A proposition is a statement that is either true or false.

Examples:  
2 + 3 = 7   F
5 - 2 = 3   T

7 is a prime number   T
Today is Friday   F

It will rain tomorrow  ?
Vanilla is the best flavor of ice cream.

Not propositions:

How are you?

Eat your vegetables.

Logical variables:  \( (p, q, r, \ldots) \)

denote an arbitrary proposition
truth value can be true or false

true:  \( p = T \)  
false:  \( p = F \)  

like variables from algebra  
except value is \( T \) or \( F \),  
instead of a number.
Logical operations can be used to combine propositions to get compound propositions.

Conjunction "AND" Symbol ∧

If \( p \) and \( q \) are propositional variables,

\[ p \land q \] is a proposition.

Truth value of \( p \land q \) depends on

truth values of \( p \) and \( q \).

\[
\begin{array}{ccc}
\text{p} & \text{q} & p \land q \\
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\]

Truth table shows

truth value of \( p \land q \) for every

possible truth value

for \( p \) and \( q \).

\( p \) : Sam is poor

\( q \) : Sam is happy

Different ways to express \( p \land q \) in English:

Sam is poor, but he is happy.
Sam is poor and happy.
Although Sam is poor, he is happy.
Disjunction: \( p \lor t \lor t \lor t \)

<table>
<thead>
<tr>
<th>p</th>
<th>t</th>
<th>( p \lor t \lor t \lor t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“inclusive” or.

Ambiguity in English:

Tonight I will go to the party or I will go to a movie.

The patient has high blood pressure or has a history of migraines.

\[ \neg p \lor q \lor p \lor \neg q \]

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \lor q \lor \neg p \lor \neg q )</th>
<th>exclusive or</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Negation: \( \neg p \).

\( p \): it is raining today
\( \neg p \): it is not raining today
\( \neg \neg p \): it is not true that it is raining today

<table>
<thead>
<tr>
<th>( p )</th>
<th>( \neg p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( F )</td>
</tr>
<tr>
<td>( F )</td>
<td>( T )</td>
</tr>
</tbody>
</table>

Renew:
\[
\begin{align*}
\neg p &= T \quad \text{?} \\
q &= F \quad \text{?} \\
r &= T \quad \text{!}
\end{align*}
\]

\( \rightarrow p \land q \) \( F \)
\( r \land p \) \( T \)
\( q \lor r \) \( T \)
\( p \lor r = F \). \( p \lor r \)
\( T \)
\( \neg q \) \( T \)
Section 1.2

Compound propositions can be built using one or more logical operations:

\[
\begin{align*}
 p \lor \neg r & \quad p = F \\
 F & \quad r = T \\
 F & \quad F \\
\end{align*}
\]

Need to specify the order in which operations are performed:

\[
\begin{align*}
 p \land q \lor r & \\
 (p \land q) \lor r & = T \quad p = F, \quad q = T, \quad r = T \\
 p \land (q \lor r) & = F \\
\end{align*}
\]

Order in which logical operations are applied:

1. \( \neg \)
2. \( \land \)
3. \( \lor \)

\[
\begin{align*}
 \neg q \lor r & \quad \neg (q \lor r) \\
 (p \land q) \lor \neg t & \\
\end{align*}
\]
Can override the default order with parens:

\[ \neg (q \lor r) \]

<table>
<thead>
<tr>
<th>F</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>\hspace{1cm} \neg T</td>
</tr>
</tbody>
</table>

\[ \neg T \Rightarrow F \]

Good to include parens as a reminder:

\[ (p \land q) \lor r \]

\[ \neg p \lor (t \land r) \]

<table>
<thead>
<tr>
<th>F</th>
<th>F</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>\hspace{1cm} \neg T</td>
<td>F</td>
</tr>
</tbody>
</table>

\[ (F \lor F) \Rightarrow F \]

\[ \neg (p \land t \land r) \]

<table>
<thead>
<tr>
<th>F</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>\hspace{1cm} \neg T</td>
<td>\hspace{1cm} T</td>
</tr>
</tbody>
</table>

\[ \neg F \Rightarrow T. \]
A truth table for a compound proposition shows the truth value for every possible combination of truth values for the propositional variables:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \lor Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

If a compound proposition has \( n \) variables, the truth table has \( 2^n \) rows.

2 variables \( \Rightarrow 2^2 = 4 \)
3 variables \( \Rightarrow 2^3 = 8 \)
\[ p: \pi > 3 \]
\[ q: 3 \text{ is a root of the equation } x^2 - 2 = 0. \]
\[ r: \text{The integer } 5 \text{ is even.} \]

\[ \neg r \quad \text{The integer } 5 \text{ is odd.} \]

\[ \neg p \lor q \]
\[ \neg p: \pi \leq 3 \]

\[ \pi \leq 3 \text{ or } 3 \text{ is a root of the eqn. } x^2 - 2 = 0. \]

\[ q \land r \]

\[ \neg (q \land r) \quad \text{It's not the case that } \pi > 3 \text{ and the integer } 5 \text{ is even.} \]
Section 1.3

Conditional operation

$p, q$ propositions  $p \rightarrow q$ false only when $p$ is true and $q$ is false.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

$p \rightarrow q$

$p$ is the hypothesis
$q$ is the conclusion

$p \rightarrow q$

$p$: you study hard
$q$: you will get an A

If you study hard, then you will get an A.

Ways to express in English: $p \rightarrow q$.

if $p$ then $q$.
if $p$, $q$.
$p$ implies $q$.
$q$, if $p$.
$p$ only if $q$.
$p$ is sufficient for $q$.
$q$ is necessary for $p$.

$p$: You have a driver's license.
$q$: You are at least 16 years old.