Announcements:

- If you haven't taken the survey yet, I encourage you to do it by Sunday!
- Chapter 7 practice problems on website.
- Discussion this week and next *for* credit.

Homework (due Mon, Feb 11th):

Chapter 7:

#44, 64 (counts double)

Review: Independent and Dependent Events

- Two events are **independent** of each other if knowing that one will occur (or has occurred) *does not change* the probability that the other occurs.
- Two events are **dependent** if knowing that one will occur (or has occurred) *changes* the probability that the other occurs.

The definitions can apply *either* …
to events *within the same random circumstance* or
to events *from two separate random circumstances.*

HOW TO DETERMINE IF TWO EVENTS ARE INDEPENDENT

1. Physical assumption

   Example: Lottery draws on different days don't affect each other.

2. See if P(B|A) = P(B). If so, A and B are independent events, otherwise they are not.

   Example: Suppose data showed that smokers and non-smokers are equally likely to get the flu.

   P(flu | smoker) = P(flu) so they are independent.

3. Events A and B are independent if and only if P(A and B) = P(A)P(B). If you know these probabilities, you can check to see if this holds.

Testing independence of myopia and lighting

TABLE 2.3 Nighttime Lighting in Infancy and Eyesight

Slept with:	No Myopia	Myopia	High Myopia	Total
Darkness	155 (90%)	15 (9%)	2 (1%)	172
Nightlight	153 (66%)	72 (31%)	7 (3%)	232
Full Light	34 (45%)	36 (48%)	5 (7%)	75
Total	342 (71%)	123 (26%)	14 (3%)	479

A = child slept in darkness as infant

P(A) = 172/479 = .36

B = child did not develop myopia

P(B) = 342/479 = .71

Does P(A and B) = P(A)P(B) = (.36)(.71) = .2556?

P(A and B) = P(darkness and no myopia) = 155/479 = .3236

NO, P(A and B) ≠ P(A)P(B), so A and B are *not* independent.

Infant sleeptime lighting and myopia are not independent.

7.4 Basic Rules for Finding Probabilities

Probability an Event Does Not Occur

Rule 1 (for "not the event"): $P(A^C) = 1 - P(A)$

Example: In the US, blood type percents* are:

O: 46%, **A**: 39%, **B**: 11%, **AB**: 4%

Suppose we randomly select someone.

P(Blood type O) = 0.46

so *P*(Blood type A, B, or AB) = 1 – 0.46 = 0.54.

*Source: www.bloodbook.com

Probability That Either One or Both of Two Events Happen

Rule 2 (addition rule for "either/or/both"):

Rule 2a (general):

P(A *or* B) = P(A) + P(B) – P(A and B)

Rule 2b (for mutually exclusive events):

If A and B are *mutually exclusive* events,

P(A or B) = P(A) + P(B)

Illustrate with Venn diagram in class.

Example 7.14 *Roommate Compatibility*

Brett will be assigned a roommate from 1000 other males. Will he get one who snores? Parties? Both?

		Snores?		
		Yes	No	Total
Likes to Party?	Yes	150	100	250
	No	200	550	750
		350	650	1000

A = likes to party   *P*(A) = **250**/1000 = 0.25

B = snores   *P*(B) = **350**/1000 = 0.35

P(A and B) = **150**/1000 = .15

Probability that Brett will be assigned a roommate who either likes to party *or* snores, *or* both is: ***P*(A *or* B)**

= ***P*(A)** + ***P*(B)** – ***P*(A and B)** = 0.25 + 0.35 – 0.15 = 0.45

So probability that his roommate is acceptable is 1 – 0.45 = 0.55

Probability That Two or More Events Occur Together

Rule 3 (multiplication rule for "and"):

Rule 3a (general):

$P(\text{A } \textit{and} \text{ B}) = P(\text{A})P(\text{B}|\text{A})$

Rule 3b (for independent events):

If A and B are *independent* events,

$P(\text{A } \textit{and} \text{ B}) = P(\text{A})P(\text{B})$

Extension of Rule 3b (> 2 independent events):

For several independent events,

$P(\text{A}_1 \textit{ and } \text{A}_2 \textit{ and } \ldots \textit{ and } \text{A}_n) = P(\text{A}_1)P(\text{A}_2)\ldots P(\text{A}_n)$

Example with *independent* events (Rule 3b)

The probability of a birth being a boy is .512. Suppose a woman has 2 children (not twins).

A = first child is a boy

B = second child is a boy

We assume these are independent events.

P(**A** *and* **B**) = P(**A**)P(**B**) = (.512)(.512) = .2621

Probability of 2 girls is (.488)(.488) = .2381

From **Rule 2b**, probability of same sex = .5002

From **Rule 1**, probability of different sex = .4998

Example with *dependent* events (Rule 3a)

Randomly select a student who takes Stat 7 with me. (Similar to Exercise 7.95 and homework.)

A = student comes to class regularly; P(**A**) = 0.7

$\mathbf{A}^{\mathbf{C}}$ = student doesn't come regularly; P($\mathbf{A}^{\mathbf{C}}$) = 0.3

B = student gets *at least* a B in the course

$$P(\mathbf{B}|\mathbf{A}) = 0.8 \qquad P(\mathbf{B}|\mathbf{A}^{\mathbf{C}}) = 0.4$$

What is the probability that the student comes to class regularly *and* gets at least a B?

$P(\mathbf{A} \text{ and } \mathbf{B}) = P(\mathbf{A})P(\mathbf{B}|\mathbf{A}) = (.7)(.8) = .56$

Extension of Rule 3b:
Blood Types Again

A blood bank needs Type O blood. What is the probability that the next 3 donors (not related) all have Type O blood? We can assume independent.

Event A = 1st donor has Type O blood, $P(\text{A}) = .46$
Event B = 2nd donor has Type O blood, $P(\text{B}) = .46$
Event C = 3rd donor has Type O blood, $P(\text{C}) = .46$

P(Next 3 donors all have Type O blood)
= P(A *and* B *and* C)
$= (.46)(.46)(.46) = (.46)^3 = .097336$

Determining a Conditional Probability

Rule 4 (conditional probability):

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

EXAMPLE OF CONDITIONAL PROBABILITY - Revisited

TABLE 2.3 Nighttime Lighting in Infancy and Eyesight

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Total	342 (71%)	123 (26%)	14 (3%)	479

A = child slept in darkness as infant

P(A) = 172/479 = .36

B = child did not develop myopia

P(B) = 342/479 = .71

P(B|A) = P(no myopia | slept in dark) = 155/172 = .90

But this is

$$\frac{P(A\ and\ B)}{P(A)} = \frac{155/479}{172/479} = \frac{155}{172}$$

Mutually exclusive versus Independent

Students sometimes confuse the definitions of **independent** and **mutually exclusive** events.

- When two events are *mutually exclusive* and one happens, it *turns the probability of the other one to **0***.
- When two events are *independent* and one happens, it *leaves the probability of the other one alone*.

In Summary (see page 236) …

When Events Are:	$P(A \text{ or } B)$ is:	$P(A \text{ and } B)$ Is:	$P(A\|B)$ Is:
Mutually exclusive	$P(A) + P(B)$	0	0
Independent	$P(A) + P(B) - P(A)P(B)$	$P(A)P(B)$	$P(A)$
Any	$P(A) + P(B) - P(A \text{ and } B)$	$P(A)P(B\|A)$	$\frac{P(A \text{ and } B)}{P(B)}$

The most important parts to remember, because they are based on the definitions:

Mutually exclusive: $P(A|B) = 0$

Independent: $P(A|B) = P(A)$

7.5 Finding Complicated Probabilities: There are multiple ways to solve a problem

Example 7.21 *Winning the Daily 3 Lottery*
Event A = winning number is 956. **What is P(A)?**

Method 1: With physical assumption that all 1000 possibilities are equally likely, $P(A) = 1/1000$.

Method 2: Define three events,
B_1 = 1st digit is 9, B_2 = 2nd digit is 5, B_3 = 3rd digit is 6
Event A occurs if and only if all 3 of these events occur.
Note: $P(B_1) = P(B_2) = P(B_3) = 1/10$. Since these events are all *independent*, we have $P(A) = (1/10)^3 = 1/1000$.

Some Hints for Finding Probabilities

- ***P*(A and B):** Sometimes you can define the event in physical terms and know the probability or find it from a two-way table.

  Example: I could classify the class into male, female and also year in school. Then, for example, probability that a randomly selected student in the class is Male *and* sophomore is the proportion of the class in that cell of the table. Don't need separate P(A) and P(B).

- Check if probability of the ***complement*** is **easier** to find, then subtract it from 1 (applying Rule 1).

  Example: Probability of at least 1 boy in family of 3 kids =

  1 – Probability of all girls = 1 – $(.488)^3$ = 1 – .116 = .884

Finding Conditional Probability in Opposite Direction: Bayes Rule

Know *P*(B|A) but want *P*(A|B): Use Rule 3a to find $P(B) = P(A \text{ and } B) + P(A^C \text{ and } B)$, then use Rule 4.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B \mid A)P(A) + P(B \mid A^C)P(A^C)}$$

Two useful tools that are much easier than using this formula!

1. Hypothetical hundred thousand table

2. Tree diagram

Solve these probability questions

Example 1:

Medical testing for a rare disease

D = person has the disease, suppose:

$$P(D) = 1/1000 = .001, \; P(D^C) = .999$$

T = test for the disease is positive, suppose:

$$P(T \mid D) = .95, \text{ so } P(T^C \mid D) = .05$$

$$P(T \mid D^C) = .05, \text{ so } P(T^C \mid D^C) = .95$$

So the test is ***95% accurate*** whether person has the disease or not

Find **P(D | T)**

= Probability of disease, *given* the test is positive

Example 2: Probability of getting a B or better

Return to example of Statistics 7 grades

A = student comes to class regularly; **P(A)** = 0.7

B = student gets at least a B in the course

$$\mathbf{P(B|A)} = 0.8 \qquad \mathbf{P(B|A^C)} = 0.4$$

Question:

What is the overall probability of getting at least a B?

Useful Tool 1: Hypothetical Hundred Thousand Table

Table of hypothetical 100,000 people who get tested

1/1000 of them have disease = 100 people

Of those, 95% = **95** people test positive, so 5 test negative

999/1000 of them don't have the disease = 99,900 people

Of those, 95% = 94,905 people test *negative*, so 4995 positive

	Test positive	Test negative	Total
Disease	**95**	5	100
No disease	4995	94,905	99,900
Total	**5090**	94,910	100,000

Read from Table: P(Disease | Test positive) = **95**/**5090** = .019

Some definitions from Section 7.7 Probability of accurate medical test

Define the events:

D = person has the disease

D^C = person does not have the disease

T = test is positive

T^C = test is negative

Sensitivity of a test = P(T | D), i.e., correct outcome if person *has* the disease.

Specificity of a test = $P(T^C \mid D^C)$, i.e. correct outcome if person *does not have* the disease.

Useful Tool 2: Tree Diagrams
Show disease example in class

Step 1: Determine first random circumstance in time sequence, and create first set of branches for possible outcomes. Create one branch for each outcome, write probability on branch.

EX: P(D) = .001, P(no D) = .999

Step 2: Determine next random circumstance and append branches for possible outcomes to each branch in step 1. Write associated *conditional probabilities* on branches.

EX: P(T | D) = .95, P(T | no D) = .05

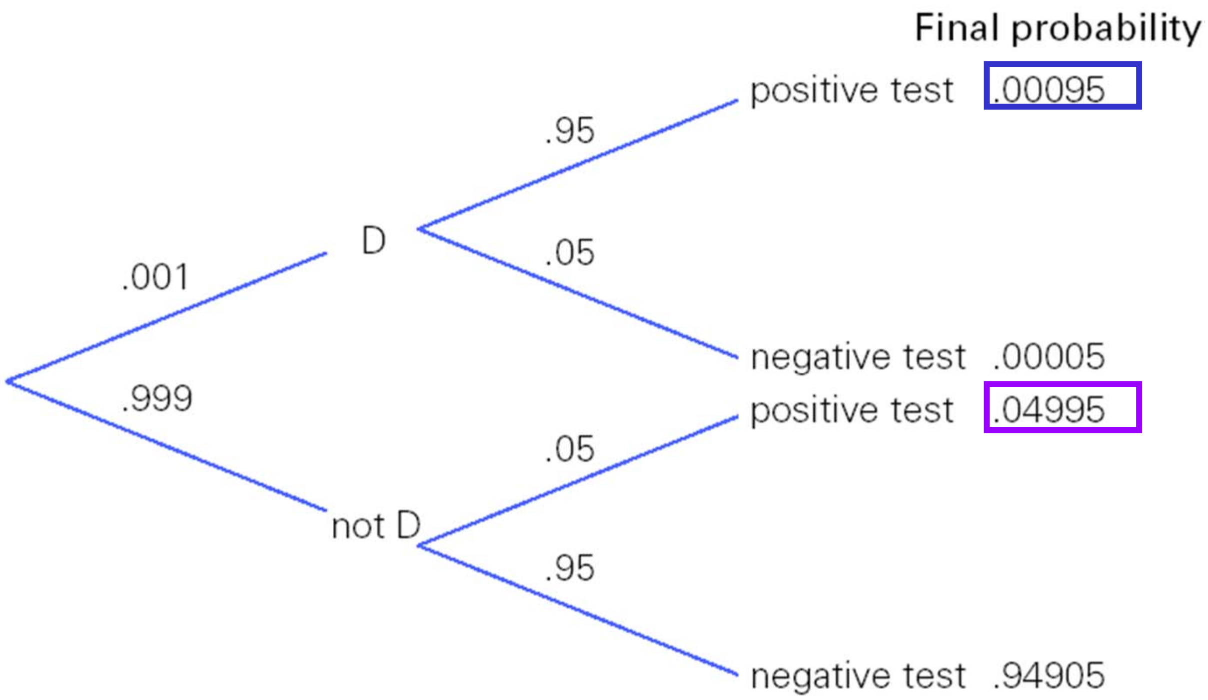
Tree Diagrams, continued

Step 3: Continue this process for as many steps as necessary. (Not needed in this 2-step example)

Step 4: To determine the probability of following any particular sequence of branches, multiply the probabilities on those branches. This is an application of Rule 3a.

Step 5: To determine the probability of any collection of sequences of branches, add the individual probabilities for those sequences, as found in step 4. This is an application of Rule 2b.

Disease probability



Sensitivity = specificity = 0.95

P(D *and* positive test) = (.001)(.95) = .00095.

P(test is positive) = .00095 + .04995 = .0509.

P(D | positive test) = .00095/.0509 = .019

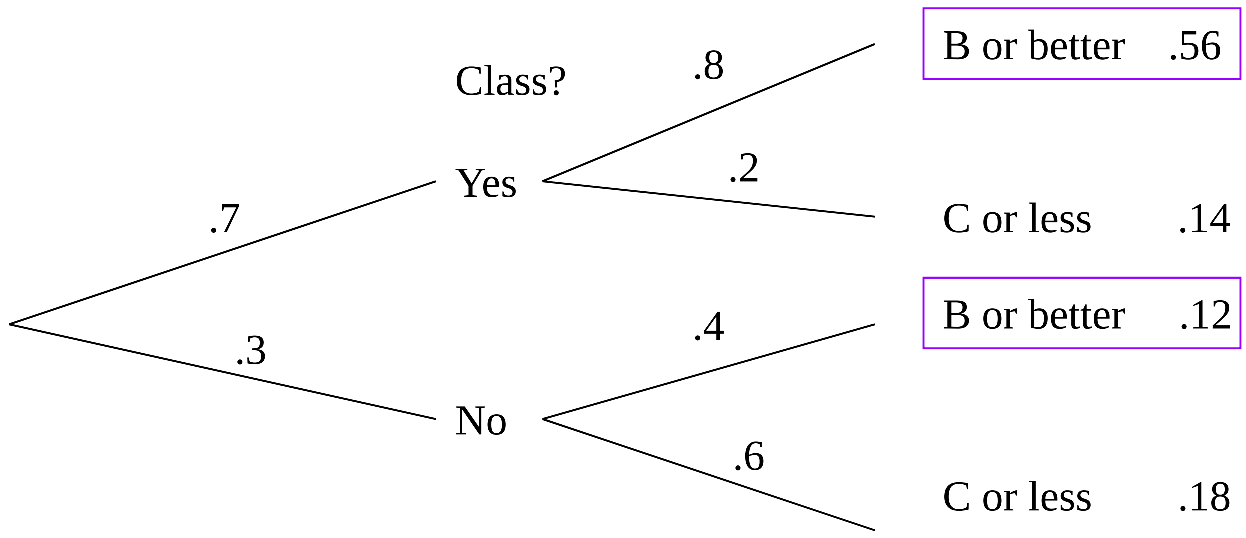
Example 2: Return to example of Stat 7 grades

A = student comes to class regularly; P(A) = 0.7, P(A^C) = 0.3

B = student gets at least a B in the course

P(B|A) = 0.8   **P(B|A^C)** = 0.4

Question: What is the overall probability of getting at least a B?



So, probability of B or better = .56 + .12 = .68 overall

More examples in class, if time. Otherwise, try on your own.

1. You drive on a certain freeway daily. Speed limit is 65. You drive over 65 all the time, but over 75 about 30% of the time.

P(ticket | over 75) = 1/50 = .02

P(ticket | 65 to 75) = 1/200 = .005.

What is the probability you get a ticket on a randomly selected day?

2. Suppose there is no relationship between two variables, e.g. listening to Mozart and increased IQ. Suppose 3 independent experiments are done, each using the 0.05 criterion for statistical significance.

What is the probability that *at least one* finds statistical significance just by chance?

Problem 1

Suppose you drive on a certain freeway daily. Speed limit is 65. You drive over 65 all the time, but over 75 about 30% of the time.

P(ticket | over 75) = 1/50 = .02

P(ticket | 65 to 75) = 1/200 = .005.

What is the probability you get a ticket on a randomly selected day?

Problem 2

Suppose there is no relationship between two variables, e.g. listening to Mozart and increased IQ. Suppose 3 independent experiments are done, each using the 0.05 criterion for statistical significance.

What is the probability that *at least one* experiment results in a statistically significant relationship just by chance?

Problem 2 Solution

Suppose there is no relationship between two variables, e.g. listening to Mozart and increased IQ. Suppose 3 independent experiments are done, each using the 0.05 criterion for statistical significance. What is the probability that *at least one* experiment results in a statistically significant relationship just by chance?

1 – P(no significant relationships) =

1 – (.95)(.95)(.95) = 1 – .857 = .143

Homework (due Mon, Feb 11th):

Chapter 7:

#44, 64 (counts double)