STATISTICS 7 MIDTERM EXAM 2

NAME:____________**KEY**____________________ Seat Number: ____________

Student ID#:_________________________________ Discussion Section: _______

You may use two pages of notes (both sides) and a calculator. *Your exam should have 5 pages.*
Free response questions: Show all work. Total of 60 points; points shown for each part.
Multiple choice questions: There are 10 questions worth 4 points each (10 x 4 pts each = 40 pts).

1. Justin needs a ride to the train station and didn't plan ahead to arrange one. He figures he can call one of his three friends who have cars and get a ride when he's ready. Suppose that the probability that each friend will be available to give him a ride is .7, and is independent for the three friends.

   a. (4 pts) What is the probability that the first friend is *not* available?

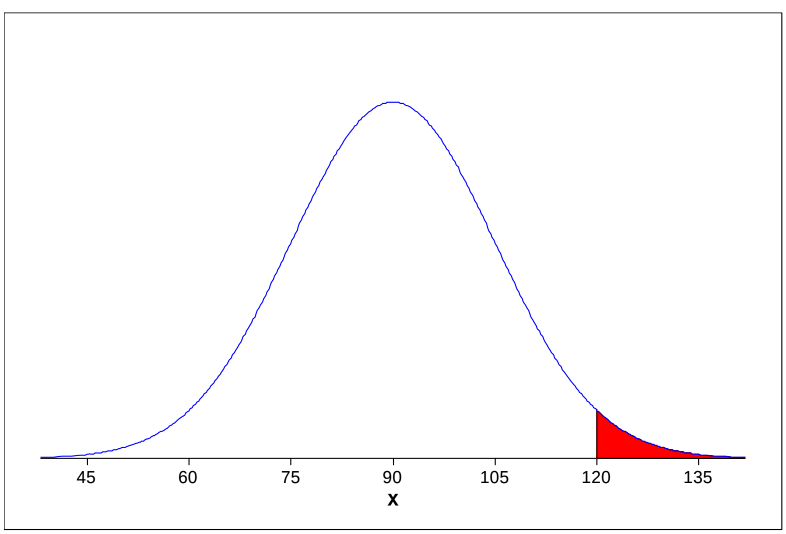
   1 – .7 = .3

   b. (4 pts) What is the probability that *at least one* of his friends is available?

   The easiest way to solve any "at least one" problem is to use 1 – P(none).
   In this case, that becomes 1 – (.3)(.3)(.3) = 1 – .027 = ***.973****.*

Questions 2 and 3 [Question 3 on next page.]**:** Draw a picture of the requested distribution, label the value for the mean, and shade in the area corresponding to the requested probability. You *do not* need to compute the probability but make sure the requested value is located in about the right spot on the picture.

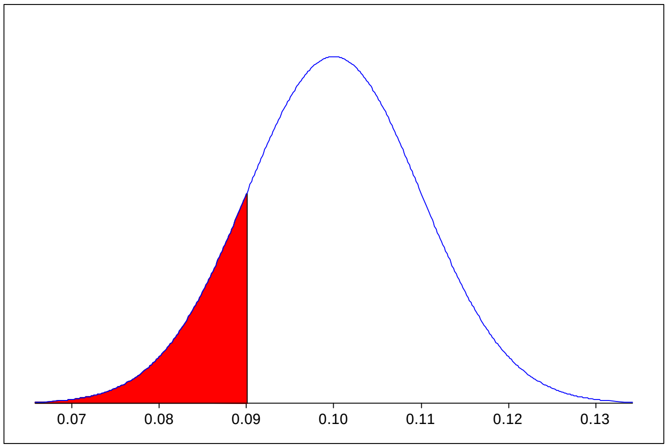
2. (5 pts) If students were to be given unlimited time, the distribution of times to complete a final exam would be normally distributed with mean = 90 minutes and standard deviation = 15 minutes. Draw a picture of this distribution, and shade in the area that represents the probability that a randomly selected student will take more than 120 minutes to complete the exam.



3. (5 pts) Suppose that 10% of college students in the United States are vegetarians. The incoming first-year class of 900 students at a college can be considered to be a representative sample of all US college students for this purpose. Define $\hat{p}$ to be the proportion of students in the incoming class that are vegetarians. Draw a picture of the possible values of $\hat{p}$. Shade the area representing the probability that this proportion will be less than .09.

This is the sampling distribution of $\hat{p}$ *and is approximately normal with mean = p = .10 and*

standard deviation $= \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.1(.9)}{900}} = .01$



4. An "instant lottery" is played by buying a ticket and scratching off a coating to reveal whether or not you have won a prize, and if so, how much. Suppose an instant lottery pays \$5 with probability 0.05 and \$100 with probability 0.006. Otherwise it pays nothing. Define X = amount won for one ticket. (You can ignore the cost of the ticket.)

   a. (5 pts) Write the probability distribution function for X.

k	0	\$5	\$100
P(X = *k*)	0.944	0.05	0.006

   b. (5 pts) Find the expected value of X.

E(X) = 0(.944) + (\$5)(.05) + (\$100)(.006) = \$0.85 or 85 cents

   c. (2 pts) Finish the following sentence by filling in an amount in the blank: If the lottery agency wants to make money over the long run, they must charge more than ____85 cents___ per ticket.

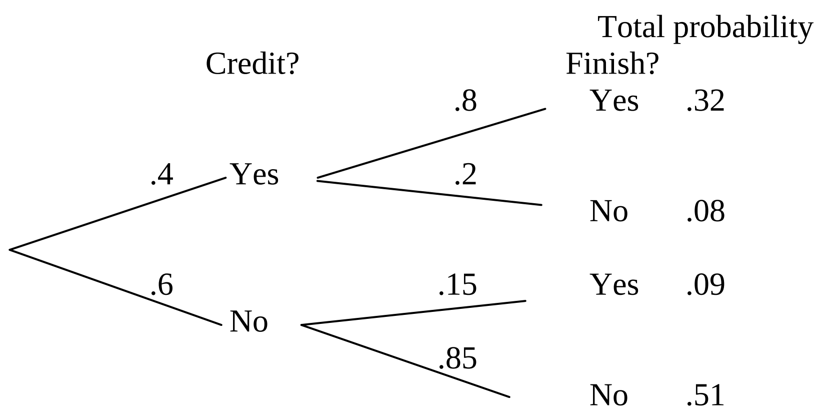
For Questions 5 to 8: An online education company offers an online Spanish course. Students can pay to take the course and receive college credit, or they can take the course for free but receive no credit. The company has noticed that 40% of students enroll for credit, while the remaining 60% enroll for the free course. Of those who enroll for credit, 80% finish the course. Of those who enroll for the free course, only 15% finish the course. Define the following events concerning a randomly selected person who enrolls in the course:

A = The person enrolls in the course for credit.
B = The person finishes the course.

5. (2 pts each) Provide values for the following:

   $P(A)$ = .40 $\qquad$ $P(B \mid A)$ = .80 $\qquad$ $P(B \mid A^C)$= .15

6. (6 pts) Draw a tree diagram for this situation *or* construct a hypothetical hundred thousand table.



	Finish	Doesn't finish	Total
For Credit	32,000	8,000	40,000
Not for Credit	9,000	51,000	60,000
Total	41,000	59,000	100,000

7. (5 pts) Using your diagram or table in Question 6, find the overall probability that a person who enrolls finishes the course.

   From the tree diagram: .32 + .09 = .41

   From the table: 41,000/100,000 = .41

8. (5 pts) Of all students who finish the course, what proportion enrolled for credit?

   From the tree diagram: .32/.41 = .78

   From the table: 32,000/41,000 = .78

Questions 9 and 10: (1 point for each blank) For each of the following, fill in the first blank with *appropriate notation*, and the second blank with *a number* for the requested value. For instance, an answer might be of the form $\underline{\ p\ }$ = $\underline{\ 0.2\ }$, $\underline{\ \bar{x}\ }$ = $\underline{\ 10\ }$, and so on.

9. In a random sample of 300 college seniors it was found that the mean credit card debt they owed was \$635. This mean can be written as:

$$\underline{\ \bar{x}\ } = \underline{\ \$635\ }$$

10. Suppose that 26% of overweight adults and 18% of normal weight adults have high blood pressure.

  a. The difference in population proportions with high blood pressure is

$$\underline{p_1 - p_2} = \underline{.26 - .18 = .08}$$

  b. Random samples of 500 overweight and 500 normal weight adults will be taken, and the proportions with high blood pressure will be found. The mean of the sampling distribution of the difference in proportions that will be found when the samples are taken is:

$$\underline{p_1 - p_2} = \underline{.26 - .18 = .08}$$

  c. The random samples of 500 each are taken, and 125 of the overweight adults have high blood pressure and 100 of the normal weight adults have high blood pressure. The point estimate for the difference in population proportions is

$$\underline{\hat{p}_1 - \hat{p}_2} = \underline{\frac{125}{500} - \frac{100}{500} = .25 - .20 = .05}$$

MULTIPLE CHOICE: Circle your answer.

Scenario for Questions 1 and 2: Suppose that 70% of the seniors at a large university have taken calculus, and 30% of the seniors have taken physics. Of the seniors who have taken calculus, 40% have taken physics. A student who is a senior at this university is randomly selected. Define events

A = {the student has taken calculus}
B = {the student has taken physics}

1. Which of the following is true about the events A and B?
   A. They are mutually exclusive.
   B. They are complements.
   C. They are independent.
   D. None of the above. [*Note that they are not independent because P(B|A) = .40 ≠ P(B) = .30**]*

2. As stated above, of the seniors who have taken calculus, 40% have taken physics. Which of the following is the correct way to write this information as a probability for the randomly selected student?
   A. P(A|B) = .40
   B. P(B|A) = .40
   C. P(A and B) = .40
   D. P(A or B) = .40

3. The formula E(X) = *np* applies in which of these situations?
   A. For all discrete random variables.
   B. For all continuous random variables.
   C. For all normal random variables.
   D. For all binomial random variables.

4. Which of the following would be true if people made decisions solely based on maximizing their "expected monetary return" (i.e., the expected value of the money they would make or save)?
   A. People wouldn't buy insurance or lottery tickets.
   B. People would buy lots of insurance.
   C. People would buy lots of lottery tickets.
   D. People would always buy an extended warranty if it was offered.

5. One purpose of statistical inference is:
   A. To describe sample data with summary statistics from the sample.
   B. To describe population data with summary statistics from the population.
   C. To make conclusions about populations based on information from a sample.
   D. To make conclusions about samples based on information from the population.

6. Which of the following is <u>not</u> a random variable?
   A. $\hat{p}$
   B. μ *[Note that μ is a parameter, not a statistic. Statistics are random variables; parameters are not.]*
   C. $\bar{x}$
   D. All of the above are random variables.

7. Which of the following situations results in paired samples as opposed to independent samples?
   A. A random sample of Democrats and a separate random sample of Republicans are asked their opinions on legalization of marijuana.
   B. Participants in a medical study are randomly assigned to receive either a drug or a placebo.
   C. Adults in a random sample are categorized as male/female and asked their opinion on an issue.
   D. None of the above situations result in paired samples, they all result in independent samples.

8. Using the Empirical Rule, the appropriate multiplier for a 68% confidence interval for a proportion is:
   A. z* = 1
   B. z* = .84
   C. z* = .68
   D. z* = .50

9. One question in the class survey asked which of the following was more likely during the Cold War:
   - An all-out nuclear war between the United States and Russia
   - An all-out nuclear war between the United States and Russia in which neither country intends to use nuclear weapons, but both sides are drawn into the conflict by the actions of a country such as Iraq, Libya, Israel, or Pakistan.

   Which of the following was illustrated by the results of that question?
   A. The historical significance heuristic.
   B. Confusion of the inverse.
   C. The conjunction fallacy.
   D. The anchoring effect.

10. Suppose that in a situation involving the difference in two proportions for independent samples, the <u>mean</u> of the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is 0. Which of the following must be true?
    A. $p_1 - p_2 = 0$. ←***This one.***
    B. $\hat{p}_1 - \hat{p}_2 = 0$.
    C. s.d.($\hat{p}_1 - \hat{p}_2$) = 0.
    D. Both A and B must be true.