**Announcements:**

- New use of clickers: to test for understanding. I will give more clicker questions, and randomly choose five to count for credit each week.
- Discussion this week is not for credit – question/answer, practice problems.
- Chapter 9 practice problems now on website
- Today: Sections 9.1 to 9.4
- **Homework (due Wed, Feb 27):**

  Chapter 9: #22, 26, 40, 144

Chapter 9

# Understanding Sampling Distributions: Statistics as Random Variables

# Recall: Sample Statistics and Population Parameters

A **statistic** is a numerical value computed from a sample. Its value may differ for different samples.
*e.g. sample mean* $\overline{x}$, *sample standard deviation s, and sample proportion* $\hat{p}$.

A **parameter** is a numerical value associated with a population. Considered fixed and unchanging.
*e.g. population mean* $\mu$, *population standard deviation* $\sigma$, *and population proportion p.*

# Statistical Inference

**Statistical Inference:** making conclusions about population parameters on basis of sample statistics.

***See picture on board in lecture***

Two most common procedures:

**Confidence interval:** an interval of values that the researcher is fairly sure will cover the true, unknown value of the population parameter.

**Hypothesis test:** uses sample data to attempt to reject (or not) a hypothesis about the population.

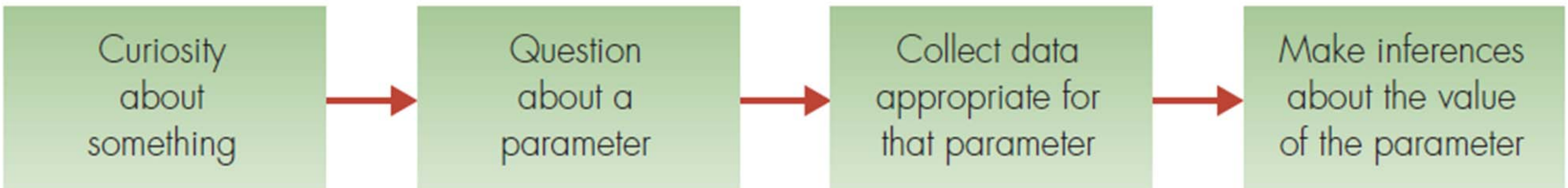
# The Plan for the Rest of the Quarter

- We will cover statistical inference for five situations; each one has a parameter of interest.

- For each of the five situations we will identify:
  - The parameter of interest
  - A sample statistic to estimate the parameter

- For each of the five situations we will learn about:
  - The *sampling distribution* for the sample statistic
  - How to construct a *confidence interval* for the parameter
  - How to *test hypotheses* about the parameter

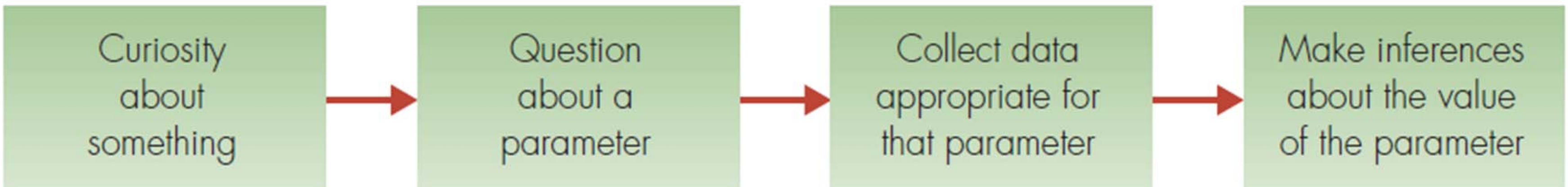
# How (Statistical) Science Works




**The Big Five Parameters (See Table on page 317)**

| Parameter Name and Description | Symbol for the Population Parameter | Symbol for the Sample Statistic |
|---|---|---|
| *For Categorical Variables* | | |
| One population proportion (or probability) | $p$ | $\hat{p}$ |
| Difference in two population proportions | $p_1 - p_2$ | $\hat{p}_1 - \hat{p}_2$ |
| *For Quantitative Variables* | | |
| One population mean | $\mu$ | $\overline{x}$ |
| Population mean of paired differences (dependent) | $\mu_d$ | $\overline{d}$ |
| Difference in two population means (independent) | $\mu_1 - \mu_2$ | $\overline{x}_1 - \overline{x}_2$ |

# How (Statistical) Science Works




## **Example**:

**Curiosity**: Do a majority of voters favor stricter gun control?

**Parameter:** $p$ = proportion of *population* of registered voters who do favor stricter gun control. What is the value of $p$?

**Collect data:** Ask a random sample of registered voters. Sample statistic = proportion of the *sample* who favor stricter gun control

**Make inferences:** Use the *sample proportion* to compute a 95% confidence interval for the *population proportion* (parameter)

# Structure for the rest of the Quarter

| Parameter name and description | Sampling Distribution | Confidence Interval | Hypothesis Test |
|---|---|---|---|
| ***For Categorical Variables:*** | Chapter 9 | Chapter 10 | Chapter 12 |
| One population proportion or binomial probability | Today & Fri. | Mon, Feb 25 | Mon, Mar 4 |
| Difference in two population proportions | Friday | Mon, Feb 25 | Wed, Mar 6 |
| ***For Quantitative Variables:*** | Chapter 9 | Chapter 11 | Chapter 13 |
| One population mean | Fri, March 8 | Mon, Mar 11 | Wed, Mar 12 |
| Population mean of paired differences (paired data) | Fri, March 8 | Mon, Mar 11 | Wed, Mar 12 |
| Difference in two population means (independent samples) | Fri, March 8 | Mon, Mar 11 | Wed, Mar 12 |

# For Situation 4, we need "Paired Data"

***Paired data*** (or ***paired samples***): when pairs of variables are collected. Only interested in population (and sample) of ***differences***, and not in the original data.

**Here are ways this can happen:**

- **Each person (unit) measured twice**. Two measurements of same characteristic or trait made under different conditions.
- Similar **individuals are paired** prior to an experiment. Each member of a pair receives a different treatment.
  Same response variable is measured for all individuals.
- **Two different variables** are measured for each individual. **Interested in amount of difference** between two variables.

# Situations 2 and 5: Independent Samples

Two samples are called **independent samples** when the measurements in one sample are not related to the measurements in the other sample.

**Here are ways this can happen:**

- **Random samples** taken separately from two populations and same response variable is recorded.
- **One random sample** taken and a variable recorded, but units are **categorized** to form two populations.
- Participants **randomly assigned** to one of two treatment conditions, and same response variable is recorded.

# Familiar Examples Translated into Questions about Parameters

## Situation 1.

***Estimate/test the proportion*** *falling into a category of a categorical variable* OR *a* ***binomial success probability***

***Example research questions:***
What proportion of American adults believe there is extraterrestrial life? In what proportion of British marriages is the wife taller than her husband?

***Population parameter:*** $p$ = proportion in the <u>population</u> falling into that category.

***Sample estimate:*** $\hat{p}$ = proportion in the <u>sample</u> falling into that category.

# Data Example for Situation 1

**Question**: What *proportion* (p) of all households with TVs watched the Super Bowl? *Get a confidence interval for p*. (Hypothesis test of no use in this example – nothing of interest to test!)

**Population parameter**:

*p* = proportion of the *population* of all US households with TVs that watched it.

**Sample statistic**:

Nielsen ratings, random *sample* of n = 25,000 households.

X = *number* in *sample* who watched the show = 11,510.

$$\hat{p} = \frac{X}{n} = \frac{11,510}{25,000}$$ = 0.46 = *proportion* of *sample* who watched. This is called "p-hat."

# Familiar Examples

## Situation 2.

*Compare two population proportions using independent samples of size $n_1$ and $n_2$.* ***Estimate*** *difference;* ***test*** *if 0.*

***Example research questions:***

- How much difference is there between the proportions that would quit smoking if taking the antidepressant buproprion (Zyban) versus if wearing a nicotine patch?
- How much difference is there between men who snore and men who don't snore with regard to the proportion who have heart disease?

***Population parameter:*** $\mathbf{p_1 - p_2}$ = difference between the two population proportions.

***Sample estimate:*** $\hat{p}_1 - \hat{p}_2$ = difference between the two sample proportions.

# Data Example for Situation 2

**Question**: Is the population proportion favoring stricter gun control laws the same now as it was in April 2012?

- Get a *confidence interval* for the population difference.
- *Test* to see if it is statistically significantly different from 0.

**Population parameter**:

$p_1 - p_2$ = *population* difference in proportions where $p_1$ is the proportion now, and $p_2$ was the proportion in April 2012

**Sample statistic**: Based on CBS News Poll, $n_1$ and $n_2$ each about 1,150; 53% favor now and only 39% did in April 2012.

Difference in *sample* proportions is $\hat{p}_1 - \hat{p}_2 = .53 - .39 = +.14$

This is read as "p-hat-one minus p-hat-two"

Note that the parameter and statistic can range from −1 to +1.

# Familiar Examples

## Situation 3.

***Estimate the population mean*** *of a quantitative variable.*
***Hypothesis test*** *if there is a logical null hypothesis value.*

***Example research questions:***

- What is the mean time that college students watch TV per day?
- What is the mean pulse rate of women?

***Population parameter:*** $\mu$ = population mean for the variable

***Sample estimate:*** $\bar{x}$ = sample mean for the variable

# Data Example for Situation 3

**Question**: Airlines need to know the average weight of checked luggage, for fuel calculations. Estimate with a confidence interval, and test to see if it exceeds airplane capacity.

**Population parameter**:
$\mu$ = mean weight of the luggage for the *population* of all passengers who check luggage.

**Sample statistic**: Study measured $n$ = 22,353 bags;
$\bar{x}$ = 36.7 pounds (st. dev. = 12.8)

Source:
http://www.easa.europa.eu/rulemaking/docs/research/Weight%20Survey%20R20090095%20Final.pdf

# Familiar Examples

## Situation 4.

***Estimate the population mean** of paired differences for quantitative variables, and **test** null hypothesis that it is 0.*

***Example research questions:***

- What is the mean difference in weights for freshmen at the beginning and end of the first quarter or semester?
- What is the mean difference in age between husbands and wives in Britain?

***Population parameter:*** $\mu_d$ = population mean of differences

***Sample estimate:*** $\bar{d}$ = mean of differences for paired sample

# Data Example for Situation 4

**Question**: How much different on average would IQ be after listening to Mozart compared to after sitting in silence?

- Find *confidence interval* for population mean difference $\mu_d$
- *Test null hypothesis* that $\mu_d = 0$.

**Population parameter**:

$\mu_d$ = *population* mean for the difference in IQ *if* everyone in the population were to listen to Mozart versus silence.

**Sample statistic**: For the experiment done with $n = 36$ UCI students, the mean difference for the sample was 9 IQ points.

$\bar{d} = 9$, read "d-bar"

# Familiar Examples

## Situation 5.

***Estimate** the difference between two population means for quantitative variables and **test** if the difference is 0.*

***Example research questions:***

- How much difference is there in mean weight loss for those who diet compared to those who exercise to lose weight?
- How much difference is there between the mean foot lengths of men and women?

***Population parameter:*** $\mu_1 - \mu_2$ = difference between the two population means.

***Sample estimate:*** $\bar{x}_1 - \bar{x}_2$ = difference between the sample means, based on independent samples of size $n_1$ and $n_2$

# Data Example for Situation 5

**Question**: Is there a difference in mean IQ of 4-year-old children for the population of mothers who smoked during pregnancy and the population who did not? If so, how much?

- Find *confidence interval* for population difference $\mu_1 - \mu_2$
- *Test null hypothesis* that $\mu_1 - \mu_2 = 0$.

**Population parameter**:

$\mu_1 - \mu_2$ = difference in the mean IQs for the two *populations*

**Sample statistic**: Based on a study done at Cornell, the difference in means for two *samples* was 9 IQ points.

$\bar{x}_1 - \bar{x}_2 = 9$, Read as "x-bar-one minus x-bar-two."

# Sampling Distributions: Some Background

Notes about statistics and parameters:

- Assuming the sample is representative of the population, the *sample statistic* should represent the *population parameter* fairly well. (Better for larger samples.)
- But… the sample statistic will have some error associated with it, i.e. it won't necessarily *exactly* equal the population parameter. Recall the "margin of error" from Chapter 5!
- Suppose repeated samples are taken from the same population and the sample statistic is computed each time. These sample statistics will *vary,* but in a *predictable way.* The possible values will have a *distribution*. It is called the **sampling distribution** for the statistic.

# Rationale And Definitions For Sampling Distributions

**Claim**: A *statistic* is a special case of a *random variable*.

**Rationale**: When a sample is taken from a population the resulting numbers are the outcome of a *random circumstance*. That's the definition of a random variable.

*Super Bowl example:*

- A random circumstance is taking a random sample of 25,000 households with TVs.
- The resulting number (statistic) is the *proportion of those households that watched the Super Bowl.* (*0.46, or 46%*)
- *A different sample would give a different proportion.*

# Rationale, Continued

Remember: a random variable is a number associated with the outcome of a random circumstance, which can change each time the random circumstance occurs.

*Example: For each different sample of 25,000 households that week, we could have had a different sample proportion (sample statistic) watching the Super Bowl.*

- Therefore, a sample statistic is a *random variable*.
- Therefore, a sample statistic has a pdf associated with it.
- The pdf of a sample statistic can be used to find the probability that the sample statistic will fall into specified intervals when a new sample is taken.

# Sampling Distribution Definition

## Statistics as Random Variables

Each new sample taken →
value of the sample statistic will change.

*The distribution of possible values of a statistic for repeated samples of the same size from a population is called the* **sampling distribution** of the statistic.

More formal definition: A sample statistic is a random variable. The probability density function (pdf) of a sample statistic is called the **sampling distribution** for that statistic.

# Sampling Distribution for a Sample Proportion

Let $p$ = population proportion of interest
or binomial probability of success.

Let $\hat{p}$ = sample proportion or proportion of successes.

If numerous random samples or repetitions of the same size $n$ are taken, the distribution of possible values of $\hat{p}$ is **approximately** a **normal** curve distribution with

- **Mean** = $p$
- **Standard deviation** = s.d.( $\hat{p}$ ) = $\sqrt{\dfrac{p(1-p)}{n}}$

This approximate distribution is **sampling distribution of** $\hat{p}$ **.**

# Conditions needed for the sampling distribution to be approx. normal

The sampling distribution for $\hat{p}$ can be applied in ***two situations***:

**Situation 1**: A random sample is taken from a population.

**Situation 2**: A binomial experiment is repeated numerous times.

In each situation, ***three conditions*** must be met:

**1:** *The Physical Situation*
There is an actual population or repeatable situation.

**2:** *Data Collection*
A random sample is obtained or the situation repeated many times.

**3:** *The Size of the Sample or Number of Trials*
The size of the sample or number of repetitions is relatively large, *np* and *np(1-p)* must be at least 5 and preferable at least 10.

# Motivation via a Familiar Example

Suppose 48% (p = 0.48) of a *population* supports a candidate.

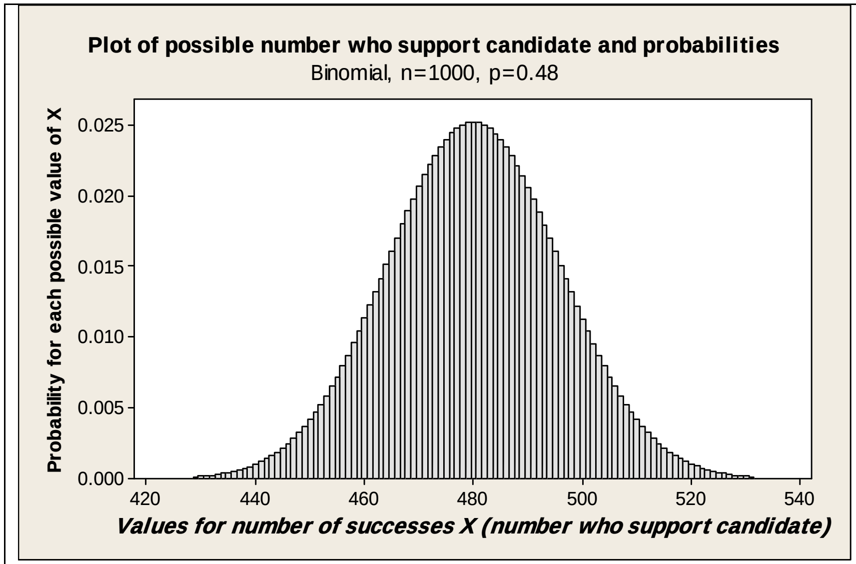
- In a poll of 1000 randomly selected people, what do we expect to get for the *sample proportion* who support the candidate in the poll?
- In the last few lectures, we looked at the pdf for X = the *number* who support the candidate. X was binomial, and also X was approx. normal with mean = 480 and s.d. = 15.8.
- Now let's look at the pdf for the *proportion* who do.

  $\hat{p} = \frac{X}{n} =$ where X is a binomial random variable.

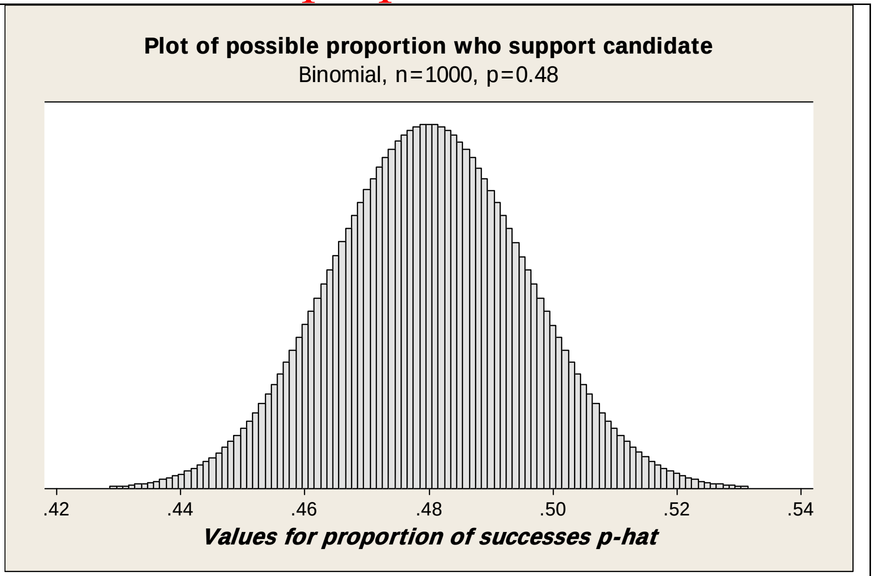
- We have seen picture of possible values of X.

  Divide all values by *n* to get picture for possible $\hat{p}$.

PDF for x = *number* of successes




PDF for $\hat{p}$ = *proportion* of successes




What's different and what's the same about these two pictures?

Everything is the same except the values on the x-axis!
On the left, values are *numbers* 0, 1, 2, to 1000
On the right, values are *proportions* 0, 1/1000, 2/1000, to 1.

# Recall: Normal approximation for the binomial

For a *binomial* random variable X with parameters $n$ and $p$ with $np$ and $n(1-p)$ at least 5 each:

- X is *approximately* a *normal* random variable with:

  mean $\mu = np$  standard deviation $\sigma = \sqrt{np(1-p)}$

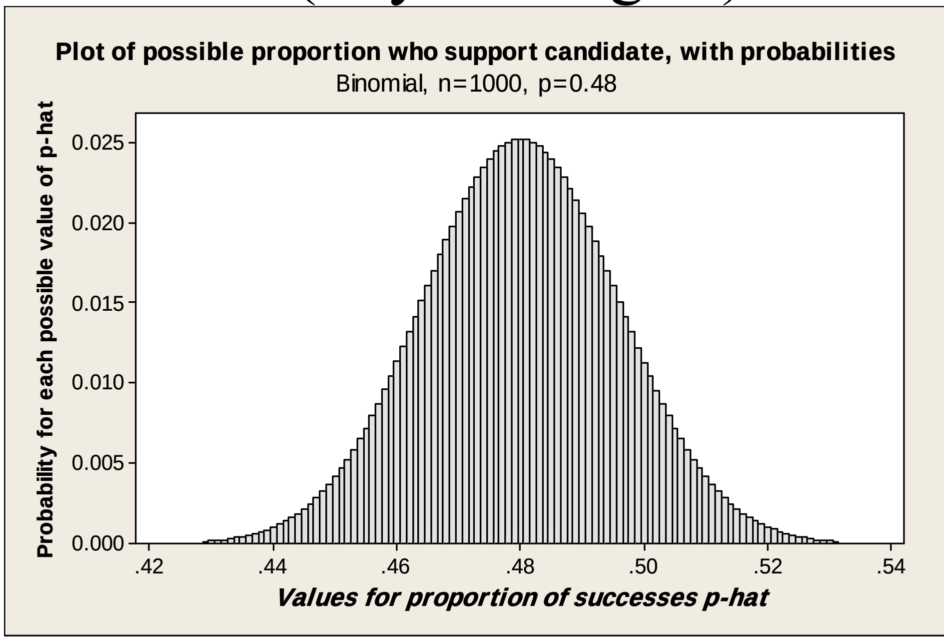
NOW: Divide everything by $n$ to get similar result for $\hat{p} = \frac{X}{n}$

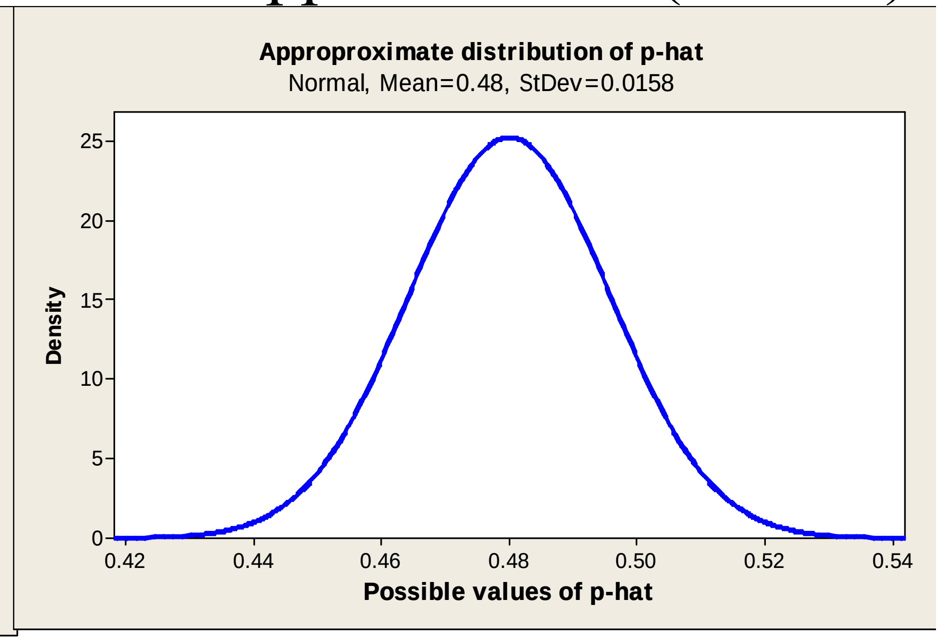
- $\hat{p}$ is *approximately* a *normal* random variable with:

  mean $\mu = p$  standard deviation $\sigma = \sqrt{\frac{p(1-p)}{n}}$

So, we can find probabilities that $\hat{p}$ will be in specific intervals if we know $n$ and $p$.

Actual (tiny rectangles) Normal approximation (smooth)




For example, to find the probability that $\hat{p}$ is at least 0.50:
Could add up areas of rectangles from .501, .502, …, 1000
but that would be too much work! P( $\hat{p}$ > 0.50)

$$\approx P(z > \frac{0.50 - .48}{.0158}) = P(z > 1.267) = .103$$

# Sampling Distribution for a Sample Proportion, Revisited

Let $p$ = population proportion of interest
or binomial probability of success.

Let $\hat{p}$ = sample proportion or proportion of successes.

If numerous random samples or repetitions of the same size $n$ are taken, the distribution of possible values of $\hat{p}$ is **approximately** a **normal** curve distribution with

- **Mean** = $p$
- **Standard deviation** = s.d.( $\hat{p}$ ) = $\sqrt{\dfrac{p(1-p)}{n}}$

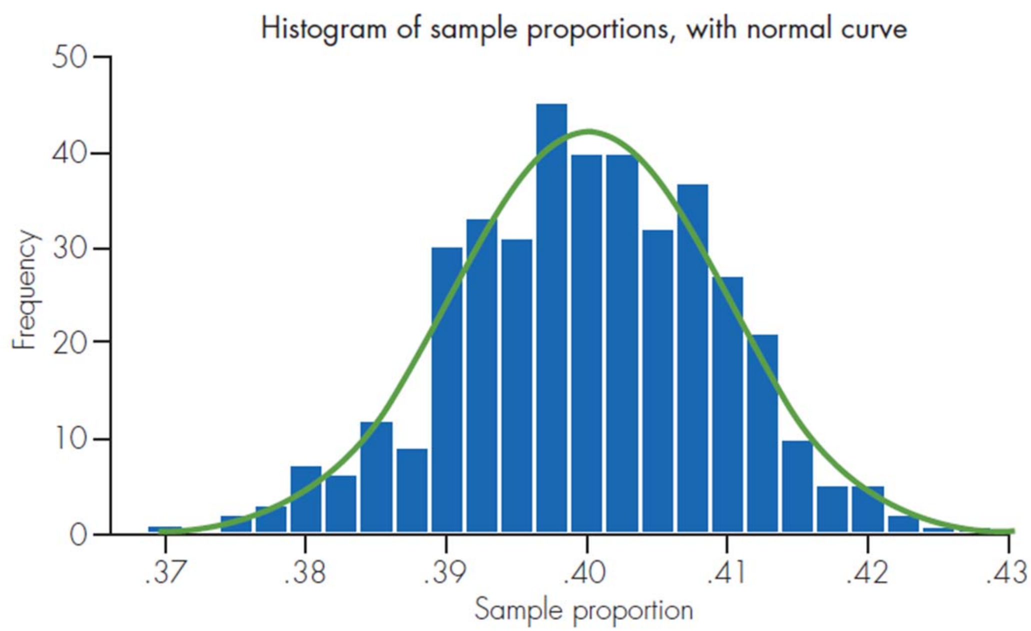
This approximate distribution is **sampling distribution of** $\hat{p}$ **.**

# Example 9.4 *Possible Sample Proportions Favoring a Candidate*

Suppose 40% all voters favor Candidate C. Pollsters take a sample of $n$ = 2400 voters. Rule states the sample proportion who favor X will have approximately a normal distribution with

$$\text{mean} = p = 0.4 \text{ and s.d.}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4(1-0.4)}{2400}} = 0.01$$

Histogram at right shows sample proportions resulting from simulating this situation 400 times.




Empirical Rule: Expect
68% from .39 to .41
95% from .38 to .42
99.7% from .37 to .43

# A Final Dilemma and What to Do

**In practice,** we don't know the true population proportion $p$, so we cannot compute the **standard deviation** of $\hat{p}$ ,

$$\text{s.d.}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}} \ .$$

Replacing $p$ with $\hat{p}$ in the standard deviation expression gives us an estimate that is called the **standard error of** $\hat{p}$ .

$$\text{s.e.}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \ .$$

The *standard error* is an excellent approximation for the *standard deviation*. We will use it to find *confidence intervals,* but will not need it for sampling distribution or hypothesis tests because we assume a specific value for $p$ in those cases.

# CI Estimate of the Population Proportion from a <u>Single</u> Sample Proportion

CBS Poll taken this month asked "*In general, do you think gun control laws should be made more strict, less strict, or kept as they are now?*

Poll based on $n$ = 1,148 adults, 53% said "more strict."

**Population parameter** is $p$ = proportion of *population* that thinks they should be more strict.

**Sample statistic** is $\hat{p} = .53$

$$\text{s.e.}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.53(.47)}{1148}} = .015$$

If $\hat{p} = 0.53$ and $n = 1148$, then the standard error is 0.015. Sample $\hat{p} = .53$ is 95% certain to be within 2 standard errors of population $p$, so $p$ is probably between .50 and .56.

# Preparing for the Rest of Chapter 9

For all 5 situations we are considering, the sampling distribution of the sample statistic:

- Is approximately normal
- Has mean = the corresponding population parameter
- Has standard deviation that involves the population parameter(s) and thus can’t be known without it (them)
- Has standard error that doesn’t involve the population parameters and is used to estimate the standard deviation.
- Has standard deviation (and standard error) that get smaller as the sample size(s) n get larger.

Summary table on page 353 will help you with these!