrt{n}}$.

Sample estimate is $\bar{d}$ = 1.8 inches. Standard error is $\text{s.e.}(\bar{d}) = \frac{s_d}{\sqrt{n}} = \frac{2.1}{\sqrt{20}} = 0.47$.

Multiplier is $t^* = 2.09$. In Table A.2, use df = $n-1$ = 20−1=19.
Interpretation: With 95% confidence, we can say that in the population of women represented by the sample, the mean difference between desired height and actual height is between 0.82 inches and 2.78 inches.
b. With a sample of this size, in theory the population of differences should be bell-shaped. In practice, it is sufficient that the data show no extreme skewness and contain no outliers. Also, the sample should be randomly selected although in practice it is sufficient if for the question of interest, the sample can be considered to represent the population.
c. The confidence interval found in part (a) does not describe the responses of *individual* women, but instead estimates only the *mean* response over the entire population. Therefore, we can conclude that *on average* the women would like to be taller, but not that *all* women have that wish. In fact, based on the sample mean of 1.8 inches and the standard deviation of 2.1 inches, using the Empirical Rule (assuming the differences are approximately bell-shaped) we can see that the population of original values extends to several inches below 0.

11.53 The purpose of this question is to see if students can apply the logic of confidence intervals in a new situation that hasn't been shown in class or the book.
a. We can conclude that the risks under the two conditions differ in the population(s) represented by the sample(s). The value 1.0 is the value of a relative risk that occurs if the risks are the same, so we can reject this value as a possibility.
b. We cannot conclude that the risks under the two conditions differ in the population(s) represented by the sample(s). The value 1.0 is the value of a relative risk that occurs if the risks are the same, and in this case we cannot reject this possibility because it is included in the confidence interval.

11.78 **a**. Using all data, deprived = "no" mean is 7.06 hours and deprived="yes" mean is 5.94 hours. The difference in these sample means = 1.12 hours. There is an outlier at 0 hours in the deprived = "yes" group. If that outlier is removed, the "yes" mean changes to 6.06 hours and the difference is 1 hour. The outlier is clearly a mistake. Students were asked how many hours they usually sleep per night, and if someone actually slept 0 hours per night they wouldn't be around to talk about it! So it is reasonable to remove the outlier.
b. With all data included, the unpooled procedure gives an interval from 0.574 to 1.658 hours. If the outlier is deleted, which it should be, the unpooled procedure gives an interval from 0.505 to 1.490 hours. See the R Commander output below.

R Commander output for 11.78:

With the outlier included, confidence interval in bold:

```
data:  SleepHrs by Deprived
t = 4.0975, df = 79.624, p-value = 9.98e-05
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.5739261 1.6580067
sample estimates:
 mean in group No mean in group Yes
        7.057143          5.941176
```

With the outlier removed, confidence interval in bold:

```
data:  SleepHrs by Deprived
t = 4.0395, df = 68.827, p-value = 0.0001373
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.5046684 1.4896173
sample estimates:
 mean in group No mean in group Yes
        7.057143          6.060000
```

Interpretation: The interval estimates the difference between mean nightly hours of sleep for the populations of students who feel they are not sleep deprived and those who feel they are. We are 95% confident that the mean amount of sleep in the population of students who do not feel sleep deprived is between half an hour and one and a half hours more than the mean amount of sleep in the population of students who do feel sleep deprived.

Assigned Monday, Nov 22

13.15 **a.** Null standard error is $\text{s.e.}(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{15}{\sqrt{50}} = 2.121$

b. $\frac{\text{Sample statistic - Null value}}{\text{Null standard error}} = \frac{102-100}{2.121} = 0.943$

c. No, to find the proper *p*-value we must know whether the alternative hypothesis is one-sided or two-sided.

13.24 **a.** $t = \frac{(4-0)}{15/\sqrt{50}} = 1.89$, df = 50 – 1 = 49, *p*-value is between 2 × .04 = .08 and 2 × .026 = .052, using Table A.3 with df = 40. Using df = 50, .05 < *p*-value < .078. Exact *p*-value (using software) is 2 × .032 = .064.

c. $t = \frac{(0-0)}{15/\sqrt{50}} = 0$; df = 49 and *p*-value = 2 × .5 = 1.0.

13.25 Step 1: $H_0: \mu_d = 0$

$H_a: \mu_d > 0$ (January weight is greater, on average)

μ_d = mean "January weight–November weight" difference for population represented by the sample.

Step 2: The sample size is sufficiently large to proceed. We must assume that the sample represents a random sample from a larger population.

Test statistic is $t = \dfrac{\text{Sample statistic - Null value}}{\text{Null standard error}} = \dfrac{0.37 - 0}{0.109} = 3.39$

Null standard error $= \dfrac{s_d}{\sqrt{n}} = \dfrac{1.52}{\sqrt{195}} = 0.109$ kg.

Step 3: p-value ≈ 0. It is the area (probability) to the right of $t = 3.39$ in a t-distribution with df = $n-1 = 195-1 = 194$. With Table A.3, it can be estimated from the df = 100 row that the p-value is less than .002. With appropriate software of calculator, it can be found that $P(t > 3.39) = .0004$.
Steps 4 and 5: Reject the null hypothesis. The conclusion about the population is that the mean difference between January and November weights is greater than 0. In the sample, the observed magnitude of the difference was $\bar{d}$ = 0.37 kg ≈ 0.8 pounds. (Note that this average gain of less than one pound may not have much *practical* importance.)

You can also use the rejection region approach. In that case, reject H_0 for $\alpha = .05$ if the test statistic t is greater than 1.66. This is found in Table A.2 using df = 100 and $\alpha = .05$, one-tailed. The test statistic 3.94 > 1.66, so reject the null hypothesis.

Assigned Wed, Nov 24

15.6 **a.** No, $\chi^2 = 2.89 < 3.84$ (the critical value from Table A.5).
b. Yes, $\chi^2 = 5.00 > 3.84$ (the critical value from Table A.5).
c. Yes, $\chi^2 = 23.60 > 9.49$ (the critical value from Table A.5).
d. No, $\chi^2 = 23.60 < 25.00$ (the critical value from Table A.5).

15.26 **b.** 250, 250 and 500, respectively, calculated as 1000(1/4), 1000(1/4) and 1000(1/2).
d. 500 for each of the 5 categories, calculated as 2500(1/5) = 500.

15.35 Step 1: H_0: $p_i = 1/10 = .1$ for all digits (equal chance for all digits)
H_a: not all $p_i = 1/10$
Step 2: Expected counts = 600 × .1 = 60 for all ten digits. All expected counts are greater than 5, so proceed with the chi-square test.
Test statistic is $\chi^2 = \dfrac{(49-60)^2}{60} + \dfrac{(61-60)^2}{60} + \ldots + \dfrac{(63-60)^2}{60} = 5.03$; df = $k-1 = 10-1 = 9$

Steps 3, 4, and 5: p-value > .50 (based on Table A.5).
With Excel, p-value = CHIDIST(5.03,9) = .832.
Do not reject the null hypothesis. There is not statistically significant evidence against the hypothesis that all digits are equally likely to be selected.
You could also use the rejection region approach. From Table A.5, the rejection region starts at 16.92. We would reject H_0 if the test statistic $\chi^2 \geq 16.92$. But $\chi^2 = 5.03$, so we do not reject the null hypothesis.