Homework 5 Solutions
Wed, Oct 27 assignment from file on website
Chapter 8: #33, 39, 87
Chapter 8: 48a, 55c and 56 (count as 1), 67a

Assigned Wed, Oct 27

1. Suppose a defense attorney is trying to convince the jury that his client's wallet, found at the scene of the crime, was actually planted there by his client's gardener. Here are two possible ways he might present this to the jury:
Statement A: The gardener dropped the wallet when no one was looking.
Statement B: The gardener hid the wallet in his sock and when no one was looking he quickly reached down and pulled off his sock, allowing the wallet to drop to the ground.
a. Explain why the second statement cannot have a higher probability of being true than the first statement.
ANSWER: The events in the second statement are a subset of the first. From the rules in Chapter 7, we know that the probability of a subset of events cannot be higher than the probability of the full set. (Notice that personal probabilities are relevant here, but the rules should still apply.)

b. Based on the material in Wednesday's lecture, to which statement are members of the jury likely to assign higher personal probabilities? Explain.
ANSWER: They are likely to assign higher probability to statement B because of the excess detail. It is more representative of how people think such things could happen.

2. Comment on the following unusual lottery events, including a probability assessment:

a. On September 11, 2002, the first anniversary of the 9/11 attack on the World Trade Center, the winning number for the New York State lottery was 911.
ANSWER: The number 911 has the same probability as any other 3-digit number, which is 1/1000. It is not surprising that once in awhile the winning numbers would have a meaningful connection with the date on which they were chosen.

b. To play the Maryland Pick 4 lottery, players choose four numbers from the digits 0 to 9. The game is played twice every day, at midday and in the evening. In 1999, holiday players who decided to repeat previous winning numbers got lucky in two separate incidents. At midday on December 24, the winning numbers were 7535, exactly the same as the previous evening. And on New Year's Eve, the evening draw produced the numbers 9521 – exactly the same as the previous evening.
ANSWER: There are 10,000 possible 4-digit winning numbers. So no matter what number is chosen in one drawing, the probability of getting the same number on the next game is 1/10,000. The game is played twice every day, or 730 times a year. Therefore, on average, we would expect to see that exact event happen once every 13 to 14 years or so (10,000/730 = 13.7). But, notice that the 2nd strange event in the Maryland lottery happened not on successive games, but on successive evenings. There are other unusual things that could have happened as well, such as having the numbers match the date 1224. So, while the specific coincidence that happened here is somewhat unlikely, that coincidence or something equally surprising should happen every few years, at least. And given that there are many lottery games played around the country, it is not surprising that events like this happen.

7.68 **a.** The probability that the person actually carries the virus is 1/11 = .0909 because in the low-risk population, for every infected person who tests positive there are 10 people who test positive and do not carry the virus.
b. Although the probability of testing positive for those with the disease is high, the reverse is not true. If there are a very large number of people who do not have the disease, then even if only a small percentage of those test positive, the result will be a large number of positive tests in healthy people. Here is a table that conveys this idea. Notice that everyone with the disease tests positive, and almost 90% of those without the disease correctly test negative, yet of every 11 people who test positive, only one has the virus.

	Test positive	Test negative	Total
HIV	10	0	10
No HIV	100	890	990
Total	110	890	1000

7.73 Your friend should not take the fact that she dreamt about a bombing the day before one actually occurred as a warning. Although the chance of this happening to your friend is small, the chance of someone, somewhere in the world dreaming about a bombing that night is actually quite high, when you consider the large population of the world. And your friend's chance of dreaming about something that happens the next day, at some point in her life, is quite high, just by chance, assuming she remembers her dreams most of the time.

Assigned Friday, October 29

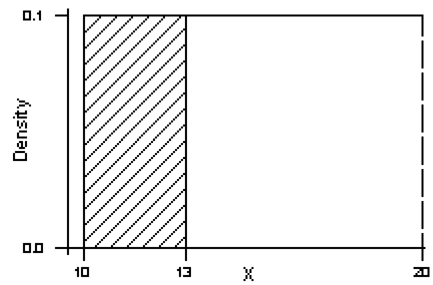
8.33 **a.** Yes. n = 10 and p = .5.
b. No. p is not the same from trial to trial.
c. No. The "trials" (cities) are not independent of each other as they will tend to have the same weather.
d. No. The "trials" (children) are not independent of each other because they are in the same class and flu is contagious.

8.39 Use a binomial distribution with n = 10, p = .5. Let X = number of games won by human. The answers can be found using any of the methods discussed in Section 8.4, including the use of R Commander or Excel.
a. $P(X = 5) = .2461$ (or 0.2460937500 if you use R Commander)
b. $P(X = 3) = .1172$ (0.1171875000 with R Commander) Note that if the computer wins 7 games, the human wins 3 games.
c. $P(X \geq 7) = .1719$, calculated as $P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10) =$
$.1172 + .0439 + .0098 + .0010 = .1719$.
Equivalently, $P(X \geq 7) = 1 - P(X \leq 6) = 1 - .8281 = .1719$
Using R Commander, you *must specify k = 6 and upper tail.* Otherwise, it gives P(X > 7) and leaves out P(X = 7). The result is 0.171875.

8.87 **a.** Using R Commander, Excel, or other software that can determine probabilities for a binomial distribution with n=200 and p=.6, the exact answer is
$P(X \geq 140) = 1 - P(X \leq 139) = 1 - .9979 = .0021$. From R Commander: 0.00213026
b. Note that 70% of 20 is 14 so the desired probability is $P(X \geq 14)$. The exact answer using a binomial distribution with n= 20 and p=0.6 is $P(X \geq 14) = 1 - P(X \leq 13) = 1 - .75 = .25$. From R Commander: 0.2500107.

Assigned Monday, Nov 1

8.48 **a.** The rectangle has height =1/10=0.1 because the range of X is 20–10 = 10.



8.55 **c.** You could use R Commander or Excel to find these without converting to z-scores. Using z-scores: For pulse =59, $z = \frac{59-75}{8} = -2$ while for pulse = 95, $z = \frac{95-75}{8} = 2.5$. Find the area under the standard normal curve between these two z-scores.
$P(-2 \le Z \le 2.5) = P(Z \le 2.5) - P(Z \le -2) = .9938 - .0228 = .9710$.

8.56 First, find the standardized score z^* for which $P(Z \le z^*) = .10$. It's $z^* = -1.28$ so the answer is 1.28 standard deviations below the mean. The answer is $(-1.28 \times 8) + 75 = 64.76$, or about 65.

8.67 **a.** Answer = .9015. For a binomial random variable with n = 1000 and p = .60,
$\mu = np = 1000(.60) = 600$, and $\sigma = \sqrt{1000(.60)(1-.60)} = 15.492$.
For X = 620, $z = \frac{620-600}{15.492} = 1.29$. $P(Z \le 1.29) = .9015$. (You could use R Commander or Excel to find the normal probability without converting to a z-score first.)