General Linear Test – Summary

Here are the details for the general linear test of
H_0: Reduced Model is sufficient
H_a: Full Model is needed

	General	Section 2.8 Test for $\beta_1 = 0$	Section 3.7 Lack of fit test c = number of different X values in sample n_j = number of Y values at X_j Y_{ij} = i^{th} value of Y at X_j
Full Model	Varies	$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$	$Y_{ij} = \mu_j + \varepsilon_{ij}$
Reduced Model	Varies	$Y_i = \beta_0 + \varepsilon_i$	$Y_{ij} = \beta_0 + \beta_1 X_j + \varepsilon_{ij}$
Degrees of freedom (full) = df_F	n – # of parameters estimated in full model	n – 2 (2 parameters are estimated)	n – c (c different means estimated)
Degrees of freedom (reduced) = df_R	n – # of parameters estimated in reduced model	n – 1 (1 parameter is estimated)	n – 2 (2 parameters are estimated)
SSE(F)	$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$ where $\hat{Y}_i$ is the predicted value using the full model	Usual SSE for simple linear regression	SSPE = Pure error sum of squares $= \sum_{j=1}^{c} \sum_{i=1}^{n_j}(Y_{ij} - \bar{Y}_j)^2$
SSE(R)	$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$ where $\hat{Y}_i$ is the predicted value using the reduced model	Usual SSTO for simple linear regression	Usual SSE for simple linear regression
Numerator of test statistic F*	$\frac{\left[\text{SSE}(\text{R}) - \text{SSE}(\text{F})\right]}{\left(\text{df}_\text{R} - \text{df}_\text{F}\right)}$	$\frac{\text{Usual SSR}}{1}$	SSLF/(c – 2) where SSLF = Lack of fit sum of squares = Usual SSE – SSPE
Denominator of test statistic F*	$\frac{\text{SSE(F)}}{\text{df}_\text{F}}$	Usual MSE for simple linear regression	$\frac{\text{SSPE}}{n - c}$
Degrees of freedom for F*	$[(df_R - df_F), df_F]$	[1, n – 2]	[c – 2, n – c]