ANNOUNCEMENTS:

- Discussion this week is for credit. Last chance!

- We will be covering the remaining chapters out of order.

Proportions (categorical data):

Today: Sections 12.1 and 12.2, except 12.1 Lesson 3
Wed: More of Chapter 12 (hypothesis tests for proportions)

Means (quantitative data):

Fri: Finish Chapter 9 (sampling distributions for means)
Mon: Chapter 11 (confidence intervals for means)
Wed: Start Chapter 13 (hypothesis tests for means)

General hypothesis testing issues, summary and wrap-up:

Fri: Finish Chapters 12 and 13 and cover Chapter 17 (wrap-up)

HOMEWORK: (Due Monday, March 11)

Chapter 12: #20, 24, 50b, 102

Reminder from when we started Chapter 9

Five situations we will cover for the rest of this quarter:

Parameter name and description	**Population parameter**	**Sample statistic**
For Categorical Variables:		
One population proportion (or probability)	p	$\hat{p}$
Difference in two population proportions	$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$
For Quantitative Variables:		
One population mean	μ	$\bar{x}$
Population mean of paired differences (dependent samples, paired)	μ_d	$\bar{d}$
Difference in two population means (independent samples)	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$

For each situation will we:

√ Learn about the *sampling distribution* for the sample statistic

√ Learn how to find a *confidence interval* for the true value of the parameter

- *Test hypotheses* about the true value of the parameter

Review: Standardized Statistics

(Use as the "test statistic" for our 5 situations)

- *If distribution of the sample statistic* is
  - ◊ Approximately normal
  - ◊ Mean = population parameter
  - ◊ St. Dev. = s.d.(sample statistic)

- Then standardize using:

$$z = \frac{\text{sample statistic - population parameter}}{\text{s.d.(sample statistic)}}$$

Review: Using Standard Error

- When standard deviation isn't known, use standard error in denominator instead.
- For hypothesis testing, will use *null standard error* in denominator (defined later)
- Standardized statistic is still z for situations with proportions.
- For means, standardized statistic is t, not z (more Fri).

Summary of sampling distributions for the 5 parameters (p. 353):

	Parameter	Statistic	Standard deviation of the statistic	Standard Error of the Statistic	Std. Stat. with s.e.
One proportion	p	$\hat{p}$	$\sqrt{\frac{p(1-p)}{n}}$	$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	z
Difference Between Proportions	$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$	$\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	z
One Mean	μ	$\bar{x}$	$\frac{\sigma}{\sqrt{n}}$	$\frac{s}{\sqrt{n}}$	t
Mean Difference, Paired Data	μ_d	$\bar{d}$	$\frac{\sigma_d}{\sqrt{n}}$	$\frac{s_d}{\sqrt{n}}$	t
Difference Between Means	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	t

Chapter 12: Significance testing = hypothesis testing.
Saw this already in Chapter 4 (chi-square test).

Today:
Hypothesis tests for *one population proportion*

Example: Media report said "A recent poll shows that more than half of adult Americans favor their own state passing an immigration law like Arizona's new law."

Based on May 2010 poll of n = 884 adults outside of Arizona, asked: *Would you favor or oppose YOUR state passing an immigration law like Arizona's new law?*

Results: More than half of the *sample*, $\hat{p} = .52$ said *favor*.

Question: Can we conclude that more than half (50%) of the *population* feels this way? Or is it plausible that less than 50% feels that way, even with 52% of the *sample* in favor?

- The *sample proportion* based on n = 884 was $\hat{p} = .52$.
- Suppose the *population proportion p* is *.50 or less*, i.e. the true p is $\leq .50$. How unlikely is such a large value of $\hat{p}$?
- In other words, is the media report correct that a *sample proportion* of .52 is convincing evidence that *more than half* of the *population* favors such a law? I.e. that $p > .50$?

We could answer this by getting a confidence interval for the true *p*, but for reasons shown later, a *hypothesis test* is better.

Five Steps to Hypothesis Testing: General, One p, Example

STEP 1: Specify null and alternative hypotheses *about the population parameter*.

General: *Alternative hypothesis* is usually what researchers hope to show. “Null is dull.” Null is often status quo, a specific value. For two proportions or two means, null is usually “no difference.”

For our 5 situations, hypotheses are written as:
Null hypothesis:
    H_0: parameter = null value (a specified value)
Alternative hypothesis is one of these:
    H_a: parameter ≠ null value (a two-sided hypothesis)
    H_a: parameter > null value (a one-sided hypothesis)
    H_a: parameter < null value (a one-sided hypothesis)

For One Proportion: p = **population** proportion

Null hypothesis is written as: H_0: $p = p_0$ (a specified value)

The "equal sign" always goes in the null hypothesis.

Alternative hypothesis is *one* of these, based on context:

H_a: $p \neq p_0$ (a two-sided hypothesis)

H_a: $p > p_0$ (a one-sided hypothesis)

H_a: $p < p_0$ (a one-sided hypothesis)

Remember how hypothesis testing works:

- *Assume* the null hypothesis is true about the population
- Compute what we *expect* if that's the case
- Compare it to what we *observed in the sample*
- For our 5 situations, we do this using standardized statistics

Example: Is the media's claim correct?

Want to know if media claim is correct that *more than half of all adults* favor a law. That is the *alternative hypothesis.*

H_0: $p = 0.5$ (could write $p \leq 0.5$)
In words: The media claim is not correct for the population: 50% or less of *all* adults favor the law

H_a: $p > 0.5$
In words: the media quote is correct; more than half of *all* adults favor such a law

The *null value* is $p_0 = 0.5$. This is a *one-sided* test (H_a has >)

STEP 2:

Verify data conditions. If met, summarize data into a test statistic.

General: For our five situations, test statistic will be a standardized statistic:

z (for situations with proportions)

t (for situations with means – more later)

When the null hypothesis is true, population parameter ≡ null value, so standardized statistic is

$$\frac{\text{sample statistic} - \text{null value}}{\text{(null) standard error}}$$

Note that here "sample statistic" = the value of the sample statistic computed from the data.

For One Proportion:

Data conditions: np_0 and $n(1 - p_0)$ are both at least 10.

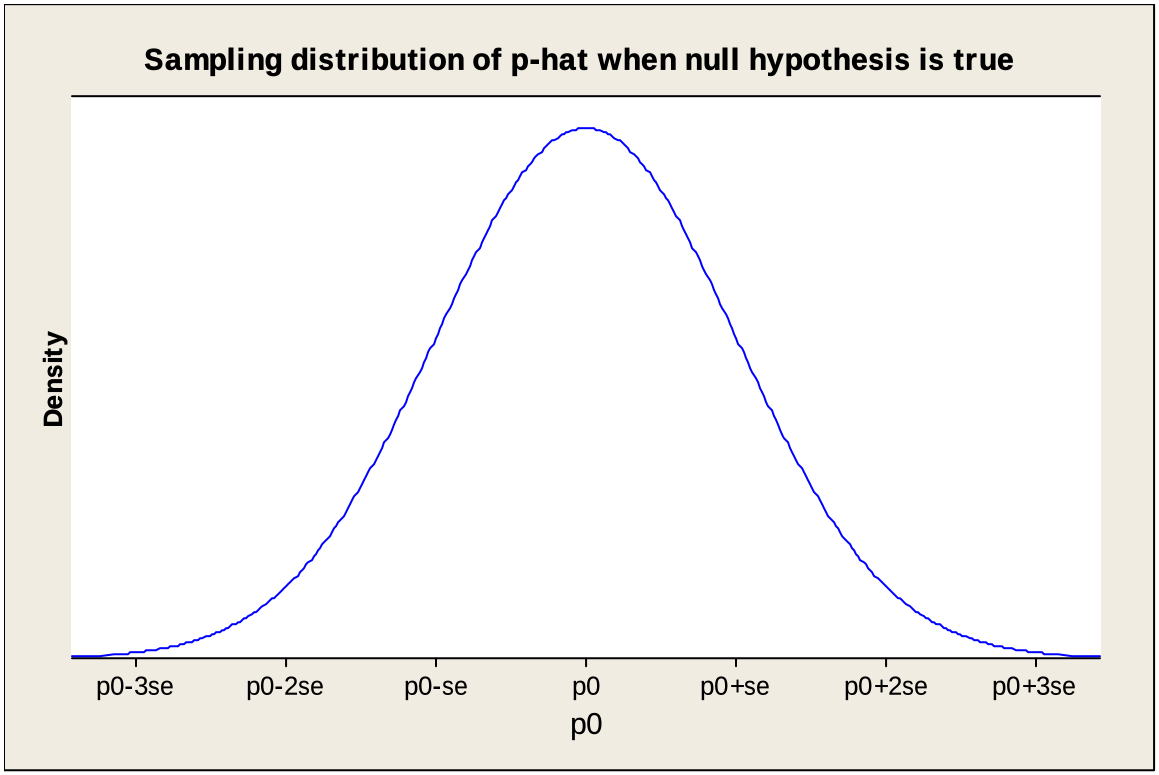
Sample statistic = $\hat{p}$, *null value* = p_0, *null standard error* = *s.d.*($\hat{p}$) with *null value* p_0 in place of p. So the *test statistic* is:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

Rationale:

Compare *observed* ($\hat{p}$) to what's *expected if null is true* (p_0).

If p_0 really is the true population proportion, where does the *observed value* of $\hat{p}$ fall in the sampling distribution of possibilities?



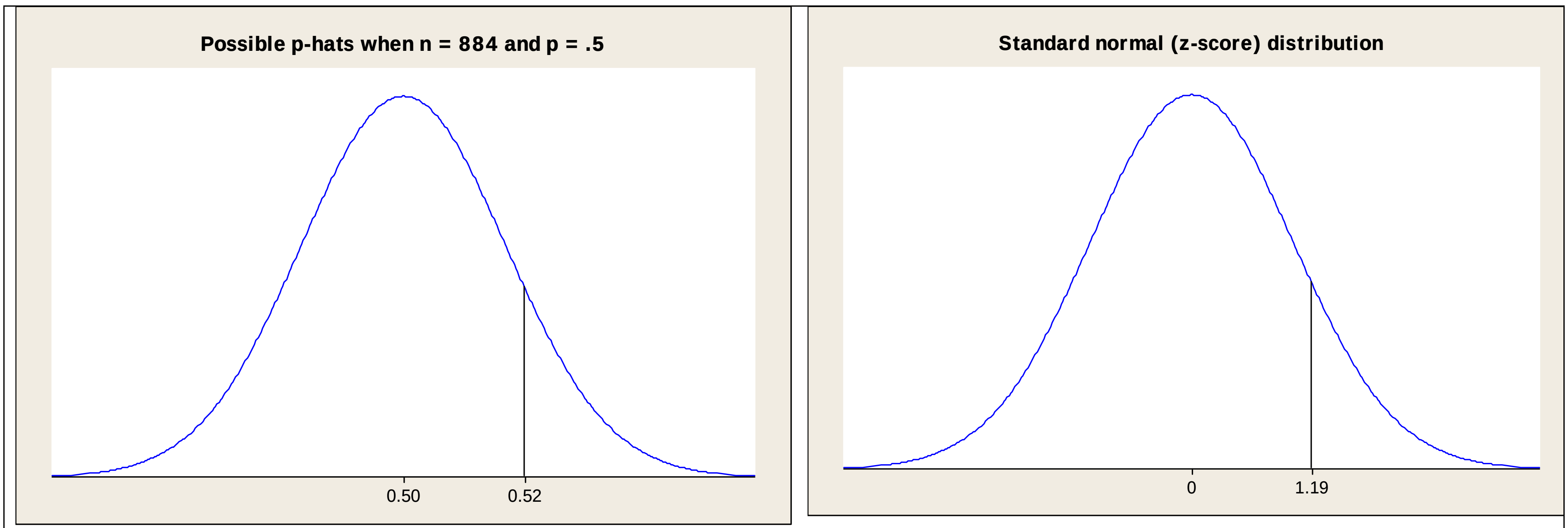
Test statistic is the z-score for the location of the observed $\hat{p}$ in this picture. *If the null hypothesis is true*, test statistic is standard normal. *If the null hypothesis is not true* picture will *not* be centered on p_0

Example (Immigration poll):

Data conditions are met: np_0 and $n(1 - p_0) = (884)(0.5) = 442$

Null standard error is $\sqrt{\frac{(.5)(.5)}{884}} = .0168$; test statistic is:

$$\frac{\text{Sample statistic - null value}}{\text{(null) standard error}} = \frac{.52-.50}{.0168} = 1.19$$



Note where $\hat{p}$ = .52 falls here = where z = 1.19 falls here.

STEP 3:
Assuming the null hypothesis is true, find the p-value.

General: *p*-value = the probability of a test statistic as extreme as the one observed or more so, in the direction of H_a, *if* the null hypothesis is true. (I.e., *conditional* on the null being true.)

One proportion: Depends on the alternative hypothesis. See pictures in class and on p. **465**

Alternative hypothesis:	p-value is:
H_a: $p > p_0$ (a one-sided hypothesis)	Area above the test statistic z
H_a: $p < p_0$ (a one-sided hypothesis)	Area below the test statistic z
H_a: $p \neq p_0$ (a two-sided hypothesis)	$2 \times$ the area above $\|z\|$ = area in tails beyond $-z$ and z

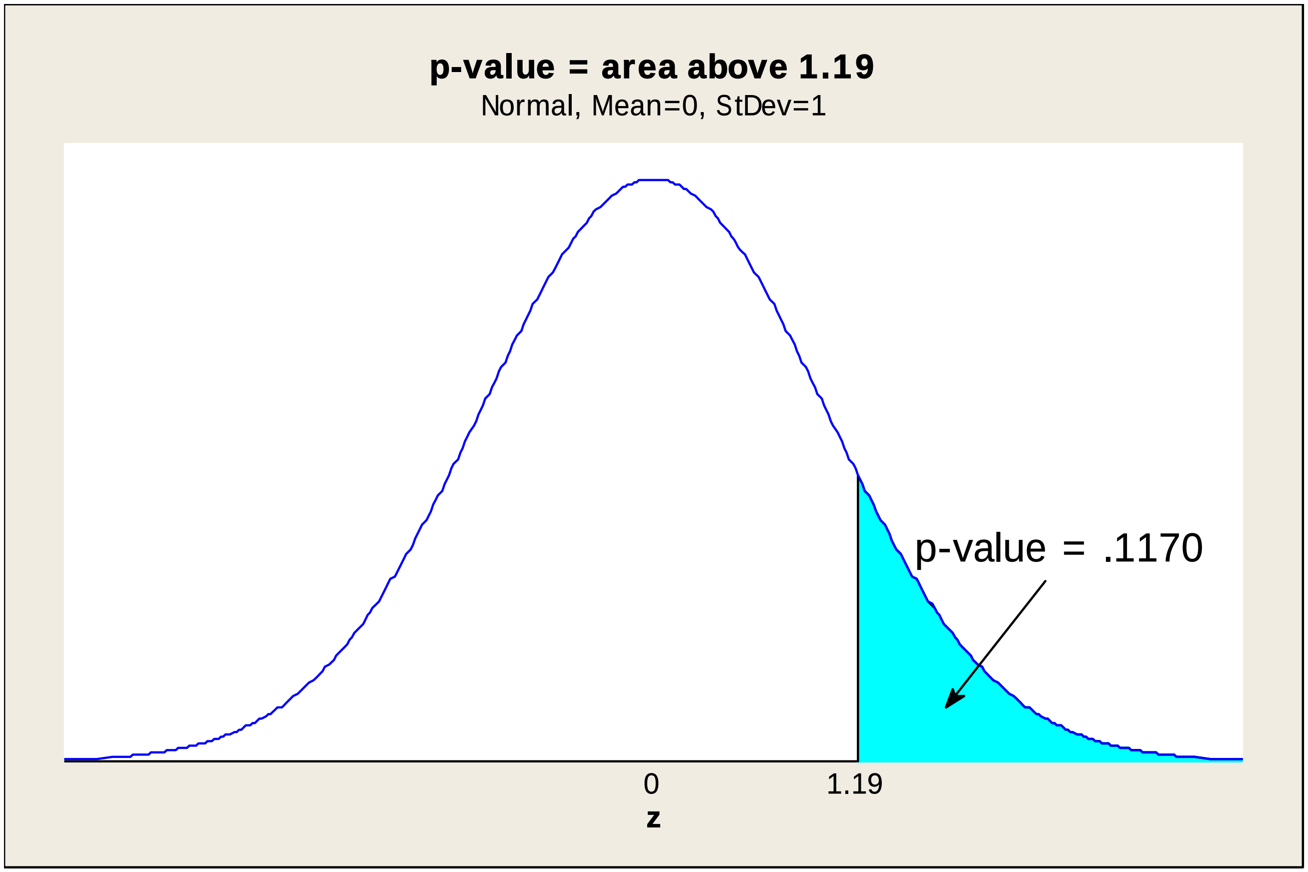
Example: p = *population* proportion who would favor a law

H_0: p = 0.5, H_a: p > 0.5

Alternative hypothesis is *one-sided*

p-value = Area above the test statistic = $P(z \geq 1.19)$

From Table A.1, *p*-value = .1170



STEP 4:
Decide whether or not the result is statistically significant based on the p-value. Two possible conclusions.

If p-value ≤ α ***(level of significance,*** usually .05), these are equivalent ways to state the conclusion:

- Reject the null hypothesis.
- Accept the alternative hypothesis
- The result is statistically significant

If p-value > α (***level of significance,*** usually .05), these are equivalent ways to state the conclusion:

- Do not reject the null hypothesis
- There is not enough evidence to support the alternative hypothesis.
- The result is not statistically significant.

Example:
p-value = .1170 > .05, so our conclusion can be stated as:

- Do *not* reject the null hypothesis.
- There is not enough evidence to support the alternative hypothesis.
- The result is *not* statistically significant.

Step 5: Report the conclusion in the context of the situation.

Example:
We could not reject the null hypothesis. There was not enough evidence to accept the alternative hypothesis. The proportion of adults who favor a law in their state (when poll was taken) is *not* statistically significantly more than half.
So the media quote was *not* justified. (Recall, quote was "A recent poll shows that more than half of adult Americans favor their own state passing an immigration law like Arizona's new law.")

TECHNICAL NOTE

Often in hypothesis testing the standardized score will be "off the charts." The bottom of Table A.1 has values for "In the extreme." Here are some of them for the lower tail:

z	area below z
-4.28	.00001
-4.75	.000001
-5.20	.0000001

Example: Suppose the test statistic is $z = -5.00$

For Ha: $p < p_0$ we would say the *p*-value < .000001.

(Area below –5.00)

For Ha: $p \neq p_0$ we would say the *p*-value < .000002.

(Area below –5.00 + Area above 5.00)

EXAMPLE 2: According to the CDC website, 59% of 18 to 25 year-olds in the United States drink alcohol. ("Base rate" = .59)

QUESTION: Is the percent different for UCI students? (Note that before looking at data, we have no reason to ask if it's in one direction or the other, just is it *different* from national proportion.)

Define the population parameter:
p = proportion of *all* UCI students who drink alcohol at least once a month.

Data: Sample of n = 149 students in Biostatistics (Stat 8).

We are assuming that the Biostatistics students are representative of all UCI students for this question.

STEP 1: Determine the null and alternative hypotheses.

For One Proportion: p = **population** proportion

Null hypothesis is H_0: $p = p_0$ (a specified value)

Alternative hypothesis is one of these, based on context:

H_a: $p \neq p_0$ (a two-sided hypothesis)

H_a: $p > p_0$ (a one-sided hypothesis)

H_a: $p < p_0$ (a one-sided hypothesis)

Example:

Is UCI proportion who drink *different* from national proportion of 0.59?

H_0: $p = 0.59$ (UCI students are same as US population.)

H_a: $p \neq 0.59$ (UCI students are different from US population.)

The *null value* is $p_0 = 0.59$. This is a *two-sided* test.

STEP 2:
Verify data conditions. If met, summarize data into test statistic.

For One Proportion:
Data conditions: np_0 and $n(1 - p_0)$ are both at least 10. Only need to find smaller: The condition is met; $n(1 - p_0) = 149(.41) = 61.1$

Test statistic: In the sample, 100 students out of 149 drink, so

$$\hat{p} = \frac{100}{149} = .671,\ \ p_0 = .59,\ \text{null standard error} = \sqrt{\frac{(.59)(1-.59)}{149}} = .04$$

$$z = \frac{\text{sample statistic - null value}}{\text{(null) standard error}} = \frac{.671 - .59}{.04} = 2.02$$

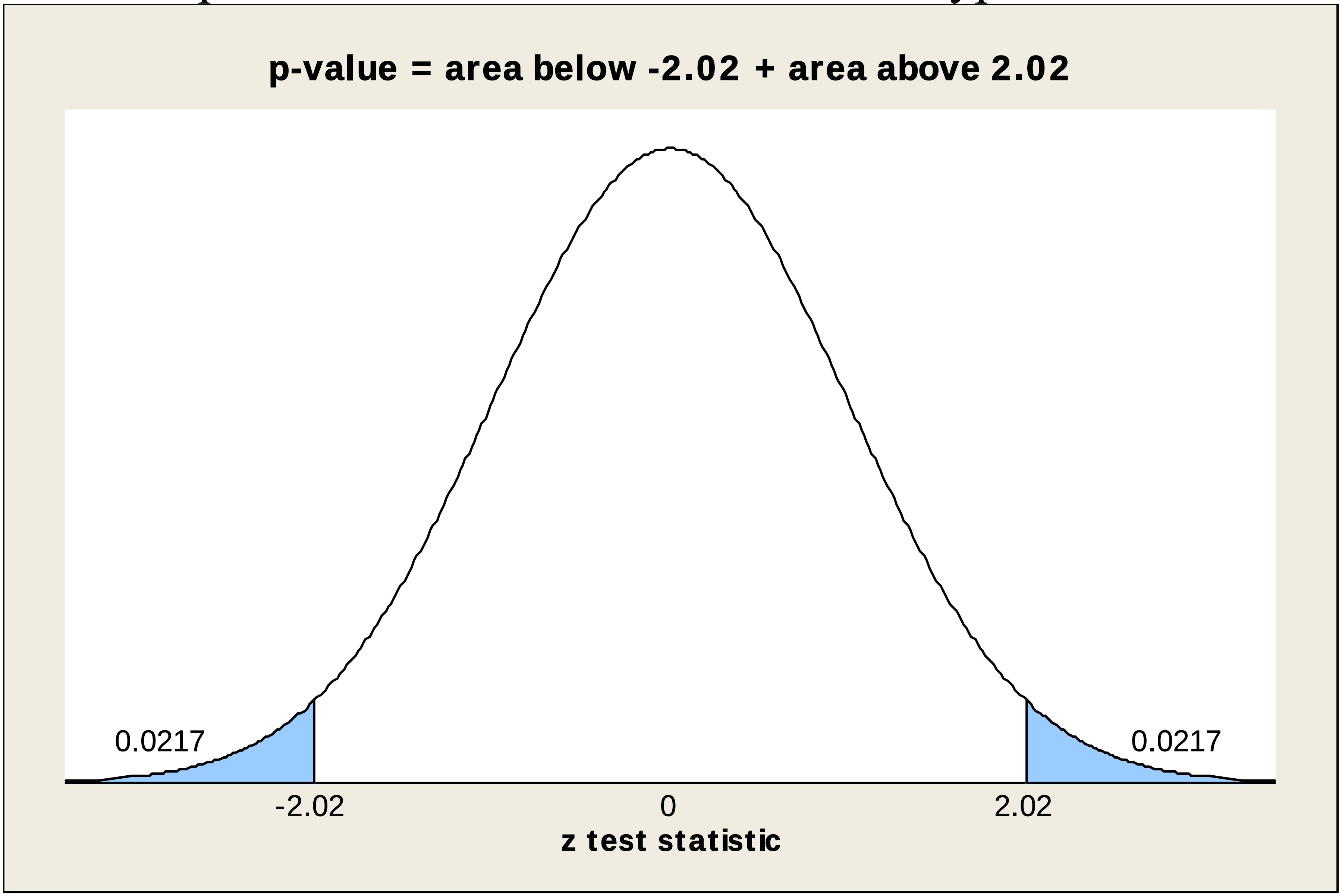
STEP 3:
Assuming the null hypothesis is true, find the p-value.

Alternative hypothesis:	p-value is:
H_a: $p > p_0$ (a one-sided hypothesis)	Area above the test statistic z
H_a: $p < p_0$ (a one-sided hypothesis)	Area below the test statistic z
H_a: $p \neq p_0$ (a two-sided hypothesis)	2 × the area above |z| = area in tails beyond –z and z

Example: In this case, H_a: $p \neq 0.59$
p-value = 2 × the area above |z| = area in tails beyond –z and z
2 × the area above 2.02 = area in tails beyond -2.02 and 2.02

From Table A.1, *p*-value = 2 × .0217 = .0434
See picture on next page.

p-value for a two-sided alternative hypothesis



STEP 4:
Decide whether or not the result is statistically significant based on the p-value.

Example: *p*-value = .0434
If p-value $\leq$ ***level of significance*** (usually .05), these are equivalent ways to state the conclusion:

- Reject the null hypothesis.
- Accept the alternative hypothesis
- The result is statistically significant

Step 5: Report the conclusion in the context of the situation.

Example: There is a statistically significant difference between the proportion of the *population* of UCI students who drink alcohol and the proportion of 18 to 25 year-olds in the national population who do so.

How much different are UCI students from national?

For *two-sided tests*, often a good idea to follow up with a confidence interval:

Example:
A 95% confidence interval for the UCI proportion is

$$.671 \pm 2(.04)$$

$$.671 \pm .08$$

$$.599 \text{ to } .751$$

Just barely misses the null value of .59!

Correspondence between 2-sided test and 95% CI:

Reject H_0: $p = p_0$ at .05 if p_0 is *not* in a 95% confidence interval. That means p_0 is *not* a *plausible* value for *p*.

Example: 0.59 is (just barely) *outside* of the 95% confidence interval, so we can *reject* 0.59 as a likely value for the true proportion of UCI students who drink alcohol.

Rejection Region Approach (instead of using p-value)

The **rejection region** is the region of possible values for the test statistic that would lead to rejection of the null hypothesis.

Substitute Step 3: Instead of computing a *p-value*, find the *rejection region.*

Substitute Step 4: Reject the null hypothesis if the test statistic falls in the rejection region.

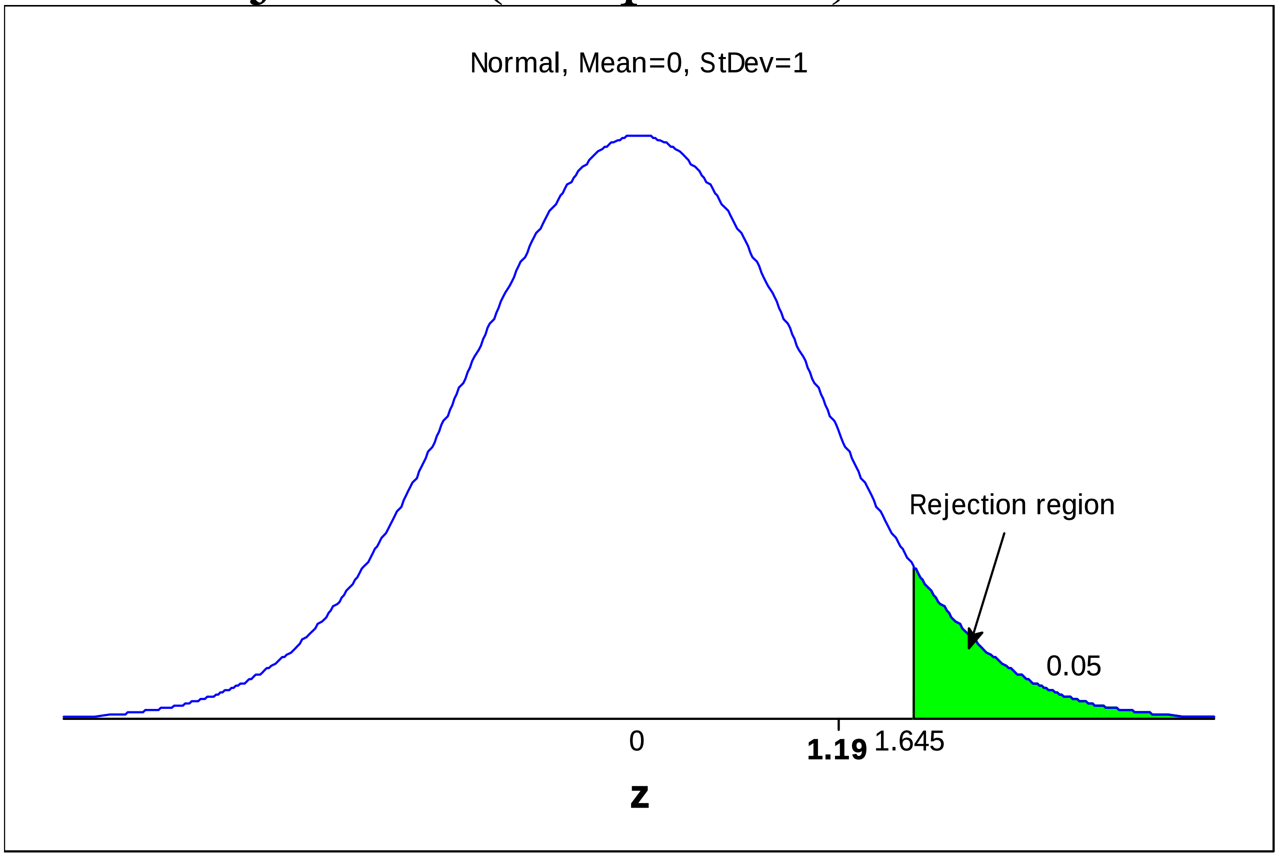
When the test statistic is a z-score and level of significance = .05:

For H_a: $p < p_0$ Rejection region is $z \leq -1.645$
For H_a: $p > p_0$ Rejection region is $z \geq +1.645$
For H_a: $p \neq p_0$ Rejection region is $z \leq -1.96$ and $z \geq +1.96$

Example 1: One-sided, H_a: $p > p_0$, test statistic $z = 1.19$

Step 3: Rejection region is $z \geq +1.645$

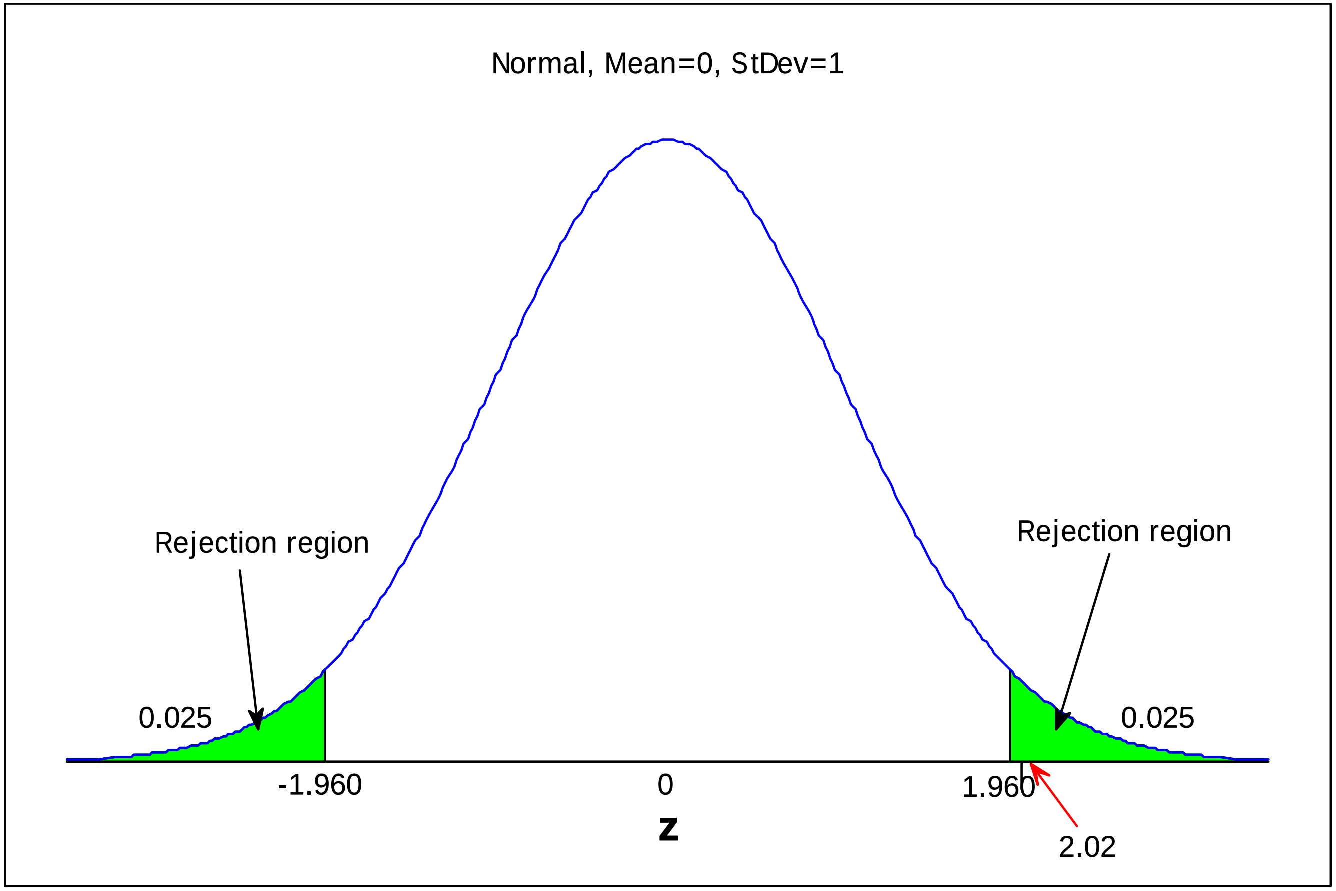
Step 4: The test statistic of 1.19 does *not* fall into the rejection region, so *do not* reject H_0. (See picture.)



Example 2: Two-sided, H_a: $p \neq p_0$, test statistic $z = 2.02$

<u>Step 3:</u> Rejection region is $z \leq -1.96$ and $z \geq +1.96$

<u>Step 4:</u> The test statistic of 2.02 *does* fall in the rejection region, so reject H_0.



Examples without context, for practice with conclusions

Use level of significance of $\alpha = .05$ (Draw pictures)

H_0: p = .2, H_a: p < .2, test statistic z = −1.75

- *p-value* = area *below* −1.75 = .0401; reject H_0 (.0401 < .05)
- Rejection region is $z \leq -1.645$; reject H_0 (−1.75 < −1.645)

H_0: p = .2, H_a: p < .2, test statistic z = +1.75

- *p-value* = area *below* +1.75 = .9509; do *not* reject H_0
- Rejection region is $z \leq -1.645$; do *not* reject H_0

H_0: p = .2, H_a: p ≠ .2, test statistic z = −1.75

- *p-value* = 2 × area *below* −1.75 = .0802; do not reject H_0
- Rejection region is $z \leq -1.96$ and ≥ 1.96; do not reject H_0