Announcements:

•Quiz #6 begins at 4pm today and ends at 3pm on Wed, Feb 27.

•Midterm Friday, Chs 7 to 10 (including additional stuff done in class). Bring 2 sheets of notes and calculator. See website for sections to skip.

•Wed: Catch up and review. I will do an overview, then ask for questions, then (if time) go over review sheet (already posted on website).

Homework: (Due Wed, Feb 27)

Chapter 10: #4, 30, 78

Chapter 10

Estimating Proportions with Confidence

Structure for the rest of the Quarter

Parameter name and description	Sampling Distribution	Confidence Interval	Hypothesis Test
For Categorical Variables:	Chapter 9	Chapter 10	Chapter 12
One population proportion or binomial probability	Today & Fri.	Mon, Feb 25	Mon, Mar 4
Difference in two population proportions	Friday	Mon, Feb 25	Wed, Mar 6
For Quantitative Variables:	Chapter 9	Chapter 11	Chapter 13
One population mean	Fri, March 8	Mon, Mar 11	Wed, Mar 12
Population mean of paired differences (paired data)	Fri, March 8	Mon, Mar 11	Wed, Mar 12
Difference in two population means (independent samples)	Fri, March 8	Mon, Mar 11	Wed, Mar 12

Reminder from when we started Chapter 9;
Five situations we will cover for the rest of this quarter:

Parameter name and description	Population parameter	Sample statistic
For Categorical Variables:		
One population proportion (or probability)	p	$\hat{p}$
Difference in two population proportions	$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$
For Quantitative Variables:		
One population mean	μ	$\bar{x}$
Population mean of paired differences (dependent samples, paired)	μ_d	$\bar{d}$
Difference in two population means (independent samples)	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$

For each situation we will:
- √ Learn about the *sampling distribution* for the sample statistic
- Learn how to find a *confidence interval* for the true value of the parameter
- *Test hypotheses* about the true value of the parameter

Example from last lecture

Gallup poll of n = 1018 adults found 39% believe in evolution. So $\hat{p}$ = .39

A **95% confidence interval** or **interval estimate** for the proportion (or percent) of *all* adults who believe in evolution is **.36** to **.42** (or **36%** to **42%**).

Confidence interval: an interval of estimates that is *likely* to capture the true population value.

Goal today: Learn to calculate and interpret confidence intervals for p and for $p_1 - p_2$ *and* learn general format.

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Remember population versus sample:

- **Population proportion:** the fraction of the ***population*** that has a certain trait/characteristic or the probability of success in a binomial experiment – denoted by p. The value of the ***parameter*** p is fixed but not known.
- **Sample proportion:** the fraction of the ***sample*** that has a certain trait/characteristic – denoted by $\hat{p}$. The ***statistic*** $\hat{p}$ is an estimate of p.

The **Fundamental Rule for Using Data for Inference:** Available data can be used to make inferences about a much larger group *if the data can be considered to be representative with regard to the question(s) of interest.*

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Some Definitions:

- **Point estimate:** A *single number* used to estimate a population parameter. For our five situations:

point estimate = **sample statistic** = **sample estimate**

= $\hat{p}$ for one proportion

= $\hat{p}_1 - \hat{p}_2$ for difference in two proportions

- **Interval estimate:** An *interval* of values used to estimate a **population parameter**. Also called a **confidence interval**. For our five situations, always the formula is:

Sample estimate ± multiplier × standard error

Details for proportions:

Sample estimate ± multiplier × standard error

Parameter	Sample estimate	Standard error
p	$\hat{p}$	$s.e.(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	See p. 386 for formula

Multiplier and Confidence Level

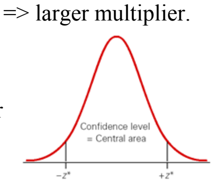
- The **multiplier** is determined by the desired confidence *level*.
- The **confidence level** is the probability that the procedure used to determine the interval *will* provide an interval that includes the population parameter. Most common is .95 (95%).
- If we consider *all possible* randomly selected samples of same size from a population, the *confidence level* is the fraction or percent of those samples for which the confidence interval includes the population parameter.
- Often express the confidence level as a percent. Common levels are 90%, 95%, 98%, and 99%.

More about the Multiplier

Confidence Level	Multiplier	Confidence Interval
90	1.645 or 1.65	$\hat{p}$ ± 1.65 standard errors
95	1.96, often rounded to 2	$\hat{p}$ ± 2 standard errors
98	2.33	$\hat{p}$ ± 2.33 standard errors
99	2.58	$\hat{p}$ ± 2.58 standard errors

Note: Increase confidence level => larger multiplier.

Multiplier, denoted as z^*, is the standardized score such that the area between $-z^*$ and $+z^*$ under the standard normal curve corresponds to the desired confidence level.



Formula for C.I. for proportion

Sample estimate ± multiplier × standard error

For one proportion: A confidence interval for a population proportion p, based on a sample of size n from that population, with sample proportion $\hat{p}$ is:

$$\hat{p} \pm (z^*) \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Example of different confidence levels

Poll on belief in evolution:

$n = 1018$

Sample proportion = .39

Standard error = $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.39(1-.39)}{1018}} = .0153$

90% confidence interval

.39 ± **1.65** (.0153) or .39 ± .025 or .365 to .415

95% confidence interval (approximate):

.39 ± **2** (.0153) or .39 ± .031 or .359 to .421

99% confidence interval

.39 ± **2.58** (.0153) or .39 ± .039 or .351 to .429

Interpreting the confidence interval:

- We are ***90% confident*** that the proportion of *all* adults in the US who believe in evolution is between **.365 and .415**.
- We are ***95% confident*** that the proportion of *all* adults in the US who believe in evolution is between **.359 and .421.**
- We are ***99% confident*** that the proportion of *all* adults in the US who believe in evolution is between **.351 and .429**.

Interpreting the confidence level of 95%:

The interval .359 to .421 ***may or may not*** capture the true proportion of adult Americans who believe in evolution

But, ***in the long run*** this ***procedure*** will produce intervals that capture the unknown population values about 95% of the time. So, we are 95% *confident* that it worked this time.

Notes about interval width

- Lower confidence <=> more narrow interval
- Larger *n* (sample size) <=> more narrow interval, because *n* is in the *denominator* of the standard error.
- So, if you want a more narrow interval you can either *reduce* your confidence, or *increase* your sample size.

- Applet to demonstrate confidence interval concepts

http://www.rossmanchance.com/applets/NewConfsim/Confsim.html

- Note that on average, about 19 out of 20 of all 95% confidence intervals should cover the true population value.

Reconciling with Chapter 5 formula for 95% confidence interval

Sample estimate ± Margin of error

where (conservative) margin of error was $\frac{1}{\sqrt{n}}$

Now, "margin of error" is $2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

These are the *same* when $\hat{p} = .5$. The new margin of error is *smaller* for *any* other value of $\hat{p}$ so we say the old version is *conservative.* It will give a *wider* interval.

Comparing three versions (when sample proportion is close to .5)

For the evolution example, $n = 1018$, $\hat{p} = .39$

- *Conservative* margin of error = .0313 ≈ .03
- *Approximate* margin of error using z* = 2
  2 × .0153 = .0306 ≈ .03
- *Exact* margin of error using z* = 1.96
  1.96 × .0153 = .029988 ≈ .03

All very close to .03, and it really doesn't make much difference which one we use!

Compare methods when sample proportion is close to 0 or 1

Marist Poll in Oct 2009 asked "How often do you text while driving?" $n = 1026$

Nine percent answered "Often" or "sometimes" so

$$\hat{p} = .09 \qquad s.e.(\hat{p}) = \sqrt{\frac{.09(.91)}{1026}} = .009$$

- *Conservative* margin of error = .0312
- *Approximate* margin of error = 2 × .009 = .018.

This time, they are quite different!

The conservative one is too conservative, it's double the approximate one! Interval is too wide.

Comparing margin of error

- Conservative margin of error $\frac{1}{\sqrt{n}}$ will be okay for sample proportions near 0.5.
- For sample proportions far from 0.5, closer to 0 or 1, don't use the conservative margin of error. Resulting interval is wider than needed.
- Note that using a multiplier of 2 is called the *approximate* margin of error; the *exact* one uses multiplier of 1.96. It will rarely matter if we use 2 instead of 1.96.

Factors that Determine Margin of Error

1. ***The sample size, n.***
   When sample size *increases*, margin of error *decreases*.
2. ***The sample proportion,*** $\hat{p}$.
   If the proportion is close to either 1 or 0 most individuals have the same trait or opinion, so there is little natural variability and the margin of error is smaller than if the proportion is near 0.5.
3. ***The "multiplier"* 2 or 1.96.**
   Connected to the "95%" aspect of the margin of error. Usually the term "margin of error" is used only when the confidence level is 95%.

Summary of the Approximate 95% CI for a Proportion

$$\hat{p} \pm 2 \text{ standard errors}$$

The standard error is $s.e.(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

***Interpretation**:* For about 95% of all randomly selected samples from the population, the confidence interval computed in this manner captures the population proportion.

***Necessary Conditions**:* $n\hat{p}$ and $n(1-\hat{p})$ are both at least 10, and the sample is randomly selected (or representative).

Finding the Formula for a 95% CI for a Proportion – use Empirical Rule:

For 95% of all samples, $\hat{p}$ is within 2 st.dev. of p

For 95% of all samples:

-2 standard deviations $< \hat{p} - p <$ 2 standard deviations

Don't know true standard deviation, so use standard error.
For approximately 95% of all samples,

-2 standard errors $< \hat{p} - p <$ 2 standard errors

which implies that for approximately 95% of all samples,
$\hat{p}$ – 2 standard errors $< p <$ $\hat{p}$ + 2 standard errors

This is the approximate 95% confidence interval formula.

Same holds for *any* confidence level; replace 2 with z*

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where:

- $\hat{p}$ is the sample proportion
- z^* denotes the multiplier.
- $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ is the standard error of $\hat{p}$.

Example 10.3 *Intelligent Life Elsewhere?*

Poll: Random sample of 935 Americans
Do you think there is intelligent life on other planets?

Results: 60% of the sample said "yes", $\hat{p} = .60$

$$s.e.(\hat{p}) = \sqrt{\frac{.6(1-.6)}{935}} = .016$$

90% Confidence Interval: .60 ± 1.65(.016), or .60 ± .026
.574 to .626 or 57.4% to 62.6%

98% Confidence Interval: .60 ± 2.33(.016), or .60 ± .037
.563 to .637 or 56.3% to 63.7%

Note: entire interval is above 50% => high confidence that a majority believe there is intelligent life.

Confidence intervals and "plausible" values

- Remember that a confidence interval is an *interval estimate* for a population parameter.
- Therefore, any value that is covered by the confidence interval is a *plausible value* for the parameter.
- Values *not* covered by the interval are still possible, but not very likely (depending on the confidence level).

Example of plausible values

- 98% Confidence interval for proportion (or percent) who believe intelligent life exists elsewhere is:

  .563 to .637 or 56.3% to 63.7%

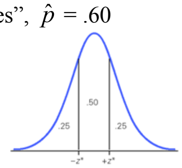
- Therefore, 56.3% is a *plausible value* for the population percent, but 50% is not very likely to be the population percent.
- Entire interval is above 50% => high confidence that a *majority* believe there is intelligent life.

New multiplier: To illustrate, let's do a confidence level of 50%

Poll: Random sample of 935 Americans
"Do you think there is intelligent life on other planets?"

Results: 60% of the sample said "yes", $\hat{p} = .60$

We want a **50% confidence interval**.
Want area between $-z^*$ and z^* to be .50, so the area to the *left* of z^* is .75.
From Table A.1 we have $z^* \approx .67$. (See next page for Table A.1)



50% Confidence Interval: .60 ± .67(.016), or .60 ± .011
.589 to .611 or 58.9% to 61.1%

Note: Lower confidence *level* results in a narrower *interval*.

Finding z* Multiplier for 50% confidence interval:

Relevant part of Table A.1. Want z* with area .7500 below it.
Closest is .7486, with z* = 0.67. Next is .7517, with z* = 0.68.
Could use average, 0.675, but use z* = 0.67 (close enough).

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133

Remember conditions for using the formula:

1. Sample is **randomly selected** from the population or at least is representative.
   Note: Available data can be used to make inferences about a much larger group *if the data can be considered to be representative with regard to the question(s) of interest.*
2. Normal curve approximation to the distribution of possible sample proportions assumes a **"large" sample size**. Both $n\hat{p}$ and $n(1-\hat{p})$ should be at least 10.

In Summary: Confidence Interval for a Population Proportion *p* (see page 393)

Exact CI for *p*, any confidence level: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Approximate 95% CI for *p*: $\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Conservative 95% CI for *p*: $\hat{p} \pm \frac{1}{\sqrt{n}}$

Section 10.3: Comparing two population proportions

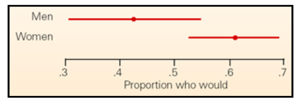
- Independent samples of size n_1 and n_2
- Use the two *sample* proportions as data.
- Could compute separate confidence intervals for the two population proportions and see if they overlap.
- Better to find a confidence interval for the *difference* in the two population proportions.

Case Study 10.3 *Comparing proportions*

Would you date someone with a great personality even though you did ***not*** find them attractive?

Women: .611 (61.1%) of 131 answered "yes."
95% confidence interval is .527 to .694.

Men: .426 (42.6%) of 61 answered "yes."
95% confidence interval is .302 to .55.



Conclusions:
- ***Higher proportion*** of ***women*** would say yes. CIs slightly overlap
- Women CI ***narrower*** than men CI due to larger sample size

More accurate to compare the two proportions by finding a CI for the difference

C.I. for difference in two population proportions:

Sample estimate ± multiplier × standard error

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Case Study 10.3 *Comparing proportions*

Would you date someone with a great personality even though you did ***not*** find them attractive?

Women: .611 of 131 answered "yes."
95% confidence interval is .527 to .694.

Men: .426 of 61 answered "yes."
95% confidence interval is .302 to .55.

Confidence interval for the difference in *population proportions* of women and men who would say yes.

$$(.611 - .426) \pm z^* \sqrt{\frac{.611(1-.611)}{131} + \frac{.426(1-.426)}{61}}$$

Case Study 10.3, continued

- A 95% confidence interval for the difference is .035 to .334 or 3.5% to 33.4%.
- We are 95% confident that the *population* proportions of men and women who would date someone they didn't find attractive *differ* by between .035 and .334, with a lower proportion for men than for women.
- We can conclude that the two *population* proportions *differ* because 0 is not in the interval.

Section 10.4: Using confidence intervals to guide decisions

- A value *not* in a confidence interval can be rejected as a likely value for the population parameter.
- When a confidence interval for $p_1 - p_2$ does not cover 0 it is reasonable to conclude that the two population proportions differ.
- When confidence intervals for p_1 and p_2 do not overlap it is reasonable to conclude they differ, but if they do overlap, no conclusion can be made. In that case, find a confidence interval for the difference.

Summary Tables

Sampling distributions, end of Chapter 9, p. 353.

Confidence intervals, end of Chapter 11, p. 439

These include:

- General formula for all 5 situations
- Specifics for each situation, including:
  - Parameter
  - Statistic
  - Relevant choices of these: standard deviation, standard error, standardized statistic, multiplier.

From the Midterm 2 review sheet for Chapter 10 - you should know these now

1. Understand how to interpret the *confidence level*
2. Understand how to interpret a *confidence interval*
3. Understand how the sampling distribution for $\hat{p}$ leads to the confidence interval formula (pg. 380-381)
4. Know how to compute a confidence interval for one proportion, including conditions needed.
5. Know how to compute a confidence interval for the difference in two proportions, including conditions needed.
6. Understand how to find the multiplier for a specified confidence level.
7. Understand how margin of error from Chapter 5 relates to the 95% confidence interval formula in Chapter 10
8. Know the general format for a confidence interval for the 5 situations defined in Chapter 9 (see summary on page 439).

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