Final – 250 points 600.488 Computational Geometry May 12, 1998

1. Please define each of the following terms using one or two sentences:

   (a) polygon triangulation

   (b) lower envelope of a set of functions

   (c) Voronoi diagram

2. 3-D Convex Hull

   (a) Draw, as best you can, the view from $(0, 0, -\infty)$ of the convex hull of the following set of points in $\mathbf{R}^3$ (that is, a projection to the xy-plane of all the triangular faces with downward pointing outward normals):

   $$(1, 1, 2)$$
   $$(5, 2, 29)$$
   $$(2, 3, 13)$$
   $$(3, 3, 18)$$
   $$(1, 4, 17)$$
   $$(4, 5, 41)$$

   (b) Please briefly describe the *gift wrapping* method for constructing the convex hull of n points in $\mathbf{R}^3$. Characterize the running time of this method in terms of the input and output sizes for this instance of the problem.

3. Trapezoidal decomposition.

   (a) Draw, as best you can, the trapezoidal decomposition of the following set of line segments (specified by their endpoints):

   $$[(0, 0), (3, 3)]$$
   $$[(1, 2), (4, 1)]$$
   $$[(3, 2), (5, 3)]$$
   $$[(2, 4), (6, 5)]$$
   $$[(-1, 6), (4, 5)]$$

   (b) Describe the winged edge, quad-edge, or doubly-connected edge list data structure.

   (c) Briefly describe an $O((n + k) \log n)$-time method for constructing the trapezoidal decomposition of n line segments in the plane, where k is the number of pairs of intersecting segments.

4. Randomized geometric algorithms.

   (a) What does it mean for an algorithm to be *ranomized*?

   (b) Describe a randomized algorithm for constructing the convex hull of n points in $\mathbf{R}^3$.

   (c) What is the expected running time of this method (you don't need to prove your claim)?

5. Sketch a fast method for constructing the arrangement of n lines in the plane. What is the worst-case running time of this method?

6. Suppose you are given n possibly-intersecting segments in the plane, each of which is viewed as an obstacle. Describe an efficient method for computing the region of the plane visible from a point p with respect to these obstacles. What is the running time of this method?

   NOTE: For the remainder of this exam you may assume that you have a subroutine for any problem we discussed in class, provided you can correctly characterize its performance bounds.

7. Describe an efficient method for computing the area of a simple polygon in the plane. What is the running time of this method?

8. Suppose you are given two sets A and B, containing n points in the plane each. Describe an efficient method for determining if there is a single line L that separates A and B (so that the points of A are entirely on one side of L and the points of B are entirely on the other). What is the running time of this method?

9. Suppose you are given a set S of n points in the plane. Describe an efficient method for finding the largest circle C whose center is inside a given rectangle R such that C contains no points of S in its interior. What is the running time of this method?

10. Given a collection S of n line segments in the plane, describe an efficient method for finding a line L that intersects the maximum number of segments from S. What is the worst-case running time of this method?