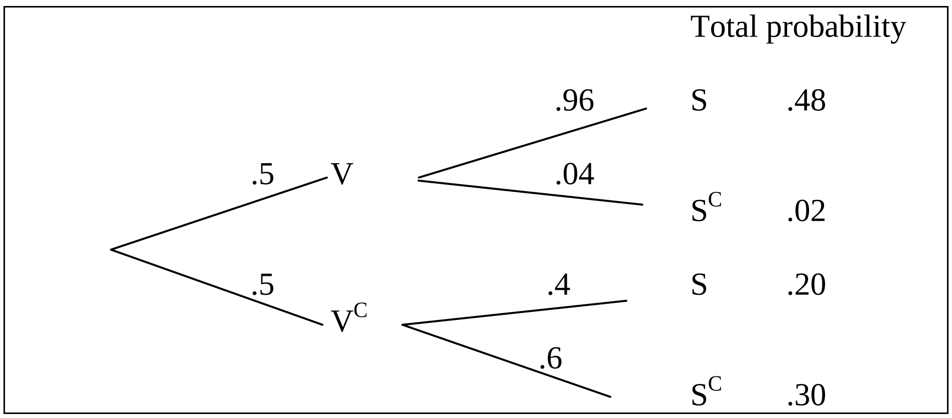
PROBLEM 1: In a study reported in Chapter 3 of the textbook, researchers were able to get information about voting behavior of a sample of registered voters. They knew whether or not these people voted in the November 1986 election. Seven months after the election, they surveyed these people and asked them whether or not they had voted in that election. Of those who actually *had* voted, 96% said that they did and 4% said they did not. Of those who had *not* voted, 40% claimed that they did vote and 60% admitted that they did not. Suppose these people are representative of the population of registered voters, and that in fact 50% of the population actually voted in the election.

Use the following abbreviations for events for a randomly selected person from this population:
V = voted, V^C = didn't vote
S = said they voted, S^C = said they did not vote

Given that someone said they voted, what is the probability that they actually did vote? In other words, find P(V | S)

SOLUTION:

It's easier to solve if you first draw a tree diagram for this situation **or** construct a "hypothetical hundred thousand" table.



	Said they voted	Said they didn't vote	Total
Voted	48,000	2,000	50,000
Did not vote	20,000	30,000	50,000
Total	68,000	32,000	100,000

From the table, it's easy to see that P(V | S) = the proportion of voters in the "Said they voted" column = $\frac{48{,}000}{68{,}000} = .706.$

From the tree diagram, we first need to find P(S). Iit's the sum of the probabilities for branches ending in "S" = .48 + .20 = .68.

Then P(V | S) = $\frac{P(V \text{ and } S)}{P(S)} = \frac{.48}{.68} = .706$

PROBLEM 2: Based on the 2000 Census, 52% (p = .52) of the California population aged 15 years old or older are married. Suppose n = 1000 persons are to be sampled from this population and the sample proportion of married persons ($\hat{p}$) is to be calculated.

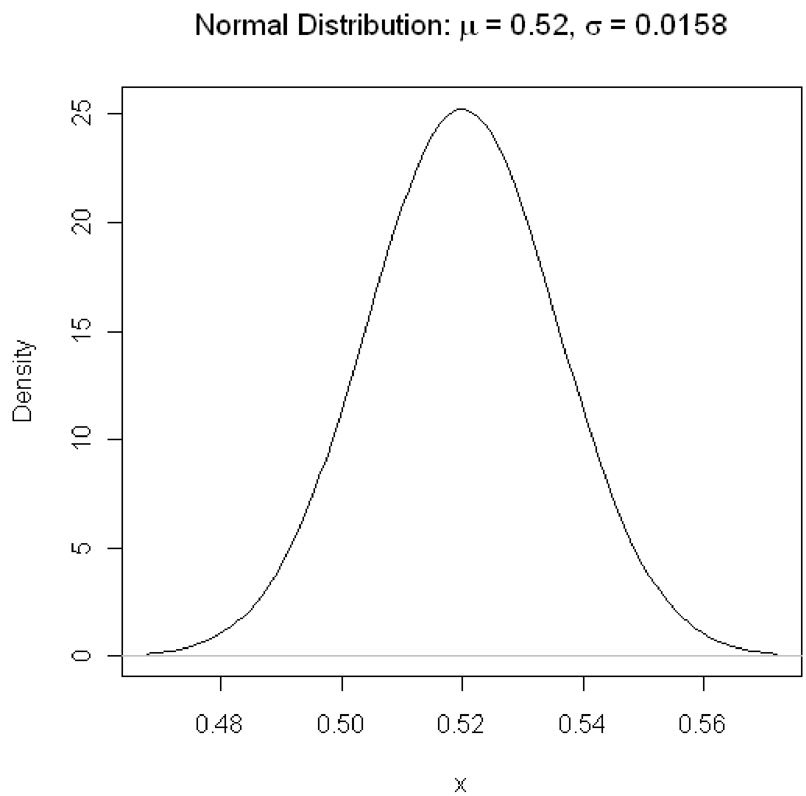
1. What is the mean of the sampling distribution of $\hat{p}$?
2. What is the standard deviation of the sampling distribution of $\hat{p}$?
3. Draw a picture of the sampling distribution of $\hat{p}$. Identify the values that have the middle 68% of the distribution between them.
4. Find the probability that less than 50% of the sample will be married. (If you don't have a table, software or calculator, then leave this in the form of a probability with a z-score.)

SOLUTION:

1. The mean is p = .52.

2. The standard deviation is $\sqrt{\dfrac{(.52)(.48)}{1000}}$ = .0158

3. Here is a picture (made using R Commander). The middle 68% is from .52 – .0158 to .52 + .0158, or .5042 to .5358.



4. The probability that less than 50% will be married is P($\hat{p}$ < .5) = P($z < \dfrac{.5 - .52}{.0158}$) = P(z < –1.266) = .1028 (from R) or about .1020 using the Table with z = –1.27.

PROBLEM 3: The speeds of cars at a certain location on an interstate highway are approximately normal with $\mu = 67$ miles per hour and $\sigma = 6$ miles per hour. When the highway patrol is looking for speed violators, they will stop cars going over 75 miles per hour.

1. What proportion of cars are going over 75 miles per hour?
2. The speed limit is 65 miles per hour. What proportion of cars are going at or under the speed limit?
3. If three cars are randomly selected at different times, what is the probability that they are all going over 67 mph?

SOLUTION

1. What proportion of cars are going over 75 miles per hour?

First, convert the value of 75 to a z-score:

$$z = (75 - 67)/6 = 1.33$$

so $P(X>75mph) = P(Z > 1.33) = P(Z < -1.33) = .0918$

2. The speed limit is 65 miles per hour. What proportion of cars are going at or under the speed limit?

First, convert the value of 65 to a z-score:

$$z = (65 - 67)/6 = -.33$$

so $P(X \leq 65mph) = P(Z \leq -.33) = .3707$

3. If two cars are randomly selected at different times, what is the probability that they are both going over the speed limit of 65 mph?

The speeds are independent, and the probability for each one is $1 - .3707 = .6293$*, so the probability is* $(.6293)^2 = .3960$

3. If three cars are randomly selected at different times, what is the probability that they are all going over 67 mph?

The speeds are independent, and the probability for exceeding 67 mph is 0.5, so the probability is $(0.5)^3 = 0.125$

PROBLEM 4: There are 2000 tickets sold for a raffle. Three winning tickets are chosen and those ticket holders each win $400. Define the random variable X = amount won by purchasing one ticket.

1. Write the pdf for X.
2. Find E(X) = the expected value of X. Show your work.
3. If you buy 2 tickets, what is the probability that you win for *both tickets*?

SOLUTION:

1. Write the pdf for X.

There are two possible values for X. If the ticket is a winning ticket, X=$400, otherwise X=0. There are 3 winning tickets, so P(X = $400) = 3/2000. So the pdf can be written as:

k	*P(X=k)*
$400	3/2000
0	1997/2000

2. Find E(X) = the expected value of X. Show your work.

E(X) = $400(3/2000) + 0 (1997/2000) =$1200/2000 = $.60 or 60 cents.

3. If you buy 2 tickets, what is the probability that you win for *both tickets*?

First, recognize that winning with Ticket 1 and winning with Ticket 2 are not independent events so you can't use Rule 3b. Instead you have to use the more general Rule 3a.
Think of this problem as if there is a deck of 2000 cards, and you are going to draw 2 without replacement. There are 3 winning cards in the deck, and 1997 losing cards. Define events:
A = your first ticket is a winner
B = your second ticket is a winner
P(A and B) = P(A) P(B | A)

P(A) = 3/2000
P(B | A) = 2/1999 because once we know A has happened, there are 1999 tickets left, and 2 of them are winning tickets.
So from Rule 3a, P(A and B) = P(A) P(B | A) = (3/2000) (2/1999) = 6/3998000 = .000015